Pearson & MINISTRY OF EDUCATION AND TECHNICAL EDUCATION

est EGYPTIAN SCHOLASTIC TEST

EST I MATH

LEVEL UP

2025 EDITION

CLASSIFIED

CONTENTS

Geometry and Measurement

Data Analysis, Statistics, and Probability

Trigonometry

Others

Algebra

And

Part of Functions

1. The acceleration of a motorcycle is a function involving time elapsed. If T represents the time lapsed and $A(T) = 15T^2 + 120T + 86$ represents the acceleration of the motorcycle, what is the acceleration of the motorcycle 10 seconds after the motorcycle started to accelerate?

Solution: Answer: 2786

$$A(10) = 15\cdot(10)^2 + 120\cdot 10 + 86$$
$$\Downarrow$$
$$A(10) = 2786$$

2. The acceleration of a motorcycle is a function involving time elapsed. If T represents the time lapsed and $A(T) = 15T^2 + 120T + 86$ represents the acceleration of the motorcycle, what is the time when the acceleration is 386?

Solution: Answer: 2

$$386 = 15T^2 + 120T + 86$$
$$\downarrow$$
$$T^2 + 8T - 20 = 0,\ (T+10)(T-2) = 0$$
$$\Downarrow$$
$$T = 2, -10$$

3. A toy seller's profit, in dollars, from the sale of t toys is given by $P(t) = 3t + 80$. How many toys must he sell in order to make a profit of $ 590?

Solution: Answer: 170

$$590 = 3t + 80$$
$$\downarrow$$
$$3t = 510$$
$$\Downarrow$$
$$t = 170$$

4. If a company charge d dollars for their product, where $0 \le d \le 100$, the income from the product will be $I(d) = 1000d - 10d^2$ dollars per day. By this model, for which of the following values of d would the company's daily income for the product be the maximum?

(A) -50
(B) 30
(C) 50
(D) -30

Solution: Answer: (C)

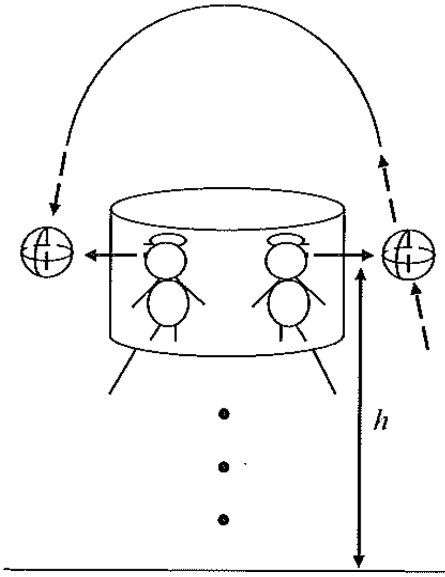
This function is a quadratic function.
$$\downarrow$$
Its graph is a parabola.

The graph opens down because $a = -10 < 0$.

When $d = -\dfrac{b}{2a}$, $I(d)$ = maximum value.

$$b = 1000,\ a = -10$$
$$\Downarrow$$
$$d = -\frac{1000}{2\cdot(-10)} = 50$$

The following information is for questions 5 and 6.



In the figure above, a projectile is launched upward from ground level with an initial speed of 800 feet per second. In the top of a building, two people, person 1 and person 2, at a height h of 9,600 feet see the projectile pass on the way up and the way down, respectively.

The formula of the relationship of initial speed s, time t, and height h is

$$h = st - 16t^2,$$

where the height is measured in feet.

5. How long will it be when the projectile travels from person1 to person 2?

(A) 12
(B) 15
(C) 30
(D) 10

Solution: Answer: (D)

$$9600 = 800t - 16t^2,\ 600 = 50t - t^2$$
$$0 = t^2 - 50t + 600$$
$$\downarrow$$
$$(t-20)(t-30) = 0,\ t_1 = 20,\ t_2 = 30$$
$$\Downarrow$$
$$x = t_2 - t_1 = 30 - 20 = 10$$

6. When will the projectile return to the ground?

(A) 50
(B) 35
(C) 25
(D) 40

Solution: Answer: (A)

$$0 = 800t - 16t^2$$
$$\downarrow$$
$$0 = 50t - t^2,\ 0 = t(50 - t)$$
$$\Downarrow$$
$$t_1 = 0,\ t_2 = \boxed{50}$$

Functions as Models

1. The acceleration of a motorcycle is a function involving time elapsed. If T represents the time lapsed and $A(T) = 15T^2 + 120T + 86$ represents the acceleration of the motorcycle, what is the acceleration of the motorcycle 10 seconds after the motorcycle started to accelerate?

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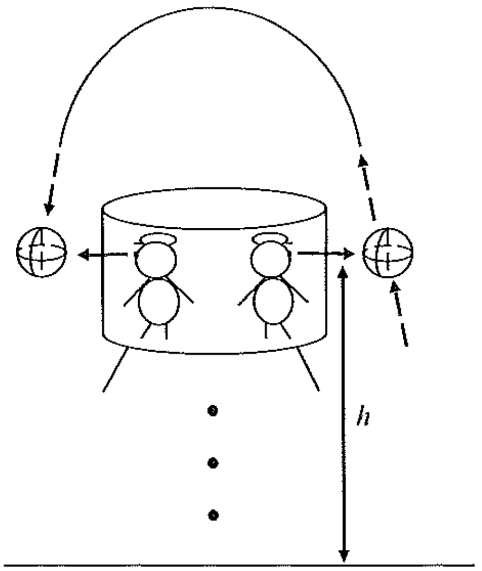

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(B) 15
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(D) 10

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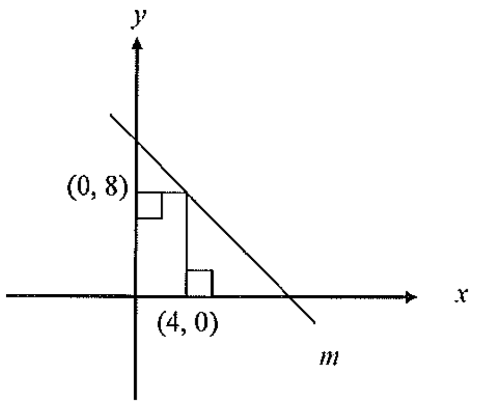
(B) 35
(C) 25
(D) 40

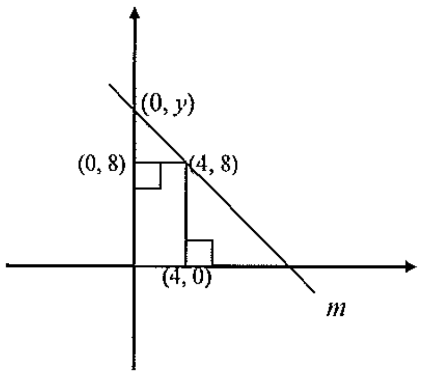


Note: Figure above not drawn to scale.

1. In the figure above, if line m has a slope -2, what is the y-intercept of m?

(A) 10
(B) 16
(C) $\frac{21}{4}$
(D) 8

Solution: Answer: (B)



In the figure above, by points (0, 8) and (4, 0), you can easily obtain the point (4, 8) on line m. Then use slope formula to obtain the y-intercept.

$$m = \frac{y_2 - y_1}{x_1 - x_2}$$

$$\downarrow$$

$$m = \frac{y-8}{0-4} = -2$$

$$\Downarrow$$

$$y - 8 = 8,\ y = 16$$

2. If the graphs of the equations $y + mx = 8$ and $y = 3mx + 10$ are parallel, where $m \geq 0$, which is the value of m?

(A) $\sqrt{3}$
(B) $\pm\frac{1}{3}$
(C) 0
(D) $\frac{\sqrt{3}}{3}$

Solution: Answer: (C)

$$\begin{cases} y - mx = 8 \\ y = 3mx + 10 \end{cases}$$

$$\downarrow$$

$$\begin{cases} y = -mx + 8 \\ y = 3mx + 10 \end{cases}$$

Their graphs are parallel.

$$\downarrow$$

$$s_1 = s_2$$

$$\Downarrow$$

$$-m = 3m,\ m = 0$$

Note: Some books use m to represent the slope, but m as letter itself is not a slope. In form $y = sx + b$, the portion of the front of x is the slope. Now $-m$ or $3m$ is a slope.

3. If the graphs of the equations $y + (m-1)x = 5$ and $y = 3mx + 10$ are parallel, where $m > 0$, which is the value of m?

(A) $\sqrt{3}$
(B) $\pm\frac{1}{3}$
(C) $\frac{1}{4}$
(D) $\frac{\sqrt{3}}{3}$

Solution: Answer: (C)

$$\begin{cases} y + (m-1)x = 5 \\ y = 3mx + 10 \end{cases}$$

$$\downarrow$$

$$\begin{cases} y = -(m-1)x + 5 \\ y = 3mx + 10 \end{cases}$$

Their graphs are parallel.

$$\downarrow$$

$$s_1 = s_2$$

$$\downarrow$$

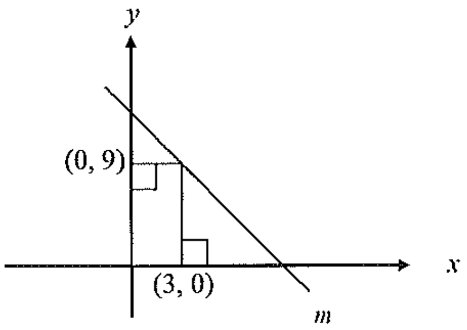
$$-(m-1) = 3m,\ -m + 1 = 3m$$

$$\Downarrow$$

$$m = \frac{1}{4}$$

Note: Some books use m to represent the slope, but m as letter itself is not a slope. In form $y = sx + b$, the portion of the front of x is the slope. Now $-(m-1)$ or $3m$ is a slope.

Linear Functions and Their Graphs

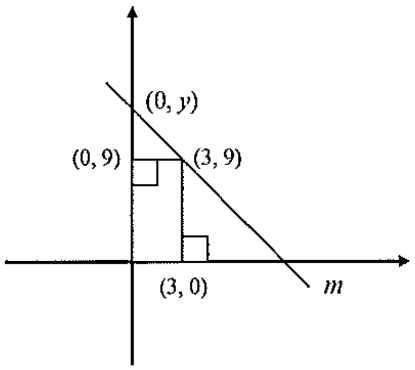


Note: Figure above not drawn to scale.

4. In the figure above, if line m has a slope –2, what is the y-intercept of m?

(A) 10
(B) 15
(C) $\frac{21}{4}$
(D) 8

Solution: Answer: (B)



In the figure above, by points (0, 9) and (3, 0), you can easily obtain point (3, 9) on line m. Then use slope formula to obtain the y-intercept.

$$\text{Slope} = \frac{y_2 - y_1}{x_1 - x_2} = \frac{y-9}{0-3} = -2$$

$$\downarrow$$

$$y - 9 = -2(-3), \quad y - 9 = 6$$

$$\Downarrow$$

$$y = 15$$

5. In the xy-plane, points (2, 1) and (4, 3) define a line, and points (-2, -3) and (-4, -2) define another line. At which of the following (x, y) points do the two lines intersect?

(A) (2, 3)
(B) $(-\frac{4}{3}, -\frac{5}{3})$
(C) (-2, -3)
(D) (3, 2)

Solution: Answer: (C)

$$y - y_0 = s(x - x_0)$$

where s = slope $= \frac{y_2 - y_1}{x_2 - x_1}$.

(1) $y - 1 = \frac{3-1}{4-2}(x-2)$

$$y = x - 1$$

(2) $y + 3 = \frac{-3+2}{-2+4}(x+2)$

$$y = -\frac{1}{2}x - 4$$

At an intersection point,

$$y = y$$

$$x - 1 = -\frac{1}{2}x - 4, \; x = -2$$

Only (C) has $x = -2$.

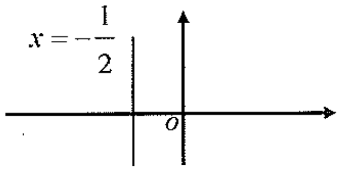
$$\Downarrow$$

Answer is (C).

6. In the xy-plane, find the slope of the line described by the equation $2x = -1$.

(A) Undefined
(B) 1/2
(C) 0
(D) -2

Solution: Answer: (A)

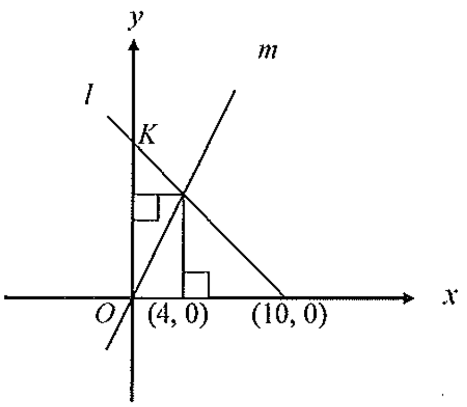


$$x = -\frac{1}{2}$$

$$\text{slope} = \frac{y_2 - y_1}{-\frac{1}{2} - (-\frac{1}{2})} = \frac{y_2 - y_1}{0}$$

$$\Downarrow$$

undefined

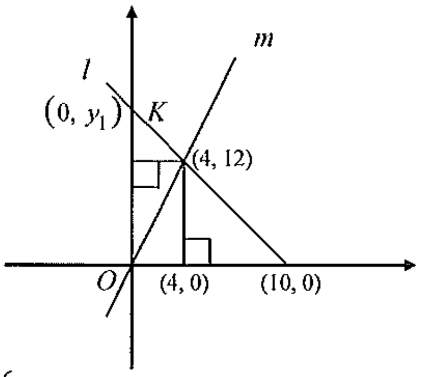


Note: Figure above not drawn to scale.

7. In the figure above, if line m has a slope 3, what are the coordinates of K?

(A) (0, 20)
(B) (4, 20)
(C) (0, 16)
(D) (4, 16)

Solution: Answer: (A)



Line m passes through origin (0, 0).
The direction of line m

$\downarrow$

$$\text{slope} = +\frac{y_0}{x_0} = 3,\ \frac{y_0}{4} = 3,\ y_0 = 12$$

$\downarrow$

The intersection point = (4, 12)
The 2 points (4, 12) and (10, 0)

$\downarrow$

$$\text{slolpe} = \frac{12-0}{4-10} = -2$$

$\downarrow$

The 2 points (4, 12) and (10, 0)

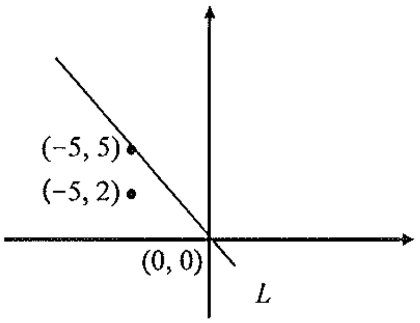
$\Downarrow$

$$\frac{y-12}{0-4} = -2,\ y = 20,\ \boxed{k = (0,\ 20)}$$

8. Line L passes through the points (0, 0) and (-5, 5). Which of the following points is located in the region between the graph of the line L and negative part of the x-axis?

(A) $x > y$ and $y < 0$
(B) $x = y$ and $y < 0$
(C) $x \geq y$ and $y < 0$
(D) $x < y$ and $y > 0$

Solution: Answer (D)



Choose one point with the same x-value, $x = -5$, between the graph of the line L and negative part of the x-axis.

Let $x = -5$ and $y = 2$. $\rightarrow$ $-5 < 2$ and $2 > 0$.

$\Downarrow$

$x < y$ and $y > 0$

9. The graph of which of the following points is below the graph of the line $y = -\frac{3}{4}x + 3$?

(A) (1, 4)
(B) (1, 3)
(C) (1, 2)
(D) (1, 5)

Solution: Answer (C)

Each value of x in choices is 1.

$\downarrow$

$$\text{When } x = 1,\ y = -\frac{3}{4}(1) + 3 = 2\frac{1}{4}.$$

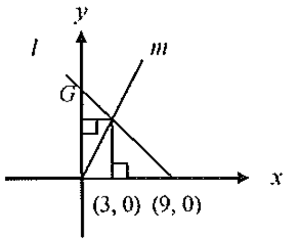
$\downarrow$

Use plug-in method.

$$2 < 2\frac{1}{4}$$

$\Downarrow$

Answer = (C)

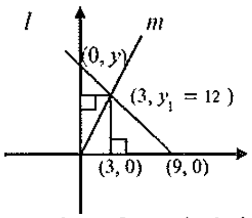


Note: Figure above not drawn to scale.

10. In the figure above, if line m has a slope 4, what are the coordinates of G?

(A) (0, 20)
(B) (4, 20)
(C) (0, 18)
(D) (4, 16)

Solution: Answer: (C)



Use slope formula 3 times.

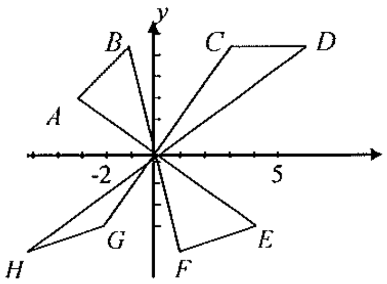
$$\frac{y_1}{3} = 4, \quad y_1 = 12$$

$$\frac{y_2 - y_1}{x_1 - x_2} = \frac{12 - 0}{3 - 9} = -2$$

$$\frac{y - 12}{0 - 3} = -2$$

$$\Downarrow$$

$$\boxed{y = 18} \text{ (when } x = 0)$$

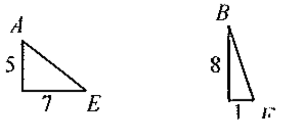


11. In the xy-coordinate system above, which of the following line segments has a slope less than -4?

(A) $\overline{HG}$
(B) $\overline{CG}$
(C) $\overline{AE}$
(D) $\overline{BF}$

Solution: Answer: (D)

Only segments $\overline{AE}$ and $\overline{BF}$ have negative slope and the slope of $\overline{BF} = \dfrac{\text{Change of } y\text{-value}}{\text{Change of } x\text{-value}} < -3$



$$\Downarrow$$

Answer is (D).

12. If the graph of a linear function passes through the points $(m, 2)$, $(n, 3)$ and $(5, 6)$, what is the value of $m - \frac{4}{3}n$?

(A) 12
(B) $-\frac{5}{3}$
(C) 4
(D)-3

Solution: Answer: (B)

A slope of a line is unique.

$$\downarrow$$

$$\text{Slope} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_3}{x_2 - x_3}$$

$$\downarrow$$

$$\frac{2 - 3}{m - n} = \frac{3 - 6}{n - 5}, \quad \frac{m - n}{-1} = \frac{n - 5}{-3}$$

$$\downarrow$$

$$3(m - n) = n - 5, \; 3m - 3n = n - 5$$

$$\downarrow$$

$$3m - 4n = -5$$

$$\Downarrow$$

$$m - \frac{4}{3}n = -\frac{5}{3}$$

13. If the graph of a linear function passes through the points $(p, 1)$, $(q, 5)$ and $(3, 2)$, what is the value of $3p + q$?

(A) 12
(B) 5
(C) 4
(D)-3

Solution: Answer: (A)

A slope of a line is unique.

$$\downarrow$$

$$\text{Slope} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_3}{x_2 - x_3}$$

$$\downarrow$$

$$\frac{1 - 5}{p - q} = \frac{5 - 2}{q - 3}, \quad \frac{p - q}{-4} = \frac{q - 3}{3}$$

$$\downarrow$$

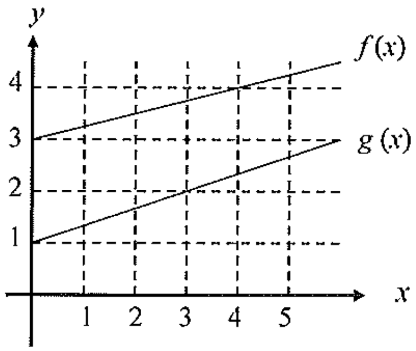
$$3(p - q) = -4(q - 2)$$

$$\downarrow$$

$$3p - 3q = -4(q - 3) = -4q + 12$$

$$\Downarrow$$

$$3p + q = 12$$



14. The graph of the functions f and g are lines, as shown above. What is the value of $f(4)-g(3)$?

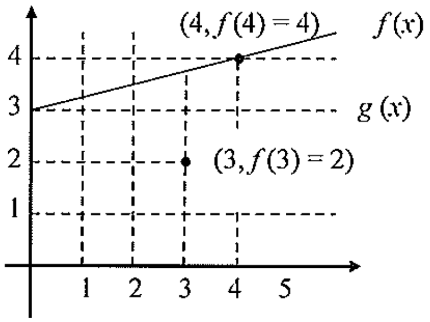
(A) 2.6

(B) 4.3

(C) 3

(D) 2

Solution: Answer: (D)



$\downarrow$

Refer to the figure above.

$\Downarrow$

$f(4)-g(3)=4-2=2$

15. If the lines with the equations $y=s_1x-5$ and $y=s_2x+3$ intersect in quardrants II or III, which of the following must be the relationship between s_1 and s_2 ?

(A) $s_1 \le s_2$

(B) $s_1 \ge s_2$

(C) $s_1 > s_2$

(D) $s_1 < s_2$

Solution: Answer: (D)

$y=s_1x-5$ and $y=s_2x+3$

intersect

$\downarrow$

$s_1x-5=s_2x+3$

$s_1x-s_2x=8$

$x(s_1-s_2)=8,\ s_1-s_2=\dfrac{8}{x}$

intersection point at the left of y-axis

$\downarrow$

$x<0$

$\Downarrow$

$\dfrac{8}{x}<0,\ s_1-s_2<0,\ \boxed{s_1<s_2}$

16. If the function g has the property: $g(k_1)=g(k_2)$ for all numbers k_1 and k_2, what is the graph of g in the xy-coordinate plane?

(A) A vertical line

(B) A line with slope $s=k_1k_2$

(C) a circle

(D) a horizontal line

Solution: Answer: (D)

$g(k_1)=g(k_2)$

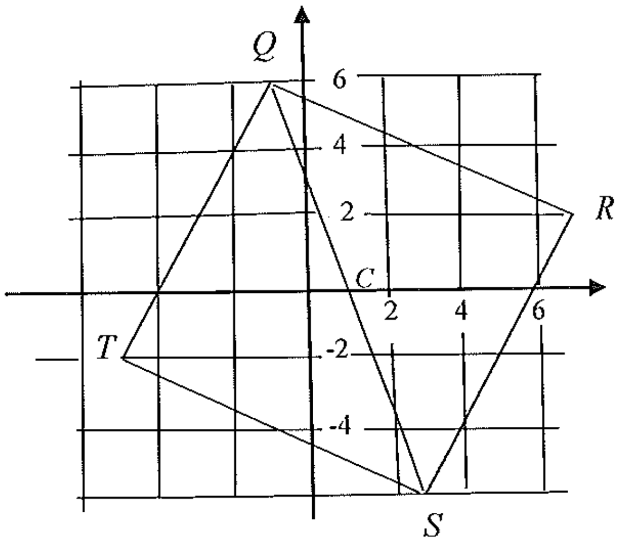
$\downarrow$

slope = 0

The slope of a horizontal line = 0

$\Downarrow$

A horizontal line



17. In the figure above, $QRST$ is a square and point C is the center of the square. The coordinates of points Q and S are (-1, 6) and (3, -6), respectively. Which of the following is an equation of the segment from points T to R ?

(A) $y = \frac{1}{3}x - \frac{1}{3}$

(B) $y = \frac{1}{3}x + \frac{1}{3}$

(C) $y = 3x - 3$

(D) $y = 3x + 3$

Solution: Answer: (A)

$$\text{Slope}_{QS} = \frac{6-(-6)}{-1-3} = -3$$

Diagonals of a square are perpendicular each other.

$\downarrow$

Slope of segment $TR = 1/3$

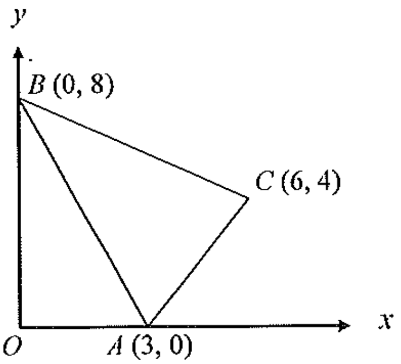
$$y = \frac{1}{3}x + b$$

Point C (1, 0) is on $\overline{TR}$.

$$0 = \frac{1}{3}\cdot 1 + b, \quad b = -\frac{1}{3}$$

$\Downarrow$

$$y = \frac{1}{3}x - \frac{1}{3}$$



18. If an endpoint of a segment is a vertex of a triangle and another endpoint of this segment is the midpoint of the opposite side of the vertex, the segment is called median of the triangle. The point of intersection of the medians of a triangle is called centroid of the triangle.

According to the figure above, what are the coordinates of the centroid of the triangle ABC ?

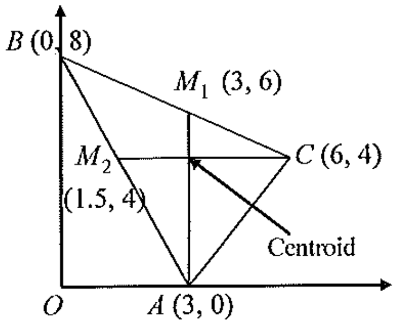
(A) (3, 4) (B) (4, 3) (C) (5, 3) (D) (3, 5)

Solution: Answer: (A)

$$M_1 = \left(\frac{0+6}{2}, \frac{8+4}{2}\right) = (3, 6)$$

$$M_2 = \left(\frac{0+3}{2}, \frac{8+0}{2}\right) = (1.5, 4)$$

$\downarrow$

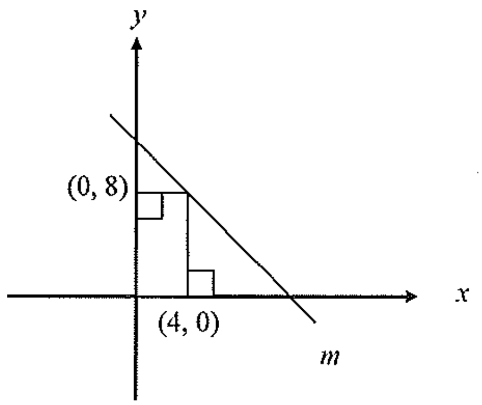


Equation of $\overline{M_1A}$ is $x = 3$. Equation of $\overline{M_2C}$ is $y = 4$.

The centroid is intersection of the two segments.

$\Downarrow$

Coordinates of the centroid = (3, 4)



Note: Figure above not drawn to scale.

1. In the figure above, if line m has a slope -2, what is the y-intercept of m?

(A) 10
(B) 16
(C) $\frac{21}{4}$
(D) 8

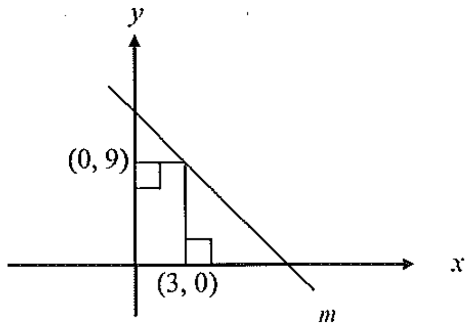
2. If the graphs of the equations $y + mx = 8$ and $y = 3mx + 10$ are parallel, where $m \geq 0$, which is the value of m?

(A) $\sqrt{3}$
(B) $\pm\frac{1}{3}$
(C) 0
(D) $\frac{\sqrt{3}}{3}$

3. If the graphs of the equations $y + (m-1)x = 5$ and $y = 3mx + 10$ are parallel, where $m > 0$, which is the value of m?

(A) $\sqrt{3}$
(B) $\pm\frac{1}{3}$
(C) $\frac{1}{4}$
(D) $\frac{\sqrt{3}}{3}$

Linear Functions and Their Graphs



Note: Figure above not drawn to scale.

4. In the figure above, if line m has a slope –2, what is the y-intercept of m?

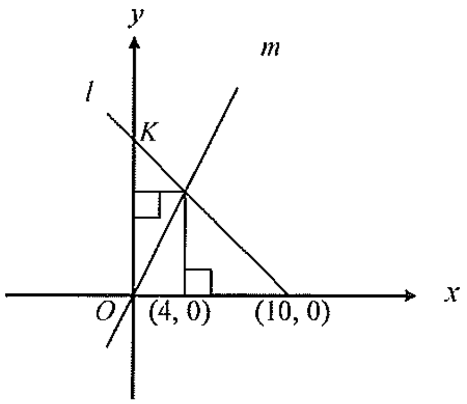
(A) 10
(B) 15
(C) $\frac{21}{4}$
(D) 8

5. In the xy-plane, points (2, 1) and (4, 3) define a line, and points (-2, -3) and (-4, -2) define another line. At which of the following (x, y) points do the two lines intersect?

(A) (2, 3)
(B) $(-\frac{4}{3}, -\frac{5}{3})$
(C) (-2, -3)
(D) (3, 2)

6. In the xy-plane, find the slope of the line described by the equation $2x = -1$.

(A) Undefined
(B) 1/2
(C) 0
(D) -2



Note: Figure above not drawn to scale.

7. In the figure above, if line m has a slope 3, what are the coordinates of K?

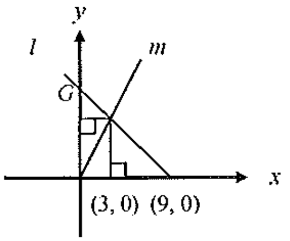
(A) (0, 20)
(B) (4, 20)
(C) (0, 16)
(D) (4, 16)

8. Line L passes through the points (0, 0) and (-5, 5). Which of the following points is located in the region between the graph of the line L and negative part of the x-axis?

(A) $x > y$ and $y < 0$
(B) $x = y$ and $y < 0$
(C) $x \geq y$ and $y < 0$
(D) $x < y$ and $y > 0$

9. The graph of which of the following points is below the graph of the line $y = -\frac{3}{4}x + 3$?

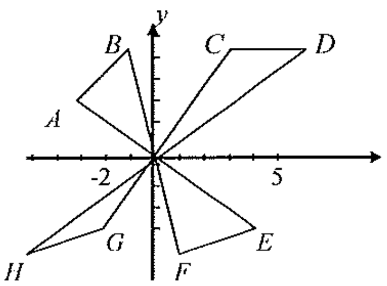
(A) (1, 4)
(B) (1, 3)
(C) (1, 2)
(D) (1, 5)



Note: Figure above not drawn to scale.

10. In the figure above, if line m has a slope 4, what are the coordinates of G?

(A) (0, 20)
(B) (4, 20)
(C) (0, 18)
(D) (4, 16)



11. In the xy-coordinate system above, which of the following line segments has a slope less than -4?

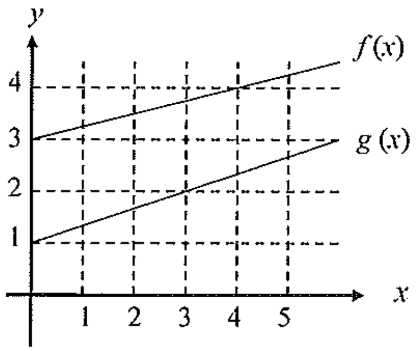
(A) $\overline{HG}$
(B) $\overline{CG}$
(C) $\overline{AE}$
(D) $\overline{BF}$

12. If the graph of a linear function passes through the points $(m, 2)$, $(n, 3)$ and $(5, 6)$, what is the value of $m - \frac{4}{3}n$?

(A) 12
(B) $-\frac{5}{3}$
(C) 4
(D)-3

13. If the graph of a linear function passes through the points $(p, 1)$, $(q, 5)$ and $(3, 2)$, what is the value of $3p + q$?

(A) 12
(B) 5
(C) 4
(D)-3



14. The graph of the functions f and g are lines, as shown above. What is the value of $f(4) - g(3)$?

(A) 2.6

(B) 4.3

(C) 3

(D) 2

15. If the lines with the equations $y = s_1x - 5$ and $y = s_2x + 3$ intersect in quardrants II or III, which of the following must be the relationship between s_1 and s_2 ?

(A) $s_1 \leq s_2$

(B) $s_1 \geq s_2$

(C) $s_1 > s_2$

(D) $s_1 < s_2$

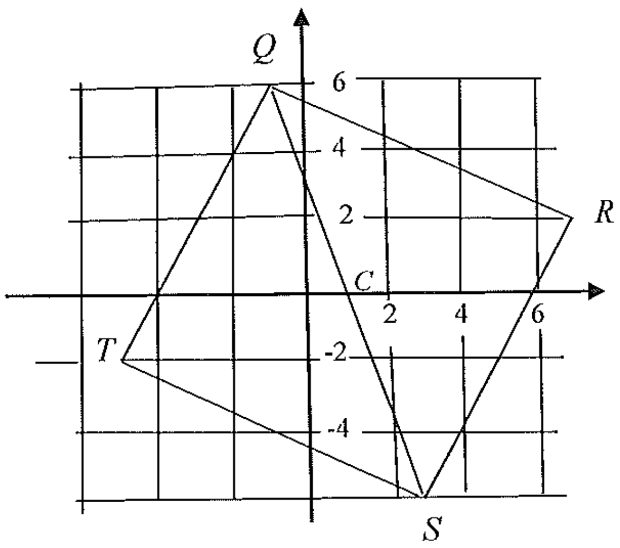
16. If the function g has the property: $g(k_1) = g(k_2)$ for all numbers k_1 and k_2, what is the graph of g in the xy-coordinate plane?

(A) A vertical line

(B) A line with slope $s = k_1k_2$

(C) a circle

(D) a horizontal line



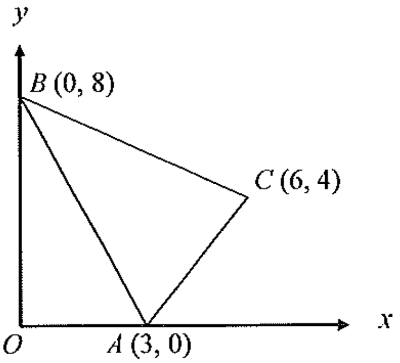
17. In the figure above, $QRST$ is a square and point C is the center of the square. The coordinates of points Q and S are (-1, 6) and (3, -6), respectively. Which of the following is an equation of the segment from points T to R ?

(A) $y = \frac{1}{3}x - \frac{1}{3}$

(B) $y = \frac{1}{3}x + \frac{1}{3}$

(C) $y = 3x - 3$

(D) $y = 3x + 3$



18. If an endpoint of a segment is a vertex of a triangle and another endpoint of this segment is the midpoint of the opposite side of the vertex, the segment is called median of the triangle. The point of intersection of the medians of a triangle is called centroid of the triangle.

According to the figure above, what are the coordinates of the centroid of the triangle ABC ?

(A) (3, 4) (B) (4, 3) (C) (5, 3) (D) (3, 5)

$g(x)$	x
4	2
20	6
n	4

1. The table above shows some values for the function g. If g is a linear function, what is the value of n?

(A) 20
(B) 8
(C) 10
(D) 12

Solution: Answer: (D)

One line only has a unique slope determined by any pair of two points on the line.

$$s = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_3 - y_2}{x_3 - x_2}$$

$$\downarrow$$

$$\frac{n-20}{4-6} = \frac{20-4}{6-2}$$

$$\Downarrow$$

$$n - 20 = -8, \quad n = 12$$

x	$g(x)$
4	2
20	6
m	4

2. The table above shows some values for the function g. If g is a linear function, what is the value of m?

(A) 20
(B) 8
(C) 10
(D) 12

Solution: Answer: (D)

One line only has a unique slope determined by any pair of two points on the line.

One line has a unique slope.

$$\text{Slope} = \frac{y_1 - y_2}{x_1 - x_2}$$

All points on the same line

$$\downarrow$$

$$\text{slope}_1 = \text{slope}_2$$

$$\Downarrow$$

$$\frac{4-6}{m-20} = \frac{2-6}{4-20}, \quad m = \boxed{12}$$

$g(x)$	x
m	2
10	4
n	6

3. The table above shows some values for the function g. If g is a linear function, what is the value of $m + n$?

(A) 20
(B) 8
(C) 10
(D) 12

Solution: Answer: (A)

One line only has a unique slope determined by any pair of two points on the line.

$$s = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_3 - y_2}{x_3 - x_2}$$

$$\downarrow$$

$$\frac{m-10}{2-4} = \frac{n-10}{6-4}$$

$$\downarrow$$

$$2(m-10) = -2(n-10)$$

$$\downarrow$$

$$m - 10 = -n + 10$$

$$\Downarrow$$

$$m + n = 20$$

$g(x)$	x
2	0
3	1
6	2
11	3

4. The table above provides the values of the quadratic function g for chosen values of x. Which of the following is g?

(A) $g(x) = x^2 + 3$
(B) $g(x) = x^2 + 2$
(C) $g(x) = x^2 - 2$
(D) $g(x) = x^2 - 1$

Solution: Answer: (B)

This table function is not a linear function. This is a mention that a function table is not always for a linear function.

Use plug-in method.

When $x = 0$, 1, 2, and 3,

$$\Downarrow$$

(B) $g(x) = x^2 + 2 = 0$, 3, 6, and 11.

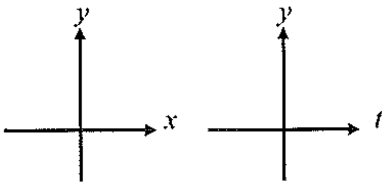
$f(x)$	x
4	3
p	5
7	6

$g(t)$	t
2	3
q	6
7	8

5. The 2 tables above show some values for the functions f and g. f and g are linear functions. What is the value of $p+q$?

(A) 9
(B) 10
(C) 11
(D) 12

Solution: Answer: (C)



Slope of any line is unique.

$$\downarrow$$

$$\text{slope} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_3 - y_2}{x_3 - x_2}$$

$$\downarrow$$

$$\frac{p-4}{5-3} = \frac{7-4}{6-3} \qquad \frac{q-2}{6-3} = \frac{7-2}{8-3}$$

$$\downarrow \qquad \downarrow$$

$$3(p-4) = 2 \cdot 3 \qquad 5(q-2) = 3 \cdot 5$$

$$\downarrow \qquad \downarrow$$

$$3p = 6 + 12 \qquad 5q = 15 - 10$$

$$\downarrow \qquad \downarrow$$

$$p = 6 \qquad q = 5$$

$$\Downarrow$$

$$p + q = 6 + 5 = 11$$

$g(x)$	x
4	2
20	6
n	4

1. The table above shows some values for the function g. If g is a linear function, what is the value of n?

(A) 20
(B) 8
(C) 10
(D) 12

x	$g(x)$
4	2
20	6
m	4

2. The table above shows some values for the function g. If g is a linear function, what is the value of m?

(A) 20
(B) 8
(C) 10
(D) 12

$g(x)$	x
m	2
10	4
n	6

3. The table above shows some values for the function g. If g is a linear function, what is the value of $m + n$?

(A) 20
(B) 8
(C) 10
(D) 12

$g(x)$	x
2	0
3	1
6	2
11	3

4. The table above provides the values of the quadratic function g for chosen values of x. Which of the following is g?

(A) $g(x) = x^2 + 3$

(B) $g(x) = x^2 + 2$

(C) $g(x) = x^2 - 2$

(D) $g(x) = x^2 - 1$

Linear Functions and Their Tables

$f(x)$	x
4	3
p	5
7	6

$g(t)$	t
2	3
q	6
7	8

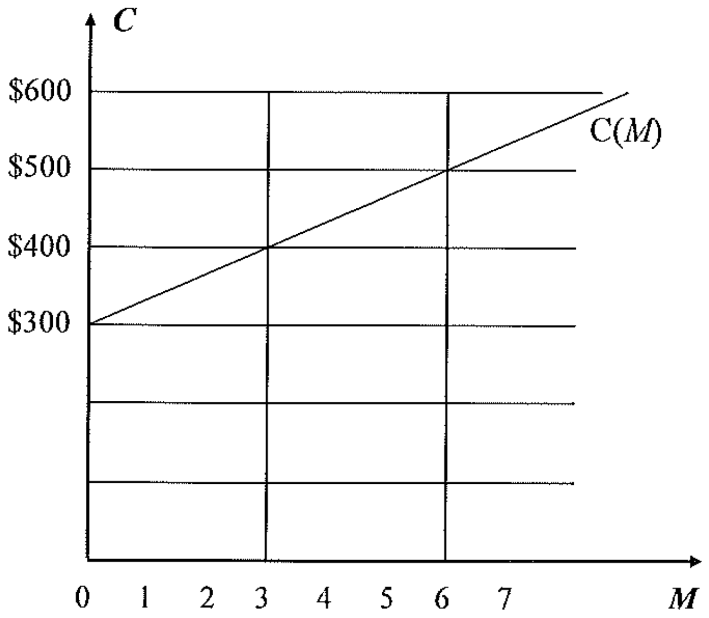
5. The 2 tables above show some values for the functions f and g. f and g are linear functions. What is the value of $p+q$?

(A) 9

(B) 10

(C) 11

(D) 12



1. The graph of the function $C(M)$ is given in the figure above. The cost of producing M units of a specific medicine is given by the function C as $C(M) = sM + b$, where s and b are positive constants. If M units are produced, which of following functions, g, represents the cost per unit, in dollars?

(A) $g(M) = 300 + 300M$

(B) $g(M) = \frac{M}{300} + 200$

(C) $g(M) = \frac{200}{3} + \frac{300}{M}$

(D) $g(M) = \frac{100}{3} + \frac{300}{M}$

Solution: Answer: (D)

You may use the slope-intercept form of linear function.

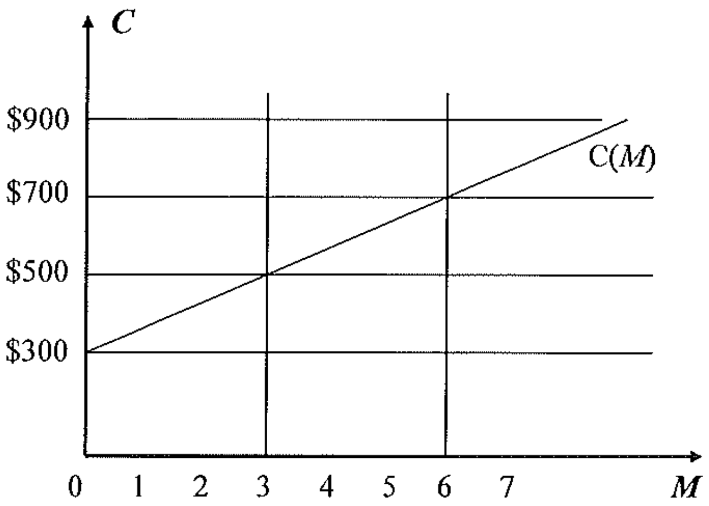
$$\left.\begin{array}{l}\text{Slope} = \frac{100}{3} \\ y\text{-intercept} = b = 300\end{array}\right\} \rightarrow C(M) = \frac{100}{3}M + 300$$

The average cost per unit for M units:

$$g(M) = \frac{C(M)}{M} \rightarrow \frac{\frac{100}{3}M + 300}{M}$$

$$\Downarrow$$

$$g(M) = \frac{100}{3} + \frac{300}{M}$$



2. The graph of the function $C(M)$ is given in the figure above. The cost of producing M units of a specific medicine is given by the function C as $C(M) = sM + b$, where s and b are positive constants. If M units are produced, which of following functions, j, represents the cost per unit, in dollars?

(A) $j(M) = \frac{M}{300} + 200$

(B) $j(M) = 300 + 300M$

(C) $j(M) = \frac{200}{3} + \frac{300}{M}$

(D) $j(M) = \frac{300}{2} + \frac{300}{M}$

Solution: Answer: (C)

You may use the slope-intercept form of linear function.

$$\left.\begin{array}{l}\text{Slope} = \frac{200}{3} \\ y\text{-intercept} = b = 300\end{array}\right\} \rightarrow C(M) = \frac{200}{3}M + 300$$

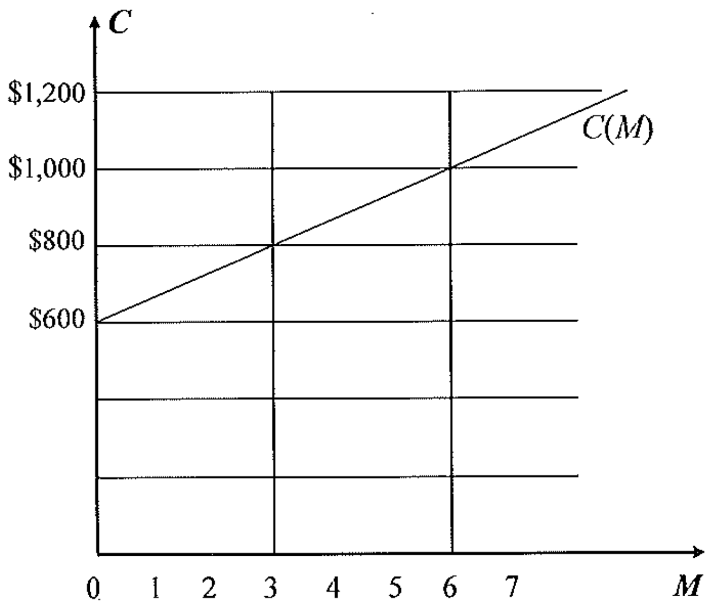
The average cost per unit for M units:

$$j(M) = \frac{C(M)}{M} \rightarrow \frac{\frac{200}{3}M + 300}{M}$$

$$\Downarrow$$

$$j(M) = \frac{200}{3} + \frac{300}{M}$$

Linear Functions Representing Cost per Unit



3. The graph of the function $C(M)$ is given above. The cost of producing M units of a specific medicine is given by the function C as $C(M) = sM + b$, where s and b are positive constants. If M units are produced, which of following functions, g, represents the cost per unit, in dollars?

(A) $g(M) = 800 + 300M$

(B) $g(M) = \frac{200}{3} + \frac{600}{M}$

(C) $g(M) = \frac{600}{M} - \frac{200}{3}$

(D) $g(M) = \frac{200}{3} + \frac{800}{M}$

Solution: Answer: (B)

You may use the slope-intercept form of linear function.

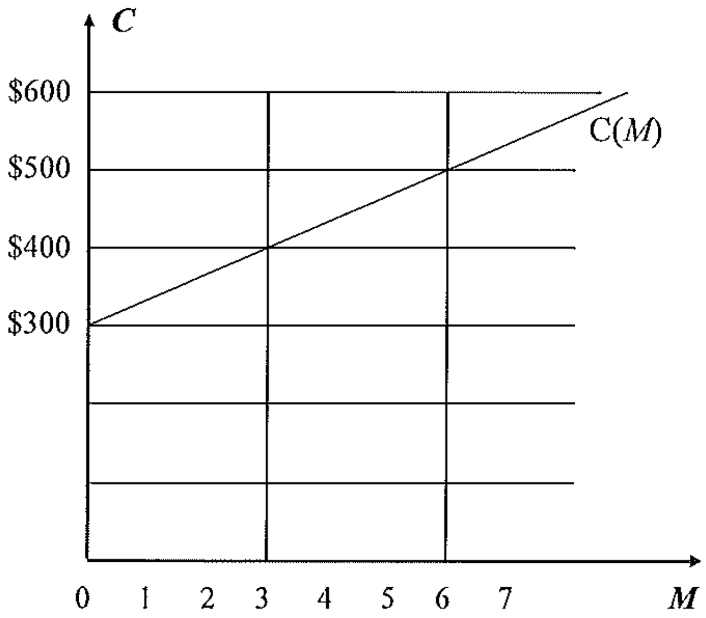
$$\left.\begin{array}{l}\text{Slope} = \frac{200}{3} \\ y\text{-intercept} = b = 600\end{array}\right\} \rightarrow C(M) = \frac{200}{3}M + 600$$

The average cost per unit for M units:

$$g(M) = \frac{C(M)}{M} \rightarrow \frac{\frac{200}{3}M + 600}{M}$$

$$\Downarrow$$

$$g(M) = \frac{200}{3} + \frac{600}{M}$$



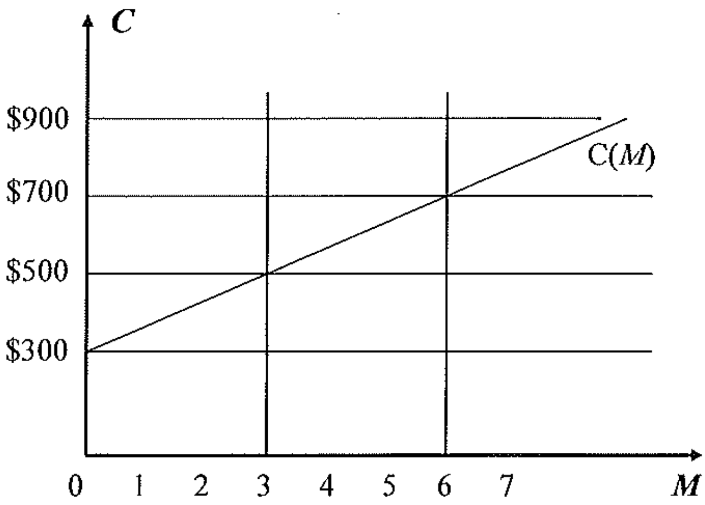
1. The graph of the function $C(M)$ is given in the figure above. The cost of producing M units of a specific medicine is given by the function C as $C(M) = sM + b$, where s and b are positive constants. If M units are produced, which of following functions, g, represents the cost per unit, in dollars?

(A) $g(M) = 300 + 300M$

(B) $g(M) = \frac{M}{300} + 200$

(C) $g(M) = \frac{200}{3} + \frac{300}{M}$

(D) $g(M) = \frac{100}{3} + \frac{300}{M}$



2. The graph of the function $C(M)$ is given in the figure above. The cost of producing M units of a specific medicine is given by the function C as $C(M) = sM + b$, where s and b are positive constants. If M units are produced, which of following functions, j, represents the cost per unit, in dollars?

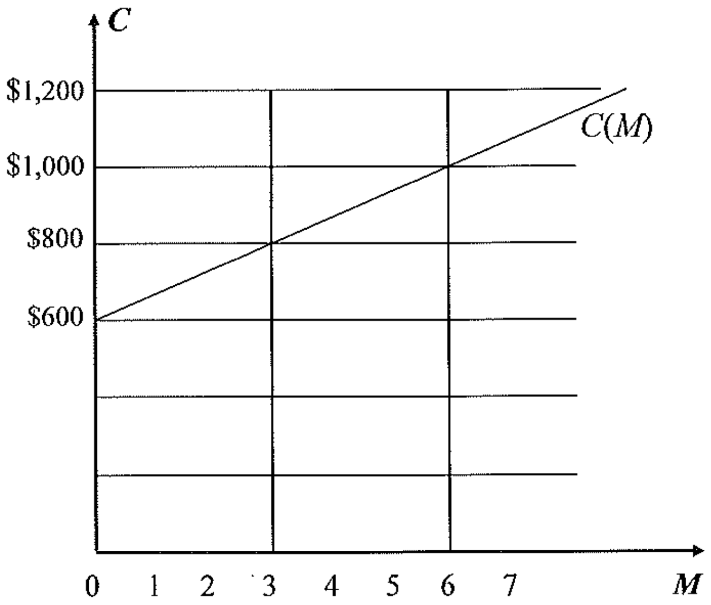
(A) $j(M) = \frac{M}{300} + 200$

(B) $j(M) = 300 + 300M$

(C) $j(M) = \frac{200}{3} + \frac{300}{M}$

(D) $j(M) = \frac{300}{2} + \frac{300}{M}$

Linear Functions Representing Cost per Unit



3. The graph of the function $C(M)$ is given above. The cost of producing M units of a specific medicine is given by the function C as $C(M) = sM + b$, where s and b are positive constants. If M units are produced, which of following functions, g, represents the cost per unit, in dollars?

(A) $g(M) = 800 + 300M$

(B) $g(M) = \frac{200}{3} + \frac{600}{M}$

(C) $g(M) = \frac{600}{M} - \frac{200}{3}$

(D) $g(M) = \frac{200}{3} + \frac{800}{M}$

Questions 1 and 2 refer to the following definitions for integers greater than 1.

$\boxed{\boxed{m}} = m^2 + 2m$

$\boxed{m} = m^2 - 2m$

1. $\boxed{\boxed{3}} + \boxed{5} =$

(A) 30
(B) 28
(C) 20
(D) 25

Solution: Answer: (A)

$$m^2 + 2m = 3^2 + 2 \cdot 3 = 15$$
$$\downarrow$$
$$m^2 - 2m = 5^2 - 2 \cdot 5 = 15$$
$$\Downarrow$$
$$\text{Sum} = 15 + 15 = 30$$

2. If n is an integer greater than 1, then $\boxed{n+1} =$

(A) $\boxed{\boxed{n}} - 2n - 1$

(B) $\boxed{\boxed{n}}$

(C) $\boxed{\boxed{n}} - 2n + 1$

(D) $\boxed{\boxed{n}} + n + 1$

Solution: Answer: (A)

$$(n+1)^2 - 2(n+1)$$
$$\downarrow$$
$$n^2 + 2n + 1 - 2n - 2$$
$$\downarrow$$
$$n^2 - 1 = n^2 - 1 + 2n - 2n$$
$$\downarrow$$
$$(n^2 + 2n) - 2n - 1$$
$$\Downarrow$$
$$\boxed{\boxed{n}} - 2n - 1$$

3. For $x \geq 2$, $\sharp x \sharp$ is defined as $\sharp x \sharp = \sqrt{x-2}$, which of the following equals $\sharp 11 \sharp + \sharp 6 \sharp$?

(A) 1
(B) 2
(C) 3
(D) 5

Solution: Answer: (D)

$$\sharp 11 \sharp = \sqrt{11-2} = \sqrt{9} = 3$$
$$\downarrow$$
$$\sharp 6 \sharp = \sqrt{6-2} = \sqrt{4} = 2$$
$$\Downarrow$$
$$\sharp 11 \sharp + \sharp 6 \sharp = 3 + 2 = 5$$

4. Let $\diamond$ be defined by $x \diamond y = x^y$. If $x = 3 \diamond p$, $y = 3 \diamond q$, and $p - q = 3$, what is the value of $\frac{x}{y}$?

(A) 3
(B) 27
(C) 9
(D) 81

Solution: Answer: (B)

$$x = 3^p, \; y = 3^q$$
$$\downarrow$$
$$\frac{x}{y} = \frac{3^p}{3^q} = 3^{p-q} = 3^3$$
$$\Downarrow$$
$$3^{p-q} = 3^3 = 27$$

New Definitions

5. Let ◙ be defined by x ◙ $y = x^y$. If x =3 ◙ p, y =3 ◙ q, and $p+q=4$, what is the value of $\dfrac{x}{y^{-1}}$?

(A) 3
(B) 6
(C) 9
(D) 81

Solution: Answer: (D)

$$x = 3 \text{ ◙ } p,\ y = 3 \text{ ◙ } q$$
$$\downarrow$$
$$x = 3^p,\ y = 3^q$$
$$\begin{cases} \dfrac{x}{y^{-1}} = xy \\ \downarrow \\ xy = 3^p \cdot 3^q = 3^{p+q} \end{cases}$$
$$\Downarrow$$
$$3^{p+q} = 3^4 = 81$$

6. If $f(x) = 2x - 1$, $g(x) = x - 3$ and an operation ♣ is defined for all real numbers A and B by the equation A ♣ $B = A + B + 2AB$, then $f(2)$ ♣ $g(2) =$

(A) -8
(B) 8
(C) 4
(D) –4

Solution: Answer: (D)

$$f(2) = 2\cdot 2 - 1 = 3, \quad g(2) = 2 - 3 = -1$$

$$f(2)\cdot g(2) = f(2) + g(2) + 2f(2)\cdot g(2)$$
$$\Downarrow$$
$$3 + (-1) + 2\cdot 3\cdot(-1) = -4$$

7. Let $a\#b$ be defined by $a\#b = a^b + (a-b)$ for all positive integers a and b. What is the value of (3#2)#2?

Solution: Answer: 108

$$3\#2 = 3^2 + (3-2) = 10$$
$$\downarrow$$
$$(3\#2)\#2 = 10\#2$$
$$\Downarrow$$
$$10^2 + (10-2) = 108$$

8. Two integers x and y that are greater than zero satisfy the relation $x\Theta y$ if and only if $x = y^2 - 2$. a, b, and c satisfy the relation $a\Theta b$ and $b\Theta c$. What is the value of $a+2$ in terms of c?

(A) $c^2 - 2$
(B) $\left(c^2-2\right)^2 + 2$
(C) $\left(c^2-2\right)^2$
(D) $\left(c^2-2\right)^2 - 4$

Solution: Answer: (C)

$$a\Theta b:$$
$$a = b^2 - 2, \rightarrow b^2 = a + 2$$
$$b\Theta c:$$
$$b = c^2 - 2, \rightarrow b^2 = \left(c^2-2\right)^2$$
$$b^2 = b^2$$
$$\Downarrow$$
$$a + 2 = \left(c^2-2\right)^2$$

Questions 1 and 2 refer to the following definitions for integers greater than 1.

$\langle m \rangle = m^2 + 2m$

$\boxed{m} = m^2 - 2m$

1. $\langle 3 \rangle + \boxed{5} =$

(A) 30
(B) 28
(C) 20
(D) 25

2. If n is an integer greater than 1, then $\boxed{n+1} =$

(A) $\langle n \rangle - 2n - 1$

(B) $\langle n \rangle$

(C) $\langle n \rangle - 2n + 1$

(D) $\langle n \rangle + n + 1$

3. For $x \geq 2$, ♪x♫ is defined as ♪x♫ $= \sqrt{x-2}$, which of the following equals ♪11♫ + ♪6♫?

(A) 1
(B) 2
(C) 3
(D) 5

4. Let ◈ be defined by x◈$y = x^y$. If $x = 3$◈p, $y = 3$◈q, and $p - q = 3$, what is the value of $\frac{x}{y}$?

(A) 3
(B) 27
(C) 9
(D) 81

New Definitions

5. Let ◎ be defined by $x \text{ ◎ } y = x^y$. If x =3 ◎ p, y =3 ◎ q, and $p+q=4$, what is the value of $\frac{x}{y^{-1}}$?

(A) 3
(B) 6
(C) 9
(D) 81

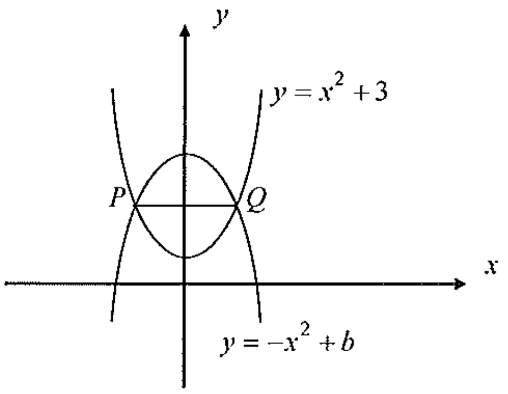
6. If $f(x)=2x-1$, $g(x)=x-3$ and an operation ♣ is defined for all real numbers A and B by the equation $A \clubsuit B = A + B + 2AB$, then $f(2) \clubsuit g(2) =$

(A) -8
(B) 8
(C) 4
(D) –4

7. Let $a \# b$ be defined by $a \# b = a^b + (a-b)$ for all positive integers a and b. What is the value of (3#2)#2?

8. Two integers x and y that are greater than zero satisfy the relation $x \Theta y$ if and only if $x = y^2 - 2$. a, b, and c satisfy the relation $a \Theta b$ and $b \Theta c$. What is the value of $a+2$ in terms of c?

(A) c^2-2
(B) $\left(c^2-2\right)^2+2$
(C) $\left(c^2-2\right)^2$
(D) $\left(c^2-2\right)^2-4$



1. The figure above shows the graph of $y = x^2 + 3$ and $y = -x^2 + b$ for some constant b. If $PQ = 6$, what is the value of b?

(A) 10
(B) 13
(C) 53
(D) 21

Solution: Answer: (D)

(1) At the intersection points,

$$y = y$$

$$x^2 + 3 = -x^2 + b$$

(2) $PQ = 6 \rightarrow x = \pm 3$

$$\downarrow$$

$$(\pm 3)^2 + 3 = -(\pm 3)^2 + b$$

$$\Downarrow$$

$$b = 21$$

2. The figure above shows the graph of $y = x^2 + 3$ and $y = -x^2 + b$ for some constant b. If $PQ = 10$, what is the value of b?

(A) 10
(B) 13
(C) 53
(D) 20

Solution: Answer: (C)

(1) At the intersection points,

$$y = y$$

$$x^2 + 3 = -x^2 + b$$

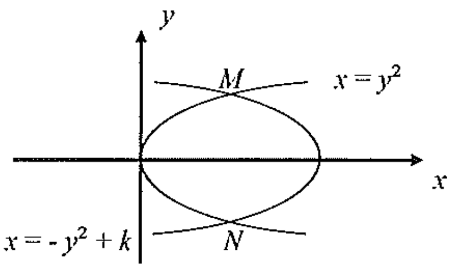
(2) $PQ = 10 \rightarrow x = \pm 5$

$$\downarrow$$

$$(\pm 5)^2 + 3 = -(\pm 5)^2 + b$$

$$\Downarrow$$

$$b = 53$$



3. The figure above shows the graph $x = y^2$ and $x = -y^2 + k$. k is a certain constant. M and N separately represent the intersecting points of the two graphs in the xy-plane. If the length of the distance between M and N is 10, what is the value of k?

(A) 50
(B) 8
(C) 63
(D) 65

Solution: Answer: (A)

(1) At the intersection points

$$x = x$$

$$y^2 = -y^2 + k$$

(2) $MN = 10 \rightarrow x = \pm 5$

$$\downarrow$$

$$(\pm 5)^2 = -(\pm 5)^2 + k$$

$$\Downarrow$$

$$k = 50$$

4. The figure above shows the graph $x = y^2$ and $x = -y^2 + k$. k is a certain constant. M and N separately represent the intersecting points of the two graphs in the xy-plane. If the length of the distance between M and N is 4, what is the value of k?

(A) 10
(B) 8
(C) 63
(D) 65

Solution: Answer: (B)

(1) At the intersection points,

$$x = x$$

$$y^2 = -y^2 + k$$

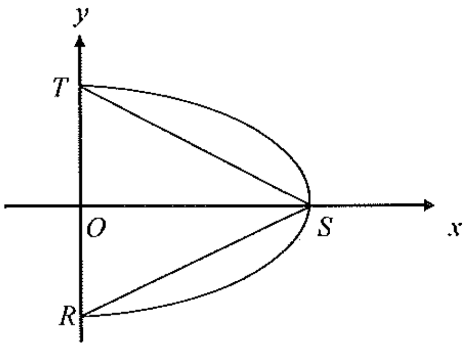
(2) $MN = 4 \rightarrow x = \pm 2$

$$\downarrow$$

$$(\pm 2)^2 = -(\pm 2)^2 + k$$

$$\Downarrow$$

$$k = 8$$



Note: figure above not drawn to scale.

14. The figure above shows the graph of $x = g - y^2$, where g is a constant. If the area of triangle RST is 8, what is the value of g?

(A)-5
(B) 3
(C)-4
(D) 4

Solution: Answer: (D)

(1) $x = 0$

$$x = -y^2 + g \rightarrow 0 = -y^2 + g$$

$$\downarrow$$

$$y^2 = g, \quad y = \pm\sqrt{g}$$

$$\downarrow$$

$$RT = 2\sqrt{g}$$

(2) $y = 0$

$$x = -y^2 + g, \rightarrow x = 0 + g$$

$$\downarrow$$

$x = g$ (By the graph above, $g > 0$.)

$$\downarrow$$

$$OS = g$$

(3) $\text{Area}_{RST} = 8$

$$8 = \frac{RT \times OS}{2} = \frac{2\sqrt{g} \times g}{2}$$

$$\downarrow$$

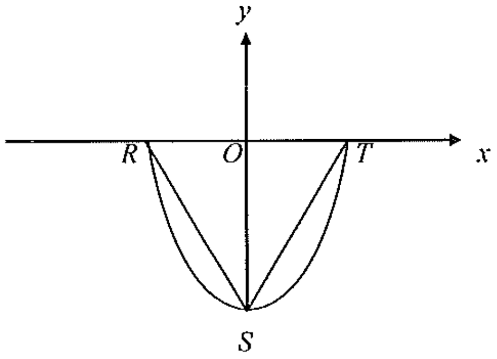
$$8 = \sqrt{g} \times g = g^{\frac{3}{2}}$$

$$\downarrow$$

$$8^{\left(\frac{2}{3}\right)} = \left(g^{\frac{3}{2}}\right)^{\left(\frac{2}{3}\right)}$$

$$\Downarrow$$

$$g = \left(\sqrt[3]{8}\right)^2 = 2^2 = \boxed{4}$$



Note: figure above not drawn to scale.

15. The figure above shows the graph of $y = g + x^2$, where g is a constant. If the area of triangle RST is 27, what is the value of g?

(A) 7
(B)-7
(C) -9
(D) 9

Solution: Answer: (C)

(1) $y = 0$

$$y = x^2 + g \rightarrow 0 = x^2 + g$$

$$\downarrow$$

$$x^2 = g, \; x = \pm\sqrt{-g}$$

$$\downarrow$$

$$RT = 2\sqrt{-g}$$

(2) $x = 0$

$$y = x^2 + g, \rightarrow y = 0 + g$$

$$\downarrow$$

$y = g$ (By the graph above, $g < 0$.)

$$\downarrow$$

$$OS = |g|$$

(3) $\text{Area}_{RST} = 27$

$$27 = \frac{RT \times OS}{2} = \frac{2\sqrt{-g} \times |g|}{2}$$

$$\downarrow$$

$$27 = \sqrt{-g} \times |g| = \sqrt{|g|} \times |g| = |g|^{\frac{3}{2}}$$

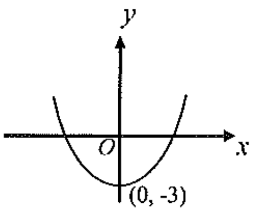
$$27^{\left(\frac{2}{3}\right)} = \left(|g|^{\frac{3}{2}}\right)^{\left(\frac{2}{3}\right)}$$

$$|g| = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$$

g is the value of y of point S.

$$\Downarrow$$

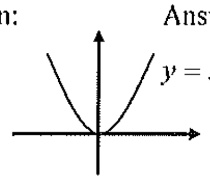
$$g = \boxed{-9}$$



16. The figure above is a parabola that is symmetric about the y-axis. Which of following could be an equation of the parabola?

(A) $x = y^2 - 3$
(B) $x = -y^2 - 3$
(C) $y = -x^2 - 3$
(D) $y = x^2 - 3$

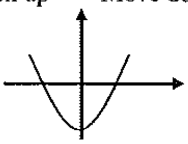
Solution: Answer: (D)

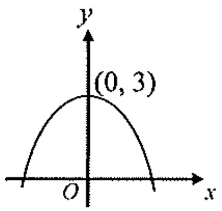


The signs of equation determine the changes of the graph of $y = x^2$.

$y = +x^2 - 3$

Open up Move down

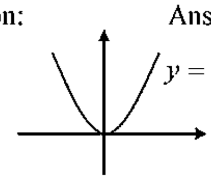




17. The figure above is a parabola that is symmetric about the y-axis. Which of following could be an equation of the parabola?

(A) $x = y^2 - 3$
(B) $x = -y^2 - 3$
(C) $y = -x^2 + 3$
(D) $y = x^2 - 3$

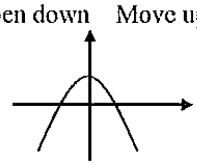
Solution: Answer: (C)



The signs of equation determine the changes of the graph of $y = x^2$.

$y = -x^2 + 3$

Open down Move up



18. In the system of equations $\begin{cases} y = -2 \\ y = ax^2 + b \end{cases}$, a and b are constants. For which of the following values of a and b does the system have exactly one real solution?

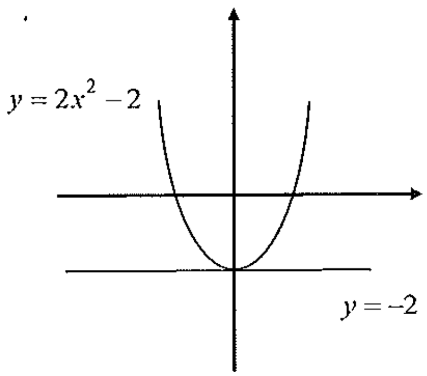
(A) $a = -1,\ b = 3$ (B) $a = -2,\ b = 2$
(C) $a = 2,\ b = -2$ (D) $a = -1,\ b = -3$

Solution: Answer: (C)

$$\begin{cases} y = ax^2 + b \\ y = -2 \end{cases} \rightarrow \begin{cases} y = 2x^2 - 2 \\ y = -2 \end{cases}$$

$\downarrow$

The sketch shown below



$\Downarrow$

Answer is (C).

19. In the xy-plane, A and B are distinct points, which have the same y-coordinate and lie on the parabola whose equation is $y = -x^2 + 5x + 30$. Which of the following is the x-coordinate of the midpoint of $\overline{AB}$?

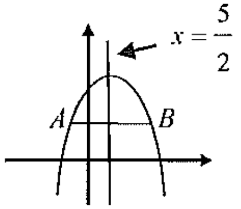
(A) $\frac{5}{2}$
(B) $-\frac{2}{5}$
(C) $\frac{2}{5}$
(D) $-\frac{5}{3}$

Solution: Answer: (A)

$$y = -x^2 + 5x + 30$$

$\Downarrow$

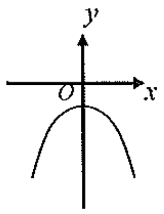
$$x = -\frac{b}{2a} = -\frac{5}{-2 \cdot 1} = \frac{5}{2}$$



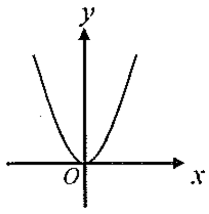
Parabolas

20. The function f is given by $f(x) = ax^2 + bx + c$, where $a > 0$, $c < 0$ and they are constants. Which of the following could be the graph of f?

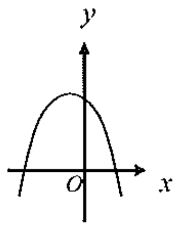
(A)



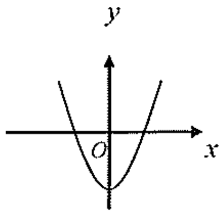
(B)



(C)



(D)



Solution: Answer: (D)

(1)

Coefficient of x^2 is a, and $a > 0$.

↓

The open direction of this parabola is up.

↓

Choices (A) and (C) are eliminated.

(2)

"Could be"

↓

Let $a = 1 > 0$ and $c = -1 < 0$. Then

$y = x^2 + bx - 1$

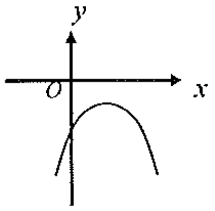
When $x = 0$, $y = -1$

⇓

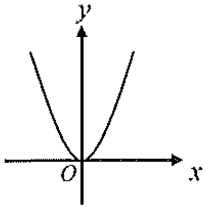
Answer is (D).

21. The function g is given by $g(x) = x^2 + bx + c$, where b and c are negative constants. Which of the following could be the graph of g?

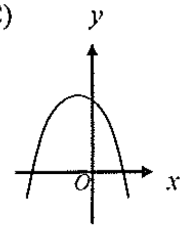
(A)



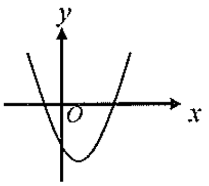
(B)



(C)



(D)



Solution: Answer: (D)

(1)

Coefficient of x^2 is 1, and $1 > 0$.

↓

The open direction of this parabola is up.

↓

Choices (A) and (C) are eliminated.

(2)

"Could be"

↓

Let $b = -2 < 0$ and $c = -1 < 0$. Then

$y = x^2 - 2x - 1$

$y = x^2 - 2x + 1 - 1 - 1$

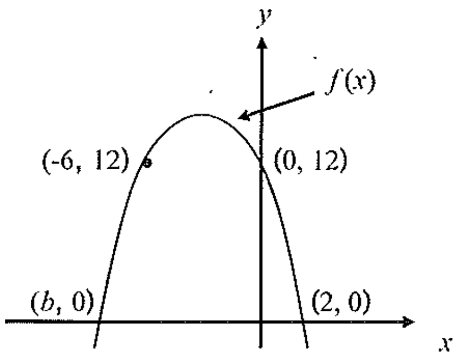
$y = (x^2 - 2x + 1) - 2$

$y = (x - 1)^2 - 2$

Negative sign ⟶ curve shifts right.

⇓

Answer is (D).



22. The diagram above shows the graph of a quadratic function f. What is the value of b?

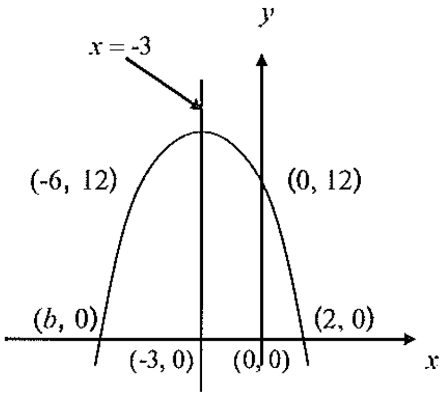
(A) -8

(B) -5

(C) -7

(D) -9

Solution: Answer: (A)



Obtain the symmetrical line first.

$$\text{symmetrical line } x = \frac{-6+0}{2} = -3$$

distance between points (-3, 0) and (2, 0) = 5

A parabola is a symmetric graph.

$$\downarrow$$

distance between points (-3, 0) and (b, 0) = 5

$$\Downarrow$$

$$b = -8$$

23. Let the function g be defined by $g(x) = k(x-a)^2$, where k is a negtive constant and a is a positive constant. For what value of x will the function g have its maximum value?

(A) k

(B) $-k$

(C) $-a$

(D) a

Solution: Answer: (D)

Method 1

Vertex form of quadratic function:

$$f(x) = k(x - x_m) + y_m$$

$$\Downarrow$$

$$x_m = a$$

Method 2

$$g(x) = k(x-a)^2 = k\left(x^2 - 2ax + a^2\right)$$

$$\downarrow$$

$$g(x) = kx^2 - 2akx + ka^2$$

$$g(x) = Ax^2 + Bx + C$$

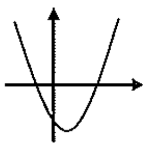
$A < 0$, when $x = \dfrac{-B}{2A}$, function g has its maximum value.

$$\Downarrow$$

$$x = \frac{-(-2ak)}{2k} = a$$

24. For how many integers m is $(3m+1)(4m-17)$ a negative number?

Solution: Answer: 5



(1). $\underbrace{(3m+1)\cdot(4m-17)=0}_{\text{to get zeros or roots}}$

$3m+1=0,\ m=-\dfrac{1}{3}$

$4m-17=0 \rightarrow m=4\dfrac{1}{4}$

(2). $(3m+1)\cdot(4m-17)<0$

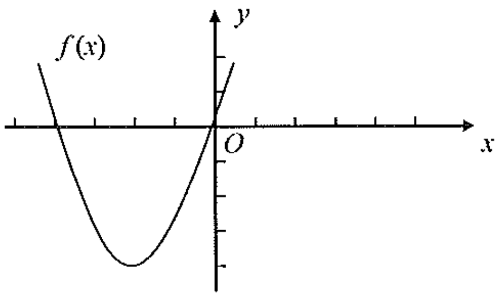
$$\downarrow$$

$y < 0$

$$\Downarrow$$

$$\underbrace{-\frac{1}{3} < m < 4\frac{1}{4}}_{\text{Under this domain, } y<0}, \quad m = 0,\ 1,\ 2,\ 3,\ 4$$

Parabolas



25. In the figure above, the graph of $f(x)$ is the result of shifting the graph of $g(x)$ 3 units to right and 3 units up. If the equation of $f(x)$ is $f(x) = x^2 + 4x$, what is the equation of $g(x)$?

(A) $(x-5)^2 + 1$

(B) $(x-5)^2 - 7$

(C) $(x+5)^2 - 7$

(D) $(x+5)^2 - 4$

Solution: Answer: (C)

$$(x+3)^2 + 4(x+3) - 3$$

$$x^2 + 6x + 9 + 4x + 12 - 3$$

$$x^2 + 10x + 18$$

$$x^2 + 10x + 5^2 - 25 + 18$$

$$\Downarrow$$

$$g(x) = (x+5)^2 - 7$$

26. How must a and b be related so that the graph of $g(x+11)$ will be symmetrical about the y-axis if $g(x) = ax^2 + bx + c$?

(A) $22a = -b$

(B) $10a = -b$

(C) $10 + a = b$

(D) $20a = -b$

Solution: Answer: (A)

$$g(x) = ax^2 + bx + c$$

When $x = -\dfrac{b}{2a}$, $g(x)$ has maximum or minimum value. And $x = -\dfrac{b}{2a}$ is the symmetrical line. The graph of $g(x+11)$ is the translation of $g(x)$ 11 units to the left. If $g(x)$ is symmetric about $x = +11$, $g(x+11)$ will be symmetric about the y-axis.

$$\Downarrow$$

$$-\frac{b}{2a} = 11 \text{ or } 22a = -b$$

27. If $f(x) = 2x^2 - 5x + 6$, what must the value of k equal so that the graph of $f(x-k)$ will be symmetric to the y-axis ?

(A) $-\dfrac{4}{5}$

(B) $\dfrac{4}{5}$

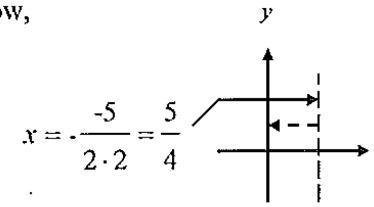
(C) $-\dfrac{5}{4}$

(D) $\dfrac{3}{2}$

Solution: Answer: (C)

The graph of $x = -\dfrac{b}{2a}$ is the symmetric line of a quadratic function $y = ax^2 + bx + c$.

Now,



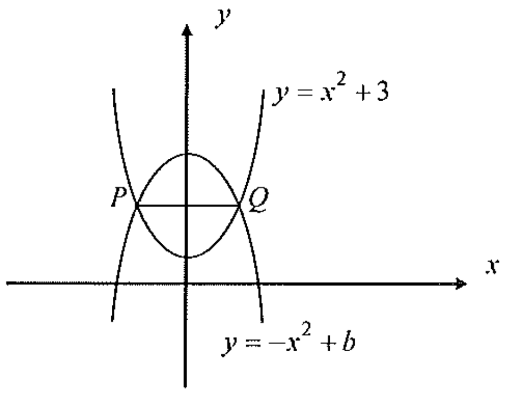
$$\downarrow$$

$$k = -\frac{5}{4}$$

$$\Downarrow$$

$$f(x-k) = f(x+\frac{5}{4})$$

(It means the graph should shift $\dfrac{5}{4}$ units to left to be symmetric to the y-axis.)

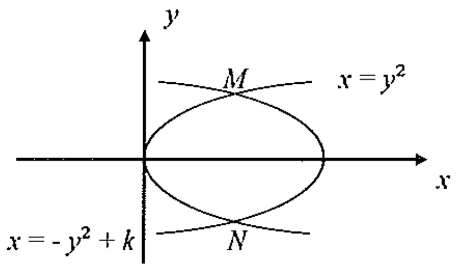


1. The figure above shows the graph of $y = x^2 + 3$ and $y = -x^2 + b$ for some constant b. If $PQ = 6$, what is the value of b?

(A) 10
(B) 13
(C) 53
(D) 21

2. The figure above shows the graph of $y = x^2 + 3$ and $y = -x^2 + b$ for some constant b. If $PQ = 10$, what is the value of b?

(A) 10
(B) 13
(C) 53
(D) 20



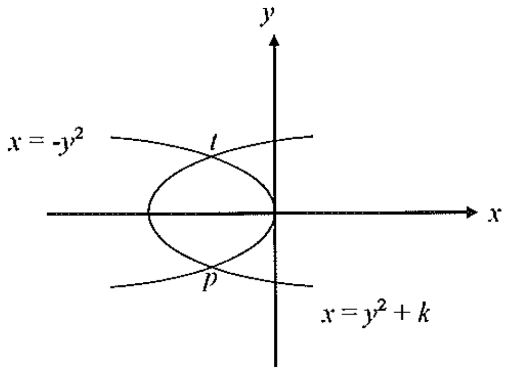
3. The figure above shows the graph $x = y^2$ and $x = -y^2 + k$. k is a certain constant. M and N separately represent the intersecting points of the two graphs in the xy-plane. If the length of the distance between M and N is 10, what is the value of k?

(A) 50
(B) 8
(C) 63
(D) 65

4. The figure above shows the graph $x = y^2$ and $x = -y^2 + k$. k is a certain constant. M and N separately represent the intersecting points of the two graphs in the xy-plane. If the length of the distance between M and N is 4, what is the value of k?

(A) 10
(B) 8
(C) 63
(D) 65

Parabolas

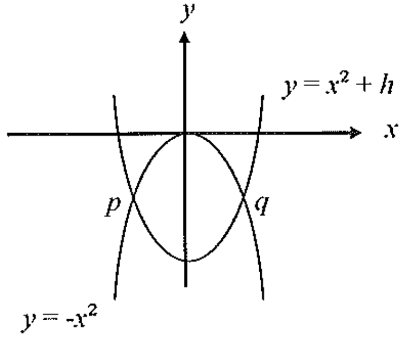


5. The figure above shows the graph $x = -y^2$ and $x = y^2 + k$. k is a certain constant. p and t separately represent the intersecting points of the 2 graphs in the xy-plane. If the length of the distance between t and p is 8, what is the value of k?

(A) -32
(B) 72
(C) 32
(D) 37

6. The figure above shows the graph $x = -y^2$ and $x = y^2 + k$. k is a certain constant. p and t separately represent the intersecting points of the 2 graphs in the xy-plane. If the length of the distance between t and p is 12, what is the value of k?

(A) -32
(B) 72
(C) 32
(D) -72

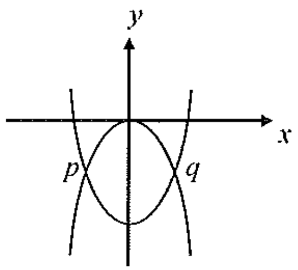


7. The figure above shows the graphs $y = -x^2$ and $y = x^2 + h$. h is a certain constant. p and q separately represent the intersecting points of the two graphs in the xy-plane. If the length of the distance between p and q is 14, what is the value of h?

(A) -82
(B) 98
(C) 82
(D) -98

8. The figure above shows the graphs $y = -x^2$ and $y = x^2 + h$. h is a certain constant. p and q separately represent the intersecting points of the two graphs in the xy-plane. If the length of the distance between p and q is 10, what is the value of h?

(A) -82
(B) -50
(C) 82
(D) -98



9. The figure above shows the graphs $y = -x^2$ and $y = x^2 + h$. h is a certain constant. p and q separately represent the intersecting points of the two graphs in the xy-plane. If the length of the distance between p and q is $2a$, what is the value of h?

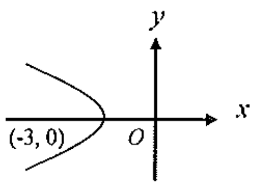
(A) $2a$
(B) a
(C) $-2a^2$
(D) $2a^2$

10. In the xy-plane, (b, y) is one of the points of intersection of the graph of $y = -x^2 + 16$ and $y = x^2 - 16$. What is the possible value of b?

(A) ± 6
(B) ± 4
(C) ± 2
(D) ± 3

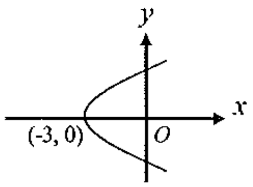
11. In the xy-plane, (b, y) is one of the points of intersection of the graph of $y = -x^2 + 4$ and $y = x^2 - 4$. What is the possible value of b?

(A) 6
(B) -4
(C) -2
(D) 3



12. The figure above is a parabola that is symmetric about the x-axis. Which of following could be an equation of the parabola?

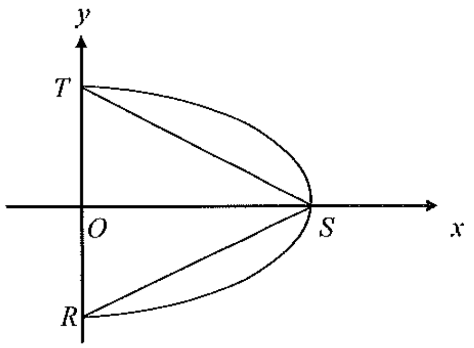
(A) $x = y^2 - 3$
(B) $x = y^2 + 3$
(C) $y = -x^2 - 3$
(D) $x = -y^2 - 3$



13. The figure above is a parabola that is symmetric about the x-axis. Which of following could be an equation of the parabola?

(A) $x = y^2 - 3$
(B) $x = -y^2 - 3$
(C) $y = -x^2 - 3$
(D) $y = x^2 - 3$

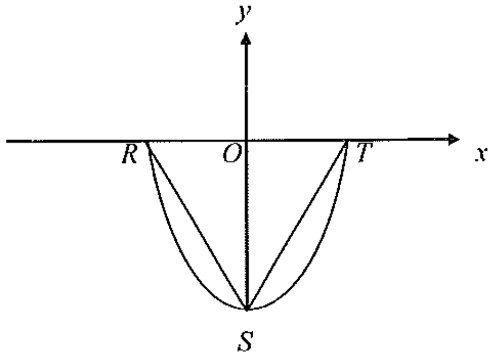
Parabolas



Note: figure above not drawn to scale.

14. The figure above shows the graph of $x = g - y^2$, where g is a constant. If the area of triangle RST is 8, what is the value of g?

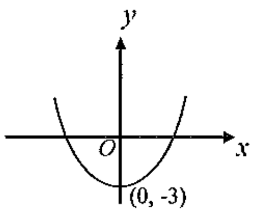
(A)-5
(B) 3
(C)-4
(D) 4



Note: figure above not drawn to scale.

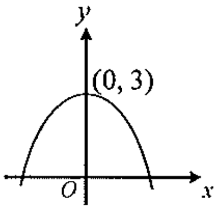
15. The figure above shows the graph of $y = g + x^2$, where g is a constant. If the area of triangle RST is 27, what is the value of g?

(A) 7
(B)-7
(C) -9
(D) 9



16. The figure above is a parabola that is symmetric about the y-axis. Which of following could be an equation of the parabola?

(A) $x = y^2 - 3$
(B) $x = -y^2 - 3$
(C) $y = -x^2 - 3$
(D) $y = x^2 - 3$



17. The figure above is a parabola that is symmetric about the y-axis. Which of following could be an equation of the parabola?

(A) $x = y^2 - 3$
(B) $x = -y^2 - 3$
(C) $y = -x^2 + 3$
(D) $y = x^2 - 3$

18. In the system of equations $\begin{cases} y = -2 \\ y = ax^2 + b \end{cases}$, a and b are constants. For which of the following values of a and b does the system have exactly one real solution?

(A) $a = -1,\ b = 3$ (B) $a = -2,\ b = 2$
(C) $a = 2,\ b = -2$ (D) $a = -1,\ b = -3$

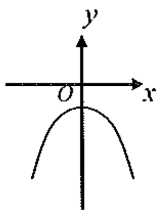
19. In the xy-plane, A and B are distinct points, which have the same y-coordinate and lie on the parabola whose equation is $y = -x^2 + 5x + 30$. Which of the following is the x-coordinate of the midpoint of $\overline{AB}$?

(A) $\frac{5}{2}$
(B) $-\frac{2}{5}$
(C) $\frac{2}{5}$
(D) $-\frac{5}{3}$

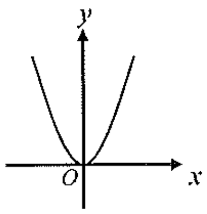
Parabolas

20. The function f is given by $f(x) = ax^2 + bx + c$, where $a > 0$, $c < 0$ and they are constants. Which of the following could be the graph of f?

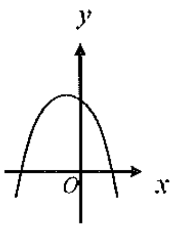
(A)



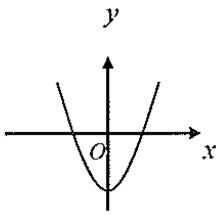
(B)



(C)

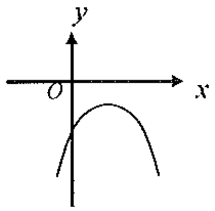


(D)

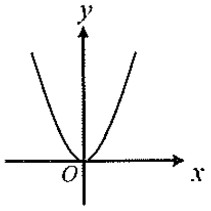


21. The function g is given by $g(x) = x^2 + bx + c$, where b and c are negative constants. Which of the following could be the graph of g?

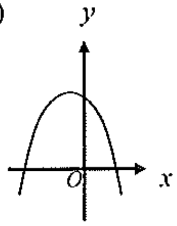
(A)



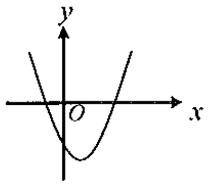
(B)

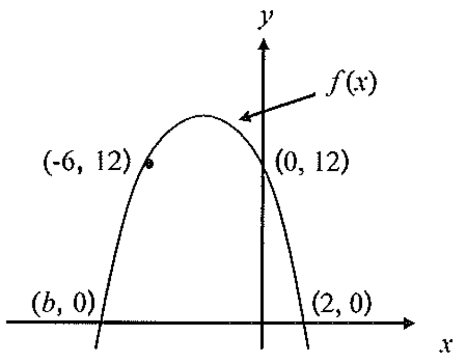


(C)



(D)





22. The diagram above shows the graph of a quadratic function f. What is the value of b?

(A) -8

(B) -5

(C) -7

(D) -9

23. Let the function g be defined by $g(x) = k(x-a)^2$, where k is a negtive constant and a is a positive constant. For what value of x will the function g have its maximum value?

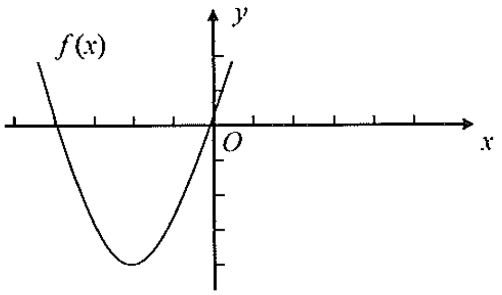
(A) k

(B) $-k$

(C) $-a$

(D) a

24. For how many integers m is $(3m+1)(4m-17)$ a negative number?



25. In the figure above, the graph of $f(x)$ is the result of shifting the graph of $g(x)$ 3 units to right and 3 units up. If the equation of $f(x)$ is $f(x) = x^2 + 4x$, what is the equation of $g(x)$?

(A) $(x-5)^2 + 1$

(B) $(x-5)^2 - 7$

(C) $(x+5)^2 - 7$

(D) $(x+5)^2 - 4$

26. How must a and b be related so that the graph of $g(x+11)$ will be symmetrical about the y-axis if $g(x) = ax^2 + bx + c$?

(A) $22a = -b$

(B) $10a = -b$

(C) $10 + a = b$

(D) $20a = -b$

27. If $f(x) = 2x^2 - 5x + 6$, what must the value of k equal so that the graph of $f(x-k)$ will be symmetric to the y-axis?

(A) $-\frac{4}{5}$

(B) $\frac{4}{5}$

(C) $-\frac{5}{4}$

(D) $\frac{3}{2}$

28. In the xy-plane, A and B are distinct points, which have the same y-coordinate and lie on the parabola whose equation is $y = -x^2 + 5x + 30$. Which of the following is the x-coordinate of the midpoint of $\overline{AB}$?

(A) $\frac{5}{2}$

(B) $-\frac{2}{5}$

(C) $\frac{2}{5}$

(D) $-\frac{5}{3}$

x	$g(x)$	$f(x)$
5	7	6
4	5	4
3	3	7
2	4	-1
1	6	-3
0	8	-5
-1	9	-7

1. The certain values of the two functions f and g, which are defined for all values of x, are given in the table above. If the function j is defined by $j(x) = 3g(x) - f(2.5x)$ for all values of x, what is the value of $j(2)$?

(A) 4

(B) 3

(C) 14

(D) 6

Solution: Answer: (D)

Refer to the table above.

$\downarrow$

$j(2) = 3g(2) - f(5)$

$\Downarrow$

$j(2) = 3 \cdot 4 - 6 = 6$

x	$g(x)$	$f(x)$
2	-1	2
1	0	-1
0	1	-2
-1	2	3
-2	3	4

2. According to chart above, what is the value of $g(-2) - f(1)$?

(A) 2

(B) 3

(C) 4

(D) -3

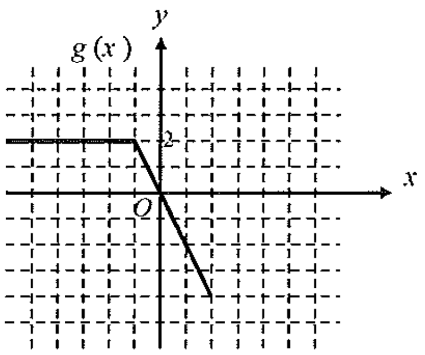
Solution: Answer: (C)

$$\begin{cases} g(-2) = 3 \\ f(1) = -1 \end{cases}$$

$\Downarrow$

$g(-2) - f(1) = 3 - (-1) = 4$

$f(x)$	x
4	-4
3	-3
2	-2
0	-1
2	0
3	1
4	2
5	3



3. The chart above shows chosen values for the function f and the figure above shows the graph of the function g. For which of the following values of x does $g(x)$ equal $f(x)$?

(A) 0

(B) -2

(C) 2

(D) 3

Solution: Answer: (B)

At point (-2, 2), the two functions f and g have the same value of y.

$\Downarrow$

When $x = -2$, $g(-2) = f(-2) = 2$.

RELATIONSHIP OF TEMPERATURE AND TIME FOR MIXTURES

Temperature T at x Second	Mixture
$T(x) = 4x - 3$	M_1
$T(x) = \frac{1}{4}x - 3$	M_2
$T(x) = -\frac{1}{4}x + 3$	M_3
$T(x) = -4x + 3$	M_4
$T(x) = 4x + 3$	M_5

4. The chart above shows the information of temperatures of several distinct chemical mixtures at x seconds when the mixture was formed. The temperature of which mixture goes down the greatest number of degrees during the interval from $x = 16$ to $x = 20$?

(A) M_5 (B) M_2 (C) M_3 (D) M_4

Solution: Answer: (D)

The slopes of M_1, M_2 and M_5 are positive. Their temperatures will not go down, but go up.

Note: When doing the operation of subtraction, ignore the same constant part. It saves your time.

$$M_3 : T(16) = -\frac{1}{4} \cdot 16 = -4$$
$$T(20) = -\frac{1}{4} \cdot 20 = -5$$
$$T(20) - T(16) = -5 - (-4) = \boxed{-1}$$

$$M_4 : T(16) = -4 \cdot 16 = -64$$
$$T(20) = -4 \cdot 20 = -80$$
$$T(20) - T(16) = -80 - (-64) = \boxed{-16}$$

$\Downarrow$

Answer is (D).

$g(t)$	t
3	-3
-2	-2
4	-1
-3	0
0	1

5. The function g is defined by the chart above. For what value of t does $t - g(t) = 0$?

(A) 2

(B) -1

(C) -2

(D) -3

Solution: Answer: (C)

Refer to the table above.

$\downarrow$

When $t = -2$, $g(-2) = -2$.

$\Downarrow$

$t - g(t) = -2 - (-2) = 0$

x	$g(x)$	$f(x)$
5	7	6
4	5	4
3	3	7
2	4	-1
1	6	-3
0	8	-5
-1	9	-7

1. The certain values of the two functions f and g, which are defined for all values of x, are given in the table above. If the function j is defined by $j(x) = 3g(x) - f(2.5x)$ for all values of x, what is the value of $j(2)$?

(A) 4

(B) 3

(C) 14

(D) 6

x	$g(x)$	$f(x)$
2	-1	2
1	0	-1
0	1	-2
-1	2	3
-2	3	4

2. According to chart above, what is the value of $g(-2) - f(1)$?

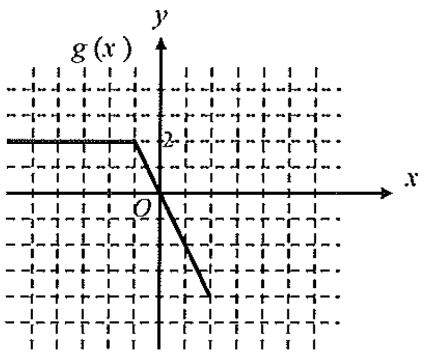
(A) 2

(B) 3

(C) 4

(D) -3

$f(x)$	x
4	-4
3	-3
2	-2
0	-1
2	0
3	1
4	2
5	3



3. The chart above shows chosen values for the function f and the figure above shows the graph of the function g. For which of the following values of x does $g(x)$ equal $f(x)$?

(A) 0

(B) -2

(C) 2

(D) 3

RELATIONSHIP OF TEMPERATURE AND TIME FOR MIXTURES

Temperature T at x Second	Mixture
$T(x) = 4x - 3$	M_1
$T(x) = \frac{1}{4}x - 3$	M_2
$T(x) = -\frac{1}{4}x + 3$	M_3
$T(x) = -4x + 3$	M_4
$T(x) = 4x + 3$	M_5

4. The chart above shows the information of temperatures of several distinct chemical mixtures at x seconds when the mixture was formed. The temperature of which mixture goes down the greatest number of degrees during the interval from $x = 16$ to $x = 20$?

(A) M_5 (B) M_2 (C) M_3 (D) M_4

$g(t)$	t
3	-3
-2	-2
4	-1
-3	0
0	1

5. The function g is defined by the chart above. For what value of t does $t - g(t) = 0$?

(A) 2

(B) -1

(C) -2

(D) -3

The fish and beef weighed 6 pounds.
The fish and chicken weighed 7 pounds.
The beef and chicken weighed 8 pounds.

1. By the information above, what is the weight of the beef?

(A) $\frac{9}{2}$ Pounds
(B) 7 Pounds
(C) $\frac{7}{2}$ Pounds
(D) $\frac{5}{2}$ Pounds

Solution: Answer: (C)

$$\begin{cases} f+b=6 \ (1) \\ f+c=7 \ (2) \\ b+c=8 \ (3) \end{cases}$$

$$\begin{cases} \text{The target is } b. \\ \text{Cancel } c \text{ and } f. \end{cases}$$

$$(2)-(1)$$

$$c-b=1 \quad (4)$$

$$(3)-(4)$$

$$\Downarrow$$

$$2b=7,\ b=\frac{7}{2}$$

2. If the system equations as below

$$2p+2q=6$$
$$3p+2q=7,$$

what is the value of p?

(A) 1
(B) 2
(C) 3
(D) 4

Solution: Answer: (A)

$$\begin{cases} 2p+2q=6 \quad (1) \\ 3p+2q=7 \quad (2) \end{cases}$$

$$(2)-(1)$$

$$\Downarrow$$

$$p=1$$

3. If the equations of 2 lines are $y=4x+3$ and $y=3x+4$, respectively, and the two lines intersect, what is the absolute value of the difference of the x-coordinate and y-coordinate?

(A) 3
(B) 6
(C) 5
(D) 4

Solution: Answer: (B)

At the intersection point, $y=y$.

$$\downarrow$$

$$4x+3=3x+4$$

$$x=1$$

By $y=4x+3$, $y=7$

$$\Downarrow$$

$$|x-y|=|1-7|=6$$

4. If the graph of the equation $ay^2+by+c=x$ passes through points (-3, 5), (7, 1), and (6, 2), what is the value of $a+b+c$?

(A) 8
(B) 7
(C) 9
(D) $\frac{3}{2}$

Solution: Answer: (B)

Because each point of the 3 points is on the graph, x and y values of each of the points should satisfy the equation $ay^2+by+c=x$.

The y value of the point $(7,1)=1$. Then

$$ay^2+by+c=x$$

$$\downarrow$$

$$a\cdot 1^2+b\cdot 1+c=7$$

$$\Downarrow$$

$$a+b+c=7$$

The fish and beef weighed 5 pounds.
The fish and chicken weighed 6 pounds.
The beef and chicken weighed 7 pounds.

5. By the information above, what is the weight of the chicken?

(A) 4 Pounds
(B) 7 Pounds
(C) 9 Pounds
(D) $\frac{5}{2}$ Pounds

Solution: Answer: (A)

$$\begin{cases} f+b=5 \ (1) \\ f+c=6 \ (2) \\ b+c=7 \ (3) \end{cases}$$

The question asks c.

$$\downarrow$$

$$\begin{cases} \text{The target is } c. \\ \text{Cancel } b \text{ and } f. \end{cases}$$

$$(2)-(1)$$

$$-b+c=1 \ (4)$$

$$(4)+(3)$$

$$\Downarrow$$

$$2c=8,\ \boxed{c=4}$$

$$\begin{cases} x = z^2 \\ x = \sqrt{y} \\ y = 8z \end{cases}$$

6. In the group of equations above, if $x > 0$, which of the following is the value of x ?

(A) 2

(B) 6

(C) 3

(D) 4

Solution: Answer: (D)

The question asks x-value.

↓

Cancel y and z.

Cannot find easier way to do elimination.

↓

Use the common substitution method.

$$\begin{cases} x = z^2 \\ x = \sqrt{y} \\ y = 8z \quad (1) \end{cases} \rightarrow \begin{cases} \pm\sqrt{x} = z \quad (2) \\ x^2 = y \quad (3) \end{cases}$$

↓

(2), (3) → (1)

↓

$$x^2 = 8(\pm\sqrt{x})$$

↓

$$\left(x^2\right)^2 = \left(8(\pm\sqrt{x})\right)^2$$

↓

$$x^4 = 64x$$

$$x > 0$$

⇓

$$x^3 = 64, \quad x = 4$$

$$\begin{cases} x = z^2 \\ x = \sqrt{y} \\ y = 9z \end{cases}$$

7. In the system of equations above, if $x > 0$, what is the value of x ?

(A) 9

(B) $3\sqrt[3]{3}$

(C) 3

(D) 4

Solution: Answer: (B)

The question asks x-value.

↓

Cancel y and z.

Cannot find easier way to do elimination.

↓

Use the common substitution method.

$$\begin{cases} x = z^2 \\ x = \sqrt{y} \\ y = 9z \quad (1) \end{cases} \rightarrow \begin{cases} \pm\sqrt{x} = z \quad (2) \\ x^2 = y \quad (3) \end{cases}$$

↓

(2), (3) → (1)

↓

$$x^2 = 9(\pm\sqrt{x})$$

↓

$$\left(x^2\right)^2 = \left(9(\pm\sqrt{x})\right)^2$$

↓

$$x^4 = 81x$$

$$x > 0$$

↓

$$x^3 = 81, \quad x = \sqrt[3]{81} = \sqrt[3]{3\times3\times3\times3}$$

⇓

$$\sqrt[3]{3\times3\times3}\cdot\sqrt[3]{3} = 3\sqrt[3]{3}$$

The fish and beef weighed 6 pounds.
The fish and chicken weighed 7 pounds.
The beef and chicken weighed 8 pounds.

1. By the information above, what is the weight of the beef?

(A) $\frac{9}{2}$ Pounds
(B) 7 Pounds
(C) $\frac{7}{2}$ Pounds
(D) $\frac{5}{2}$ Pounds

2. If the system equations as below

$$2p + 2q = 6$$
$$3p + 2q = 7,$$

what is the value of p?

(A) 1
(B) 2
(C) 3
(D) 4

3. If the equations of 2 lines are $y = 4x + 3$ and $y = 3x + 4$, respectively, and the two lines intersect, what is the absolute value of the difference of the x-coordinate and y-coordinate?

(A) 3
(B) 6
(C) 5
(D) 4

4. If the graph of the equation $ay^2 + by + c = x$ passes through points (-3, 5), (7, 1), and (6, 2), what is the value of $a + b + c$?

(A) 8
(B) 7
(C) 9
(D) $\frac{3}{2}$

The fish and beef weighed 5 pounds.
The fish and chicken weighed 6 pounds.
The beef and chicken weighed 7 pounds.

5. By the information above, what is the weight of the chicken?

(A) 4 Pounds
(B) 7 Pounds
(C) 9 Pounds
(D) $\frac{5}{2}$ Pounds

$$\begin{cases} x = z^2 \\ x = \sqrt{y} \\ y = 8z \end{cases}$$

6. In the group of equations above, if $x > 0$, which of the following is the value of x ?

(A) 2

(B) 6

(C) 3

(D) 4

$$\begin{cases} x = z^2 \\ x = \sqrt{y} \\ y = 9z \end{cases}$$

7. In the system of equations above, if $x > 0$, what is the value of x ?

(A) 9

(B) $3\sqrt[3]{3}$

(C) 3

(D) 4

Geometry

And

Measurement

Geometry and Measurement

1. In the xy-plane, the distance between points P (3, 12) and point Q (b, 5) is $\sqrt{74}$. What is one possible value of b?

Solution: Answer: 8

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$\downarrow$$
$$74 = (3-b)^2 + 49$$
$$25 = (3-b)^2, \quad \pm 5 = 3 - b$$
$$\downarrow$$
$$b = 3 \pm 5$$
$$\Downarrow$$
$$b = -2 \quad , \quad b = \boxed{8}$$

No space for negative value

2. In the xy-plane, the distance between points Q (k, 12) and point P (3, 5) is $\sqrt{50}$. What is one possible value of k?

Solution: Answer: 2 or 4

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$\downarrow$$
$$50 = (3-k)^2 + 49$$
$$1 = (3-k)^2, \quad \pm 1 = 3 - k$$
$$\downarrow$$
$$k = 3 \pm 1$$
$$\Downarrow$$
$$k = \boxed{2}, \quad k = \boxed{4}$$

3. In the xy-plane, the distance between points Q (3, t) and point R (2, 5) is $\sqrt{37}$. What is one possible value of t?

Solution: Answer: 11

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$\downarrow$$
$$37 = 1^2 + (t-5)^2$$
$$\downarrow$$
$$36 = (t-5)^2$$
$$\downarrow$$
$$\pm 6 = t - 5, \quad t = \pm 6 + 5$$
$$\Downarrow$$
$$t = -1 \quad , \quad t = \boxed{11}$$

No space for negative value

4. In the xy-plane, the distance between points R (1, 12) and point Q (4, h) is 13. What is one possible value of h?

(A) $4\sqrt{10} - 12$

(B) $12 + 4\sqrt{10}$

(C) $\sqrt{160} - 12$

(D) $12 + 2\sqrt{10}$

Solution: Answer: (B)

$$169 = 9 + (12-h)^2, \quad 160 = (12-h)^2$$
$$\downarrow$$
$$\pm\sqrt{160} = 12 - h, \quad h = 12 \pm \sqrt{4^2 \times 10}$$
$$\Downarrow$$
$$h = 12 \pm 4\sqrt{10}$$

Distance Formula

1. In the xy-plane, the distance between points P (3, 12) and point Q (b, 5) is $\sqrt{74}$. What is one possible value of b?

2. In the xy-plane, the distance between points Q (k, 12) and point P (3, 5) is $\sqrt{50}$. What is one possible value of k?

3. In the xy-plane, the distance between points Q (3, t) and point R (2, 5) is $\sqrt{37}$. What is one possible value of t?

4. In the xy-plane, the distance between points R (1, 12) and point Q (4, h) is 13. What is one possible value of h?

(A) $4\sqrt{10} - 12$

(B) $12 + 4\sqrt{10}$

(C) $\sqrt{160} - 12$

(D) $12 + 2\sqrt{10}$

1. If segment PQ in the xy-coordinate plane has endpoints P (4, 6) and Q (8, 10), then midpoint of $\overline{PQ}$ has coordinates

(A) (7, 7)
(B) (6, 8)
(C) (2, 8)
(D) (8, 6)

Solution: Answer: (B)

$$M = \left(\frac{x_1 + x_2}{2} = x_m, \ \frac{y_1 + y_2}{2} = y_m \right)$$

$$\downarrow$$

$$M = \left(\frac{4+8}{2} = 6, \ \frac{6+10}{2} = 8 \right)$$

$$\Downarrow$$

$$M = (6, \ 8)$$

2. If segment PQ in the xy-coordinate plane has endpoints P (-4, 6) and Q (8, 10), then midpoint of $\overline{PQ}$ has coordinates

(A) (7, 7)
(B) (6, 8)
(C) (2, 8)
(D) (8, 6)

Solution: Answer: (C)

$$M = \left(\frac{x_1 + x_2}{2} = x_m, \ \frac{y_1 + y_2}{2} = y_m \right)$$

$$\Downarrow$$

$$M = \left(\frac{-4+8}{2} = 2, \ \frac{6+10}{2} = 8 \right)$$

3. If segment PQ in the xy-coordinate plane has endpoint P (-8, 6) and midpoint (4, 10), then endpoint Q of $\overline{PQ}$ has coordinates
(A) (0, 26)
(B) (0, 14)
(C) (12, 8)
(D) (16, 14)

Solution: Answer: (D)

$$M = \left(\frac{x_1 + x_2}{2} = x_m, \ \frac{y_1 + y_2}{2} = y_m \right)$$

$$M = \left(\frac{-8 + x_2}{2} = 4, \ \frac{6 + y_2}{2} = 10 \right)$$

$$\downarrow$$

$$x_2 = 8 + 8 \qquad y_2 = 20 - 6$$

$$\Downarrow$$

$$x_2 = 16 \qquad y_2 = 14$$

4. If segment PQ in the xy- coordinate plane has endpoint P (-8, 6) and midpoint (-4, 10), then endpoint Q of $\overline{PQ}$ has coordinates

(A) (0, 26)
(B) (0, 14)
(C) (12, 8)
(D) (16, 14)

Solution: Answer: (B)

$$M = \left(\frac{x_1 + x_2}{2} = x_m, \ \frac{y_1 + y_2}{2} = y_m \right)$$

$$\downarrow$$

$$M = \left(\frac{-8 + x_2}{2} = 4, \ \frac{6 + y_2}{2} = 10 \right)$$

$$\downarrow$$

$$x_2 = -8 + 8 \qquad y_2 = 20 - 6$$

$$\Downarrow$$

$$x_2 = 0 \qquad y_2 = 14$$

5. In the xy-coordinate plane, the point (5, 4) is the midpoint of the line segment with endpoints $(2, y)$ and $(x, 5)$, what is the value of xy?

(A) 20
(B) 24
(C) 22
(D) 16

Solution: Answer: (B)

$$M = \left(\frac{x_1 + x_2}{2} = x_m, \ \frac{y_1 + y_2}{2} = y_m \right)$$

$$\downarrow$$

$$M = \left(\frac{2 + x}{2} = 5, \ \frac{y + 5}{2} = 4 \right)$$

$$\downarrow$$

$$x = 8, \ y = 3$$

$$\Downarrow$$

$$xy = 8 \cdot 3 = 24$$

Midpoint Formula

1. If segment PQ in the xy-coordinate plane has endpoints P (4, 6) and Q (8, 10), then midpoint of $\overline{PQ}$ has coordinates

(A) (7, 7)
(B) (6, 8)
(C) (2, 8)
(D) (8, 6)

2. If segment PQ in the xy-coordinate plane has endpoints P (-4, 6) and Q (8, 10), then midpoint of $\overline{PQ}$ has coordinates

(A) (7, 7)
(B) (6, 8)
(C) (2, 8)
(D) (8, 6)

3. If segment PQ in the xy-coordinate plane has endpoint P (-8, 6) and midpoint (4, 10), then endpoint Q of $\overline{PQ}$ has coordinates

(A) (0, 26)
(B) (0, 14)
(C) (12, 8)
(D) (16, 14)

4. If segment PQ in the xy- coordinate plane has endpoint P (-8, 6) and midpoint (-4, 10), then endpoint Q of $\overline{PQ}$ has coordinates

(A) (0, 26)
(B) (0, 14)
(C) (12, 8)
(D) (16, 14)

5. In the xy-coordinate plane, the point (5, 4) is the midpoint of the line segment with endpoints $(2, y)$ and $(x, 5)$, what is the value of xy?

(A) 20
(B) 24
(C) 22
(D) 16

Legend :

Before reflecting, $y = s_1x + b_1$

After reflecting, $y = s_2x + b_2$

s_1 and s_2 represent slopes; b_1 and b_2 represent y-intercepts.

$s_2 = -s_1$

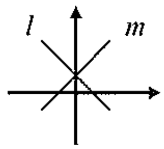
$b_2 = b_1$ if reflecting about y-axis

$b_2 = -b_1$ if reflecting about x-axis

1. In the xy-coordinate plane, the equation of line m is $y = x + 1$. If line l is the reflection of line with respect to the y-axis, what is the equation of line l?

(A) $y = -x + 1$
(B) $y = -x - 1$
(C) $y = x + 1$
(D) $y = x - 2$

Solution: Answer: (A)



(1) $m_2 = -m_1$, $m_1 = 1$

$\downarrow$

$m_2 = -1$

(2) Reflecting about y-axis

$\downarrow$

$b_2 = b_1 = 1$

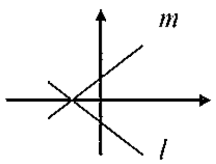
$\Downarrow$

$y = m_2x + b_2 = -x + 1$

2. In the xy-coordinate plane, the equation of line m is $y = x + 1$. If line l is the reflection of line m with respect to the x-axis, what is the equation of line l?

(A) $y = -x - 1$
(B) $y = -x + 1$
(C) $x = y + 1$
(D) $y = x - 2$

Solution: Answer: (A)



(1) $m_2 = -m_1$, $m_1 = 1$

$\downarrow$

$m_2 = -1$

(2) Reflecting about x-axis

$\downarrow$

$b_2 = -b_1 = -1$

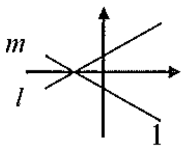
$\Downarrow$

$y = m_2x + b_2 = -x - 1$

3. In the xy-coordinate plane, the equation of line m is $y = \left(-\frac{1}{3}\right)x - 1$. If line l is the reflection of line m with respect to the x-axis, what is the equation of line l?

(A) $y = 3x + 1$
(B) $x = 3(y - 1)$
(C) $y = -\frac{1}{3}x + 1$
(D) $y = \frac{1}{3}x - 1$

Solution: Answer: (B)



(1) $m_2 = -m_1$, $m_1 = -\frac{1}{3} \rightarrow m_2 = \frac{1}{3}$

(2) Reflecting in x – axis $\rightarrow$ $b_1 = -1$, $b_2 = 1$

$\downarrow$

(3) $y = m_2x + b_2 = \frac{1}{3}x + 1$

But, you may not directly find out the matching one from the five choices now .

(4) Try to get the format $x =$ expression with y

$y = \frac{1}{3}x + 1 \rightarrow 3y = x + 3$

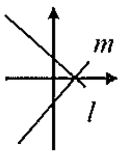
$\Downarrow$

(B) $x = 3y - 3 = 3(y - 1)$

4. In the xy-coordinate plane, the equation of line m is $y = \frac{1}{4}x - 1$. If line l is the reflection of line m with respect to the x-axis, what is the equation of line l?

(A) $y = -4x - 1$
(B) $x = -4(y + 1)$
(C) $x = -4y + 4$
(D) $y = \frac{1}{4}x + 1$

Solution: Answer: (C)



(1) $m_2 = -m_1$, $m_1 = \frac{1}{4}$, $\boxed{m_2 = -\frac{1}{4}}$

(2) Reflecting in x – axis

$b_1 = -1$, $\boxed{b_2 = 1}$

(1) and (2)

(3) $y = m_2x + b_2 = -\frac{1}{4}x + 1$

(4) Try to get the format $x =$ expression with y

$y = -\frac{1}{4}x + 1 \rightarrow 4y = -x + 4$

$\Downarrow$

(C) $x = -4y + 4$

Reflections in xy-Plane

Legend :

Before reflecting, $y = s_1x + b_1$

After reflecting, $y = s_2x + b_2$

s_1 and s_2 represent slopes; b_1 and b_2 represent y-intercepts.

$s_2 = -s_1$

$b_2 = b_1$ if reflecting about y-axis

$b_2 = -b_1$ if reflecting about x-axis

1. In the xy-coordinate plane, the equation of line m is $y = x + 1$. If line l is the reflection of line with respect to the y-axis, what is the equation of line l?

(A) $y = -x + 1$
(B) $y = -x - 1$
(C) $y = x + 1$
(D) $y = x - 2$

2. In the xy-coordinate plane, the equation of line m is $y = x + 1$. If line l is the reflection of line m with respect to the x-axis, what is the equation of line l?

(A) $y = -x - 1$
(B) $y = -x + 1$
(C) $x = y + 1$
(D) $y = x - 2$

3. In the xy-coordinate plane, the equation of line m is $y = \left(-\frac{1}{3}\right)x - 1$. If line l is the reflection of line m with respect to the x-axis, what is the equation of line l?

(A) $y = 3x + 1$
(B) $x = 3(y - 1)$
(C) $y = -\frac{1}{3}x + 1$
(D) $y = \frac{1}{3}x - 1$

4. In the xy-coordinate plane, the equation of line m is $y = \frac{1}{4}x - 1$. If line l is the reflection of line m with respect to the x-axis, what is the equation of line l?

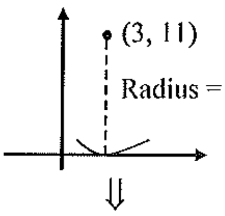
(A) $y = -4x - 1$
(B) $x = -4(y + 1)$
(C) $x = -4y + 4$
(D) $y = \frac{1}{4}x + 1$

1. In the xy-plane, the center of a circle has coordinates (3, 11), and the circle touches the x-axis at one point. What is the radius of the circle?

(A) 3
(B) 11
(C) 8
(D) 14

Solution: Answer: (B)

Method: Draw a sketch.



⇓

Answer is (B).

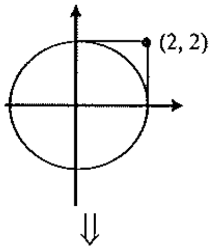
2. The area of a circle whose center is at (0, 0) is 4π. The circle passes through each of the following points except

(A) (0, 2)
(B) (2, 2)
(C) (0, -2)
(D) (-2, 0)

Solution: Answer: (B)

$A = r^2\pi,\ 4\pi = r^2\pi,\ r = +2$

Method: Draw a sketch.



⇓

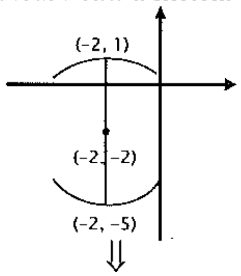
Answer is (B).

3. In the xy-plane, the center of a circle has coordinates (-2, -2).If one endpoint of a diameter of the circle is (-2, 1), what are the coordinates of the other end point of the diameter?

(A) (-4, -5)
(B) (-2, -4)
(C) (-4, -2)
(D) (-2, -5)

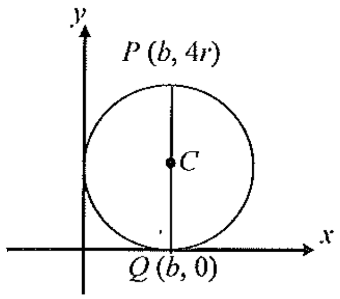
Solution: Answer: (D)

Method: Draw a sketch.



⇓

Answer is (D).



4. In the figure above, C is the center of the circle and lies on the segment PQ. What is the area of the circle?

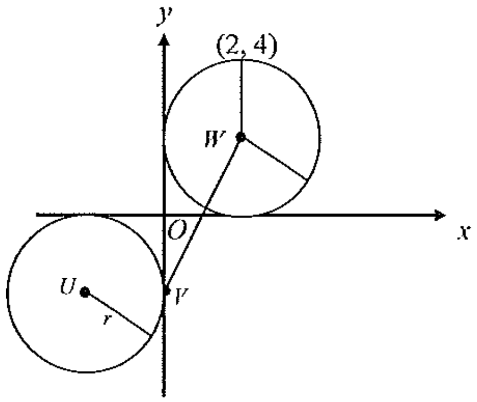
(A) $3r^2\pi$
(B) $4r^2\pi$
(C) $2r^2\pi$
(D) $9r^2\pi$

Solution: Answer: (B)

radius = $2r$

⇓

$A_{\bigcirc} = (2r)^2\pi = 4r^2\pi$



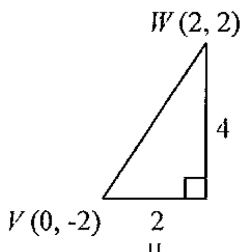
Note: Figure above not drawn to scale.

5. In the xy-plane above, points W and U are the centers of the two identical circles, which are both tangent to the x-axis and y-axis. V is the tangent point. What is the slope of $\overline{WV}$?

(A) 4
(B) 6
(C) 5
(D) 2

Solution: Answer: (D)

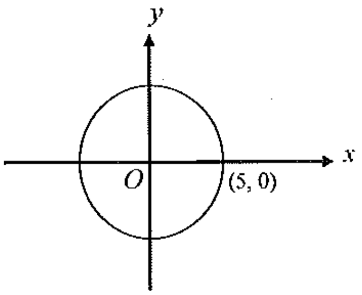
Get the value of radius first.



⇓

$\text{Slope}_{\overline{WV}} = \frac{4}{2} = 2$

Circles in xy-Plane

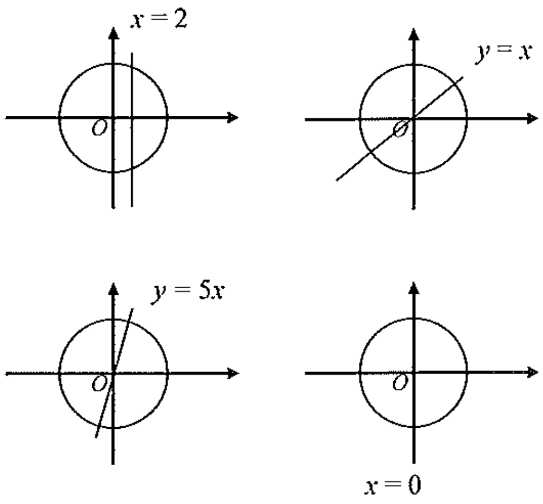


6. In the xy-coordinate plane above, the circle has center O. Which of the following lines divides the circle into 2 semicircles?

I. The line with equation $x = 2$
II. The line with equation $y = x$
III. The line with equation $y = 5x$
IV. The line with equation $x = 0$

(A) I only
(B) I and II only
(C) I, II, and III only
(D) II, III, and IV only

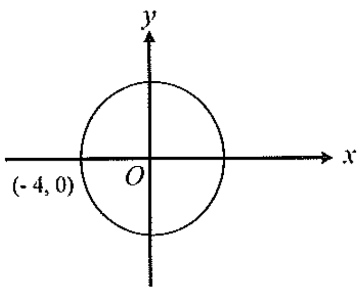
Solution: Answer: (D)



The graphs of equations of II, II, and IV pass through center of a circle, but I.

⇓

Answer = (D) II, III, and IV only

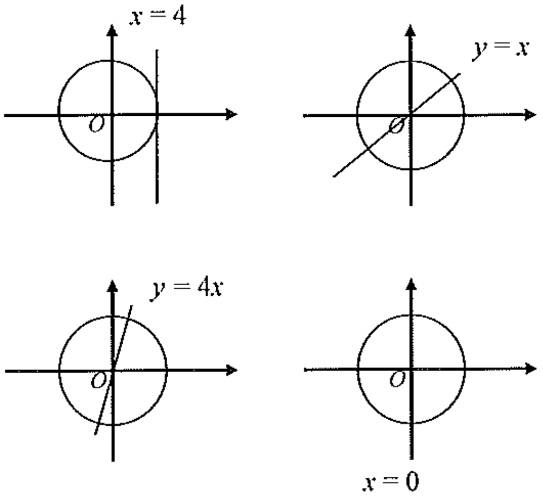


7. In the xy-coordinate plane above, the circle has center O. Which of the following lines divides the circle into 2 semicircles?

I. The line with equation $x = 4$
II. The line with equation $y = x$
III. The line with equation $y = 4x$
IV. The line with equation $y = 0$

(A) I only
(B) I and II only
(C) I, II, and III only
(D) II, III, and IV only

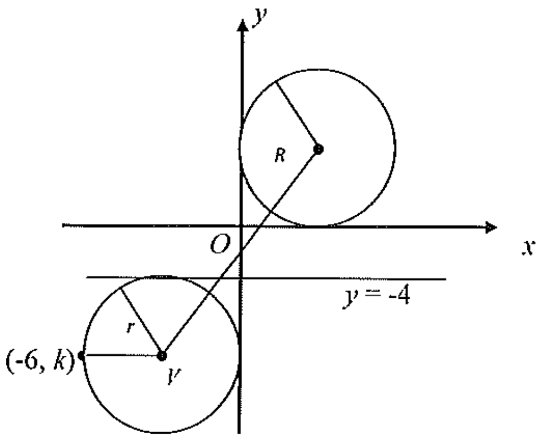
Solution: Answer: (D)



The graphs of equations of II, II, and IV pass through center of a circle, but I.

⇓

Answer = (D) II, III, and IV only



Note: Figure above not drawn to scale.

8. In the xy-plane above, points W and V are the centers of the identical circles, which are both tangent to the y-axis, one of them tangent to x-axis, and one tangent to line $y = -4$. What is the slope of $\overline{WV}$?

(A) -5

(B) $\frac{5}{3}$

(C) 3

(D) 2

Solution: Answer: (B)

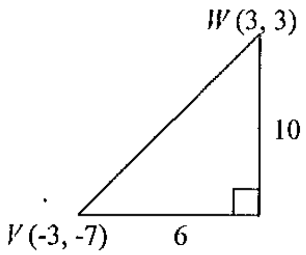
Get the value of radius first.

$x = -6$

$\downarrow$

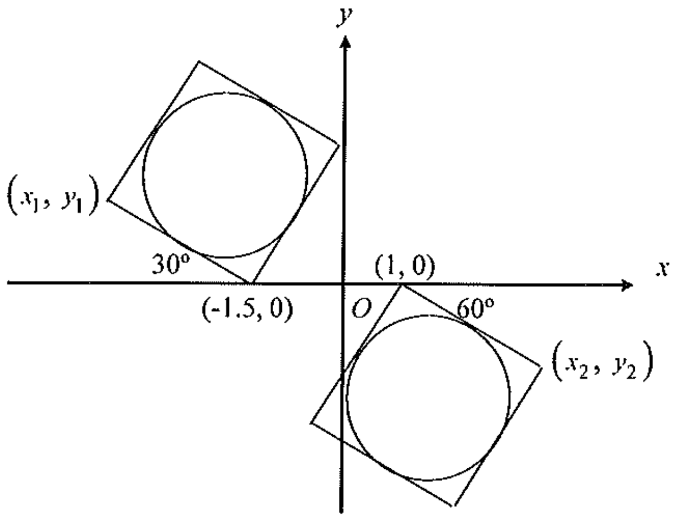
$r = R = 3$

$\downarrow$



$\Downarrow$

$$\text{Slope}_{\overline{WV}} = \frac{10}{6} = \frac{5}{3}$$



9. In the figure above, 2 identical circles with radius 3 are inscribed in 2 squares, respectively. Which of the following are the values of x_2 and y_1 ?

(A) $x_2 = 4,\ y_1 = -3$

(B) $x_2 = -4,\ y_1 = 3$

(C) $x_2 = 4\sqrt{3},\ y_1 = 3$

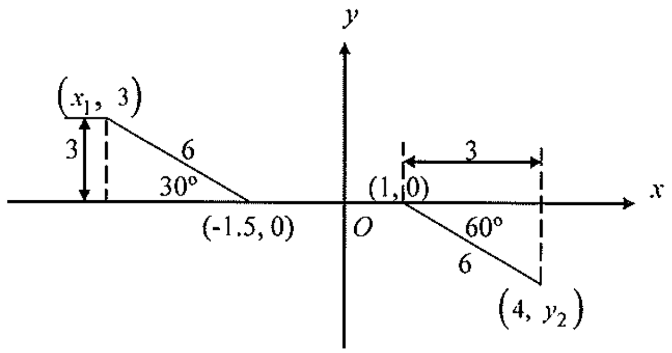
(D) $x_2 = 4,\ y_1 = 3$

Solution: Answer: (D)

Use the conclusion:

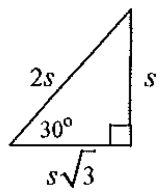
If a circle is inscribed in a square,
diameter of circle = side of square

$r = 3,\ d = 6$



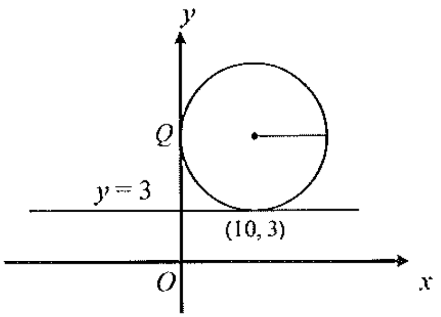
$\Downarrow$

$x_2 = 1 + 3 = 4,\quad y_1 = 3$



Circles in xy-Plane

Note: Figure below not drawn to scale.



10. In the xy-plane above, the circle is tangent to the line $y=3$ and y-axis. What are the coordinates (x, y) of point Q?

(A) (0, 10)
(B) (10, 0)
(C) (0, 3)
(D) (0, 13)

Solution: Answer: (D)

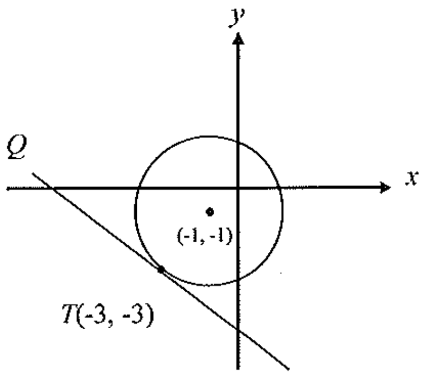
point (10, 3)

$\downarrow$

radius = 10

$\Downarrow$

Point $Q = (0, 13)$



11. In the xy-coordinate plane, $\overline{QR}$ is tangent to the circle at point T. If point (-1, -1) is the center of the circle, what is the slope of $\overline{QR}$?

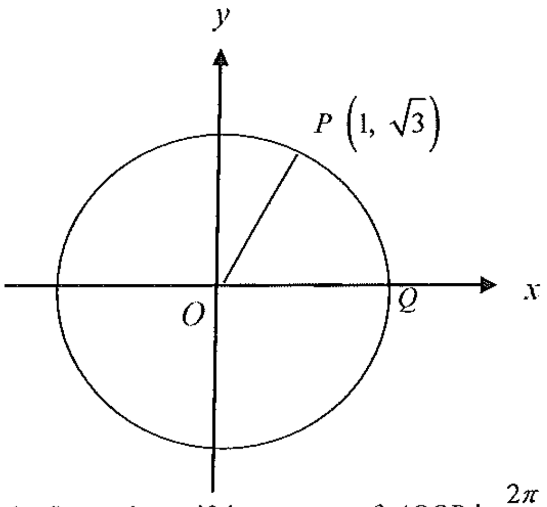
(A) 1
(B) 2
(C) -1
(D) $\frac{1}{2}$

Solution: Answer: (C)

$$S_1 = -\frac{-3-(-1)}{-3-(-1)} = \frac{-2}{-2} = 1$$

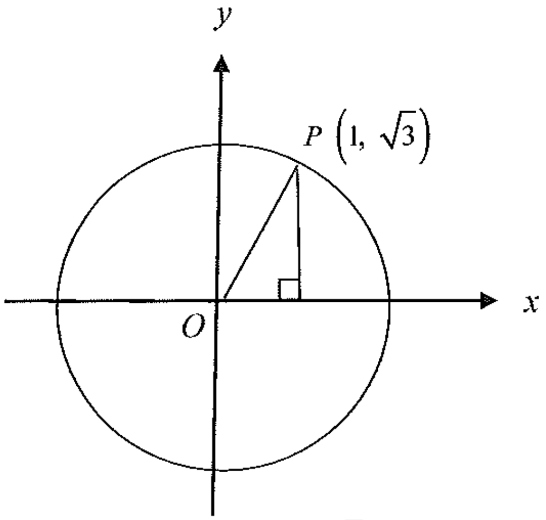
$\Downarrow$

$$S_2 = -\frac{1}{S_1} = -\frac{1}{1} = -1$$



12. In the figure above, if the measure of $\angle QOP$ is $\frac{2\pi}{x}$ radians, what is the value of x?

Solution: Answer: 6



$$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{1}$$

$\downarrow$

$$\theta = 60^\circ = 60^\circ \cdot \frac{\pi}{180} = \left(\frac{\pi}{30}\right)^r$$

$\downarrow$

$$\angle QOP = \theta, \quad \frac{\not{\pi}}{3} = \frac{2\not{\pi}}{x}, \quad \frac{3}{1} = \frac{x}{2}$$

$\Downarrow$

$$x = 6$$

1. In the xy-plane, the center of a circle has coordinates (3, 11), and the circle touches the x-axis at one point. What is the radius of the circle?

(A) 3

(B) 11

(C) 8

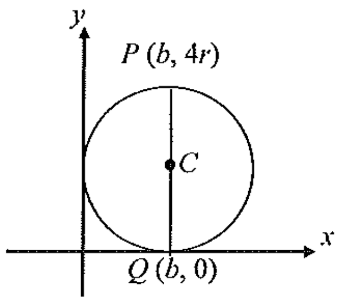
(D) 14

2. The area of a circle whose center is at (0, 0) is 4π. The circle passes through each of the following points except

(A) (0, 2)
(B) (2, 2)
(C) (0, -2)
(D) (-2, 0)

3. In the xy-plane, the center of a circle has coordinates (-2, -2).If one endpoint of a diameter of the circle is (-2, 1), what are the coordinates of the other end point of the diameter?

(A) (-4, -5)
(B) (-2, -4)
(C) (-4, -2)
(D) (-2, -5)



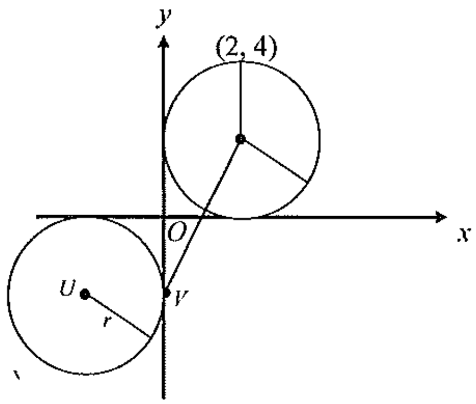
4. In the figure above, C is the center of the circle and lies on the segment PQ. What is the area of the circle?

(A) $3r^2\pi$

(B) $4r^2\pi$

(C) $2r^2\pi$

(D) $9r^2\pi$



Note: Figure above not drawn to scale.

5. In the xy-plane above, points W and U are the centers of the two identical circles, which are both tangent to the x-axis and y-axis. V is the tangent point. What is the slope of $\overline{WV}$?

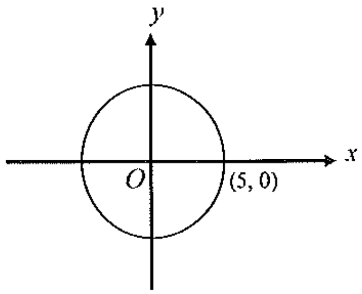
(A) 4

(B) 6

(C) 5

(D) 2

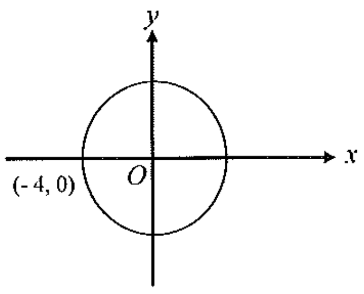
Circles in xy-Plane



6. In the xy-coordinate plane above, the circle has center O. Which of the following lines divides the circle into 2 semicircles?

I. The line with equation $x = 2$
II. The line with equation $y = x$
III. The line with equation $y = 5x$
IV. The line with equation $x = 0$

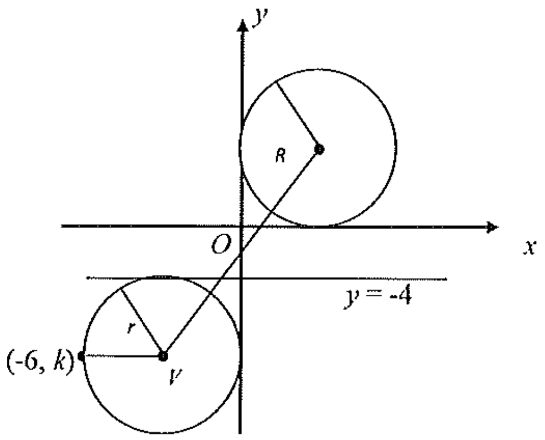
(A) I only
(B) I and II only
(C) I, II, and III only
(D) II, III, and IV only



7. In the xy-coordinate plane above, the circle has center O. Which of the following lines divides the circle into 2 semicircles?

I. The line with equation $x = 4$
II. The line with equation $y = x$
III. The line with equation $y = 4x$
IV. The line with equation $y = 0$

(A) I only
(B) I and II only
(C) I, II, and III only
(D) II, III, and IV only



Note: Figure above not drawn to scale.

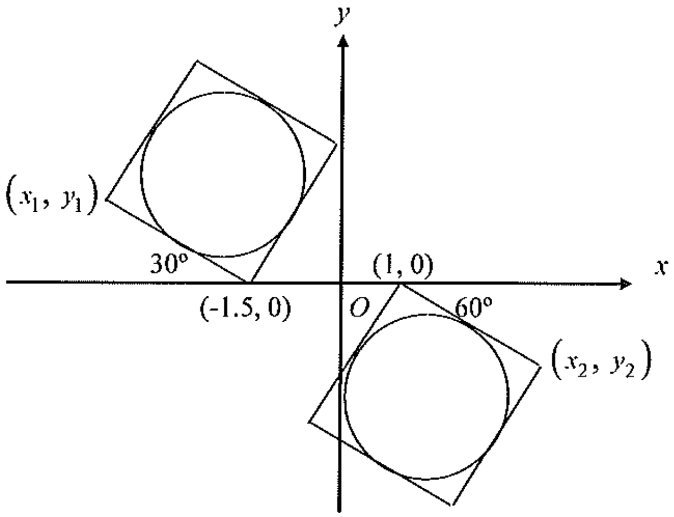
8. In the xy-plane above, points W and V are the centers of the identical circles, which are both tangent to the y-axis, one of them tangent to x-axis, and one tangent to line $y = -4$. What is the slope of $\overline{WV}$?

(A) -5

(B) $\frac{5}{3}$

(C) 3

(D) 2



9. In the figure above, 2 identical circles with radius 3 are inscribed in 2 squares, respectively. Which of the following are the values of x_2 and y_1 ?

(A) $x_2 = 4,\ y_1 = -3$

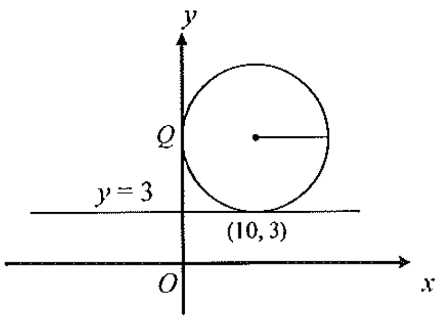
(B) $x_2 = -4,\ y_1 = 3$

(C) $x_2 = 4\sqrt{3},\ y_1 = 3$

(D) $x_2 = 4,\ y_1 = 3$

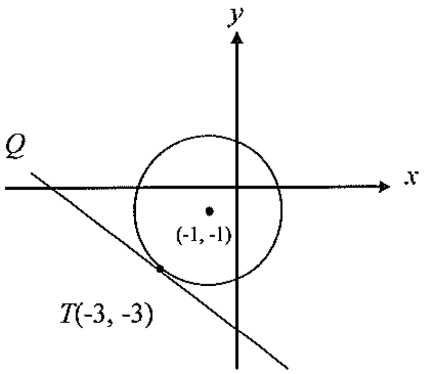
Circles in xy-Plane

Note: Figure below not drawn to scale.



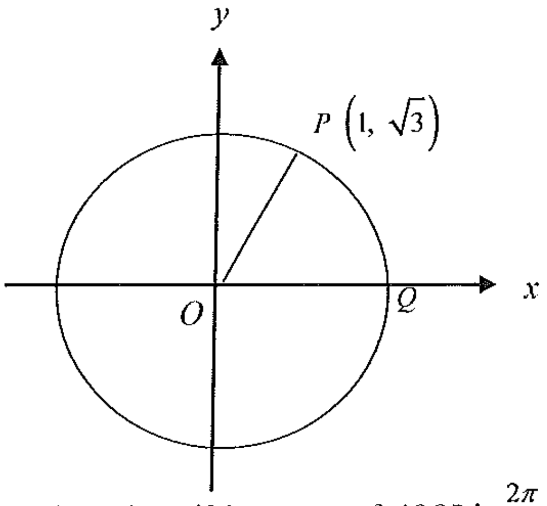
10. In the xy-plane above, the circle is tangent to the line $y=3$ and y-axis. What are the coordinates (x, y) of point Q?

(A) (0, 10)
(B) (10, 0)
(C) (0, 3)
(D) (0, 13)

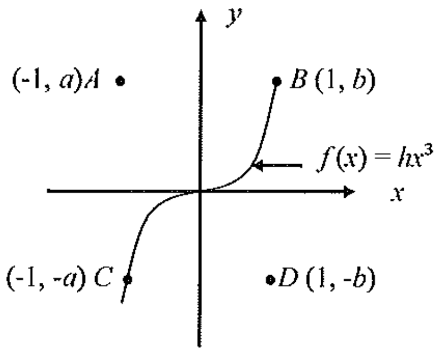


11. In the xy-coordinate plane, $\overline{QR}$ is tangent to the circle at point T. If point (-1, -1) is the center of the circle, what is the slope of $\overline{QR}$?

(A) 1
(B) 2
(C) -1
(D) $\frac{1}{2}$



12. In the figure above, if the measure of $\angle QOP$ is $\frac{2\pi}{x}$ radians, what is the value of x?



Note: Figure above not drawn to scale

1. In the figure above, A, B, C, and D are the points of the vertices of a rectangle (not shown). Points B and C lie on the curve of $f(x) = hx^3$, where h is a constant. If area of $ABCD$ is 10, what is the value of h?

(A) 5.5
(B) 4.5
(C) 3.5
(D) 2.5

Solution: Answer: (D)

$$A = 10 = L \cdot W = (1+1) \cdot 2b$$

$$10 = 2 \cdot 2b, \quad b = 2.5$$

$$\downarrow$$

$$\text{point } B = (1,\ 2.5)$$

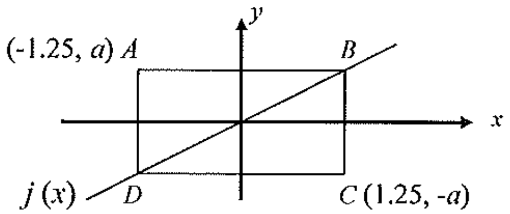
point B is on the graph of $y = hx^3$

$$\downarrow$$

$$y = 2.5,\ x = 1$$

$$\Downarrow$$

$$2.5 = h(1)^3,\ h = 2.5$$



Note: Figure above not drawn to scale

2. In the figure above, A, B, C, and D are the points of the vertices of the rectangle. Points D and B lie on the curve of $j(x) = px$, where p is a constant. If area of $ABCD$ is 10, what is the value of p?

Solution: Answer: 8/5 or 1.6

$ABCD$ is a rectangle. So, point $B = (1.25, a)$.

$$\text{Area} = L \cdot W,\ 10 = (1.25+1.25) \cdot 2a$$

$$10 = 2.5 \cdot 2a,\ a = 2$$

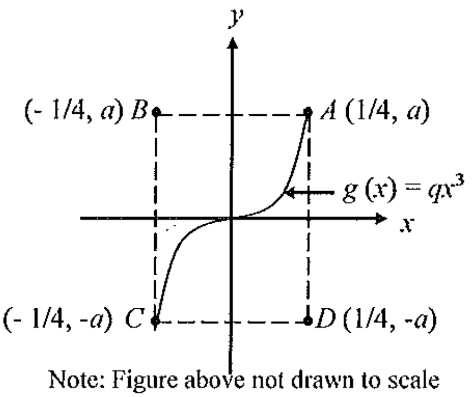
$$\downarrow$$

$$\text{Point } B = (1.25,\ 2)$$

Point B is on the graph of $y = px$

$$\Downarrow$$

$$2 = p \cdot (1.25), \qquad p = 1.6 \text{ or } 8/5$$



Note: Figure above not drawn to scale

3. In the figure above, A, B, C, and D are the points of the vertices of the rectangle. Points A and C lie on the curve of $g(x) = qx^3$, where q is a constant. If area of $ABCD$ is 1, what is the value of q?

Solution: Answer: 64

$$\text{Area} = L \cdot W = 1 = (1/4 + 1/4) \cdot (a + a)$$

$$1 = (1/2) \cdot 2a,\ a = 1$$

$$\downarrow$$

$$\text{point } A = (1/4,\ 1)$$

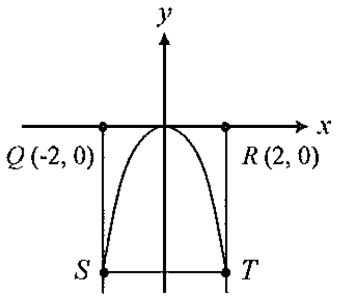
Point A is on the graph of $y = qx^3$.

$$\downarrow$$

$$y = 1, x = 1/4$$

$$\Downarrow$$

$$1 = q(1/4)^3,\ q = 64$$

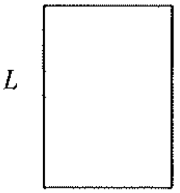


4. In the figure above, $SQRT$ is a rectangle, and points S and T lie on the graph $y = ax^2$, where a is a constant. If the perimeter of $SQRT$ is 20, what is the value of a?

(A) 1.5
(B) 3
(C) -1.5
(D) 2

Solution: Answer: (C)

Q (-2, 0) R (2, 0)



Perimeter $= 2W + 2L$

$20 = 2 \cdot 4 + 2L$

$L = 6$

Point T lies on the curve.

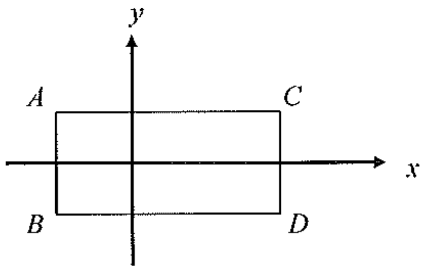
$y = ax^2,\ -6 = a2^2$

$\Downarrow$

$W = 4$ T (2, -6)

$\boxed{a = -1.5}$

Rectangles in xy-Plane



Note: Figure above not drawn to scale.

5. In the figure above, A, B, C, and D are the points of the vertices of arectangle, where $A=(A_1, A_2)$, $B=(B_1, B_2)$, $C=(C_1, C_2)$, and $D=(D_1, D_2)$. What is the area of the rectangle?

(A) $(A_1+B_1)(A_2+C_2)$
(B) $(B_1+|D_1|)(C_2+D_2)$
(C) $(|A_1|+C_1)(A_2+|D_2|)$
(D) $(C_2+D_2)(B_1+D_1)$

Solution: Answer: (C)

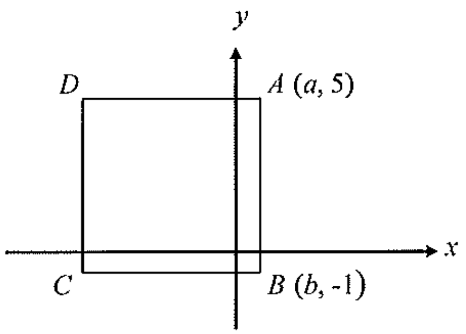
Area = A = Length · Width

$|A_1|+C_1$ = Width

$A_2+|D_2|$ = Length

$\Downarrow$

$A=(|A_1|+C_1)(A_2+|D_2|)$



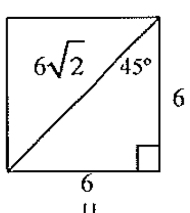
Note: Figure above not drawn to scale.

6. In the figure above, A, B, C and D are the points of vertices of the square. What is the diagonal of the square?

(A) $6\sqrt{2}$
(B) $5\sqrt{2}$
(C) $7\sqrt{3}$
(D) $5\sqrt{3}$

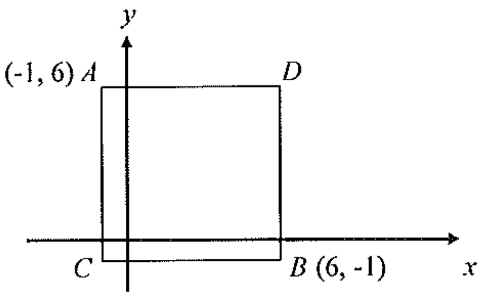
Solution: Answer: (A)

S = $5+|-1|=6$



$\Downarrow$

Answer is (A).



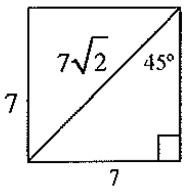
Note: Figure above not drawn to scale.

7. In the figure above, A, B, C and D are the points of vertices of the square. What is the diagonal of the square?

(A) $7\sqrt{2}$
(B) $5\sqrt{2}$
(C) $7\sqrt{3}$
(D) $5\sqrt{3}$

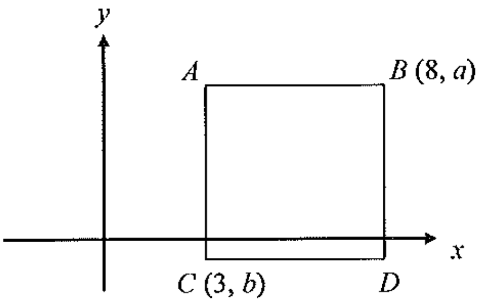
Solution: Answer: (A)

$6+|-1|=7$ or $|-1|+6=7$



$\Downarrow$

Answer is (A).



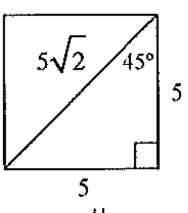
Note: Figure above not drawn to scale.

8. In the figure above, A, B, C and D are the points of vertices of the square. What is the diagonal of the square?

(A) $6\sqrt{2}$
(B) $5\sqrt{2}$
(C) $7\sqrt{3}$
(D) $5\sqrt{3}$

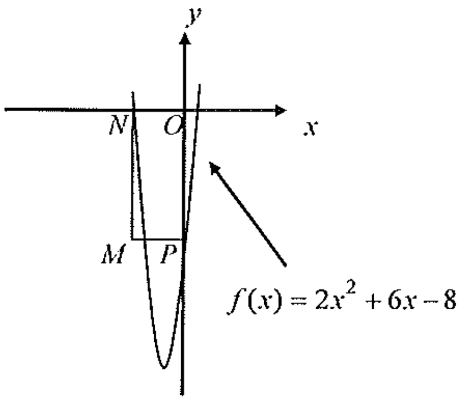
Solution: Answer: (B)

Side length = $8-3=5$



$\Downarrow$

Answer is (B).



9. In the figure above, the graph of $f(x)$ intersects the y-axis at point P and the x-axis at point N. What is the area of rectangle $MNOP$?

(A) 16

(B) 20

(C) 32

(D) 36

Solution: Answer: (C)

$$f(x) = y = 2x^2 + 6x - 8$$

(1) $\underbrace{\text{When } x = 0,\ y = -8}_{\text{for point } P}$.

(2) $\underbrace{\text{When } y = 0,\ 0 = 2x^2 + 6x - 8.}_{\text{for point } N}$

$$x^2 + 3x - 4 = 0$$

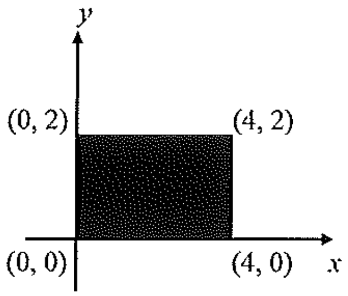
$$(x + 4)(x - 1) = 0$$

$x = 1$, $x = -4$

It is not on the rectangle.

$\Downarrow$

(3) $A_{MNOP} = |-8| \cdot |-4| = 32$



10. In the xy-plane above, if the shaded rectangle contains all points (x, y), what is the area of another rectangle (not shown) that contains all points $(3x, y + 1)$?

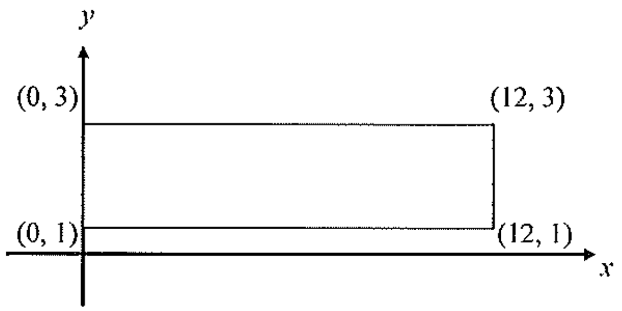
(A) 36

(B) 24

(C) 48

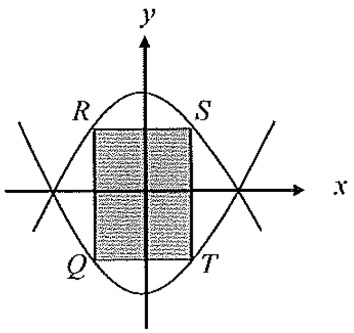
(D) 18

Solution: Answer: (B)



$$A = 12 \times (3 - 1) = 24$$

Rectangles in *xy*-Plane



Note: Figure above not drawn to scale.

11. The figure above shows two parabolas and one rectangle $QRST$. Points Q and T are on the parabola $y = \frac{1}{3}x^2 - 5$, and points R and S are on the parabola $y = -\frac{1}{5}x^2 + 6$. If point Q has coordinates (-2, -3), around what is the diagonal QS of rectangle $QRST$?

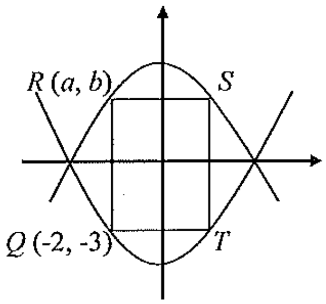
(A) 9

(B) 7

(C) 6

(D) 11

Solution: Answer: (A)



$$b = f(-2) = -\frac{1}{5}\cdot(-2)^2 + 6 = 5.2$$

$$QR = 5.2 + |-3| = 8.2$$

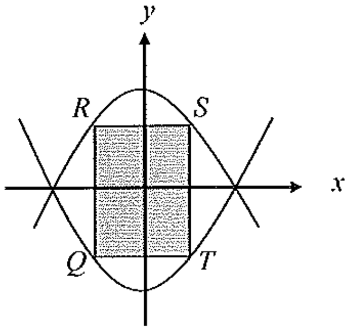
$$\begin{cases} -2 \text{ and } a \text{ both are on } x = -2. \\ a = -2 \end{cases}$$

$$\downarrow$$

$$RS = 2a = 2\cdot|-2| = 4$$

$$\Downarrow$$

$$QS = \sqrt{QR^2 + RS^2} = \sqrt{4^2 + 8.2^2} \approx 9$$



Note: Figure above not drawn to scale.

12. The figure above shows two parabolas and one rectangle $QRST$. Points Q and T are on the parabola $y = \frac{1}{2}x^2 - 5$, and points R and S are on the parabola $y = -\frac{1}{4}x^2 + 7$. If point T has coordinates (2, -3), what is the perimter of rectangle $QRST$?

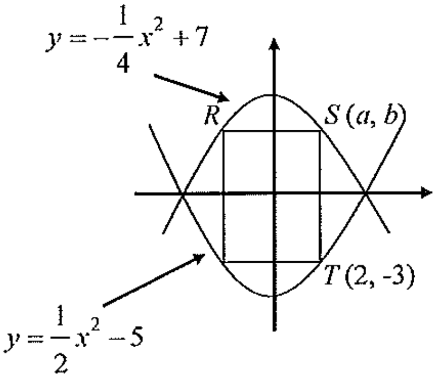
(A) 36

(B) 31

(C) 62

(D) 26

Solution: Answer: (D)



$$b = f(2) = -\frac{1}{4}\cdot 2^2 + 7 = 6$$

$$ST = 6 + |-3| = 9$$

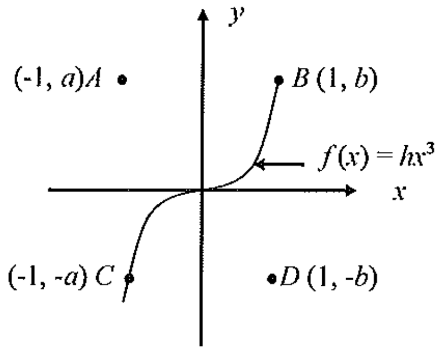
$$\begin{cases} 2 \text{ and } a \text{ both on } x = 2. \\ a = 2 \end{cases}$$

$$\downarrow$$

$$RS = 2a = 2\cdot 2 = 4$$

$$\Downarrow$$

$$P = 2\cdot(RS + ST) = 2\cdot(9 + 4) = 26$$



Note: Figure above not drawn to scale

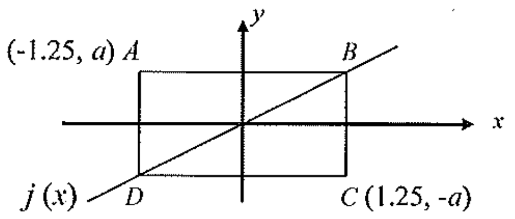
1. In the figure above, A, B, C, and D are the points of the vertices of a rectangle (not shown). Points B and C lie on the curve of $f(x) = hx^3$, where h is a constant. If area of $ABCD$ is 10, what is the value of h?

(A) 5.5

(B) 4.5

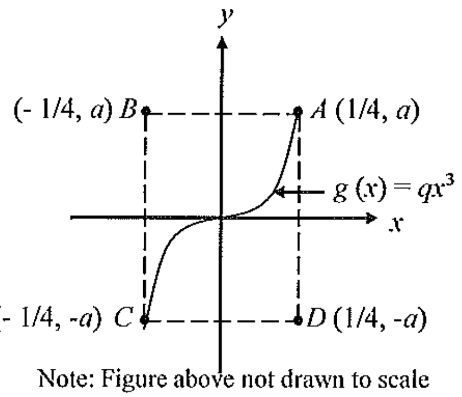
(C) 3.5

(D) 2.5



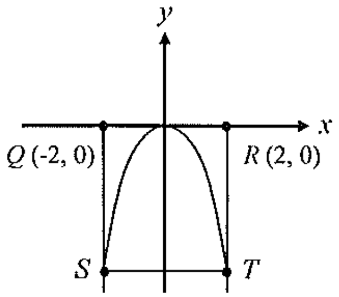
Note: Figure above not drawn to scale

2. In the figure above, A, B, C, and D are the points of the vertices of the rectangle. Points D and B lie on the curve of $j(x) = px$, where p is a constant. If area of $ABCD$ is 10, what is the value of p?



Note: Figure above not drawn to scale

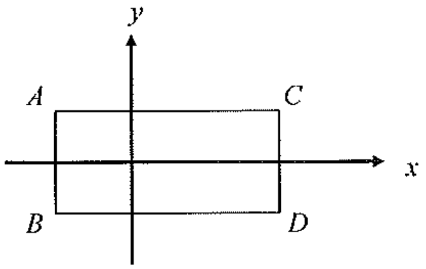
3. In the figure above, A, B, C, and D are the points of the vertices of the rectangle. Points A and C lie on the curve of $g(x) = qx^3$, where q is a constant. If area of $ABCD$ is 1, what is the value of q?



4. In the figure above, $SQRT$ is a rectangle, and points S and T lie on the graph $y = ax^2$, where a is a constant. If the perimeter of $SQRT$ is 20, what is the value of a?

(A) 1.5
(B) 3
(C) -1.5
(D) 2

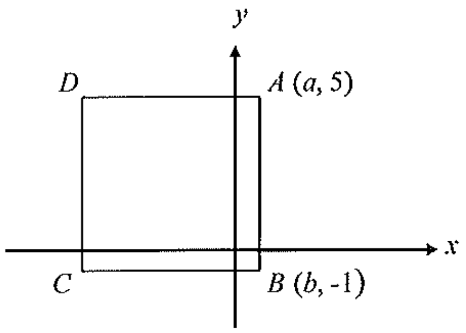
Rectangles in xy-Plane



Note: Figure above not drawn to scale.

5. In the figure above, A, B, C, and D are the points of the vertices of arectangle, where $A=(A_1, A_2)$, $B=(B_1, B_2)$, $C=(C_1, C_2)$, and $D=(D_1, D_2)$. What is the area of the rectangle?

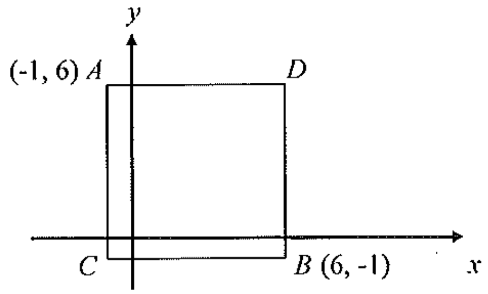
(A) $(A_1+B_1)(A_2+C_2)$
(B) $(B_1+|D_1|)(C_2+D_2)$
(C) $(|A_1|+C_1)(A_2+|D_2|)$
(D) $(C_2+D_2)(B_1+D_1)$



Note: Figure above not drawn to scale.

6. In the figure above, A, B, C and D are the points of vertices of the square. What is the diagonal of the square?

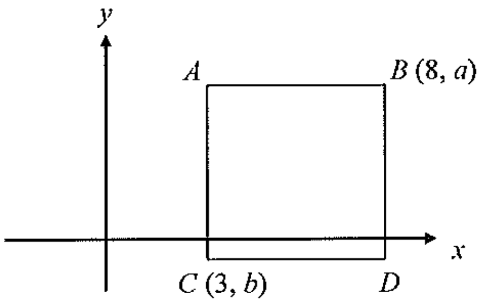
(A) $6\sqrt{2}$
(B) $5\sqrt{2}$
(C) $7\sqrt{3}$
(D) $5\sqrt{3}$



Note: Figure above not drawn to scale.

7. In the figure above, A, B, C and D are the points of vertices of the square. What is the diagonal of the square?

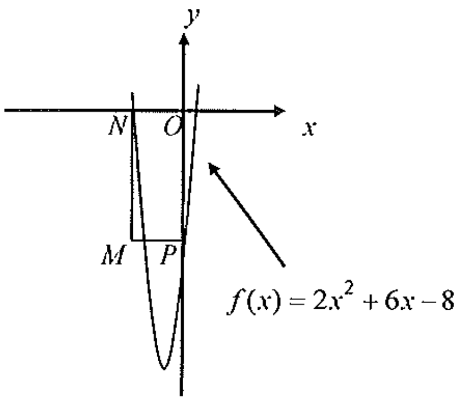
(A) $7\sqrt{2}$
(B) $5\sqrt{2}$
(C) $7\sqrt{3}$
(D) $5\sqrt{3}$



Note: Figure above not drawn to scale.

8. In the figure above, A, B, C and D are the points of vertices of the square. What is the diagonal of the square?

(A) $6\sqrt{2}$
(B) $5\sqrt{2}$
(C) $7\sqrt{3}$
(D) $5\sqrt{3}$



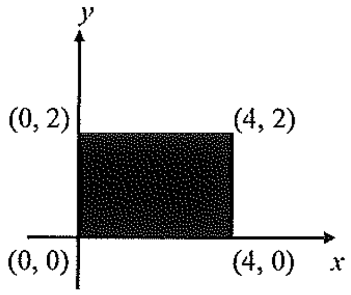
9. In the figure above, the graph of $f(x)$ intersects the y-axis at point P and the x-axis at point N. What is the area of rectangle $MNOP$?

(A) 16

(B) 20

(C) 32

(D) 36



10. In the xy-plane above, if the shaded rectangle contains all points (x, y), what is the area of another rectangle (not shown) that contains all points $(3x, y + 1)$?

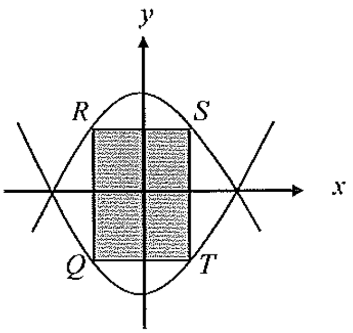
(A) 36

(B) 24

(C) 48

(D) 18

Rectangles in xy-Plane



Note: Figure above not drawn to scale.

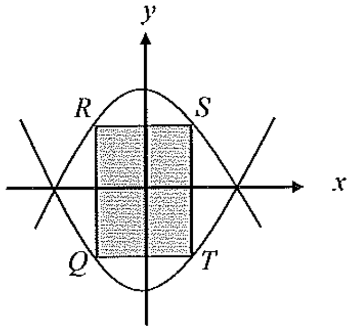
11. The figure above shows two parabolas and one rectangle $QRST$. Points Q and T are on the parabola $y = \frac{1}{3}x^2 - 5$, and points R and S are on the parabola $y = -\frac{1}{5}x^2 + 6$. If point Q has coordinates (-2, -3), around what is the diagonal QS of rectangle $QRST$?

(A) 9

(B) 7

(C) 6

(D) 11



Note: Figure above not drawn to scale.

12. The figure above shows two parabolas and one rectangle $QRST$. Points Q and T are on the parabola $y = \frac{1}{2}x^2 - 5$, and points R and S are on the parabola $y = -\frac{1}{4}x^2 + 7$. If point T has coordinates (2, -3), what is the perimter of rectangle $QRST$?

(A) 36

(B) 31

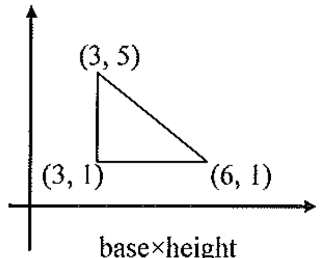
(C) 62

(D) 26

1. The vertices of a triangle are (3, 1), (3, 5), and (6, 1). The area of the triangle is

(A) 27
(B) 54
(C) 18
(D) 6

Solution: Answer: (D)



$$\text{Area} = \frac{\text{base}\times\text{height}}{2}$$

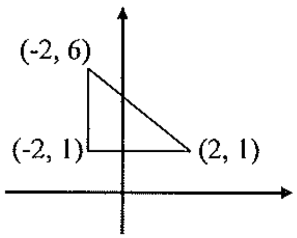
$$\Downarrow$$

$$\frac{(6-3)\cdot(5-1)}{2} = 6$$

2. The vertices of a triangle are (-2, 6), (-2, 1), and (2, 1). The area of the triangle is

(A) 14
(B) 54
(C) 10
(D) 12

Solution: Answer: (C)



$$\text{Area} = \frac{\text{base}\times\text{height}}{2}$$

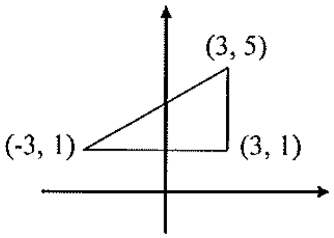
$$\Downarrow$$

$$\frac{(2+2)\cdot(6-1)}{2} = 10$$

3. The vertices of a triangle are (-3, 1), (3, 5), and (3, 1). The area of the triangle is

(A) 12
(B) 18
(C) 22
(D) 16

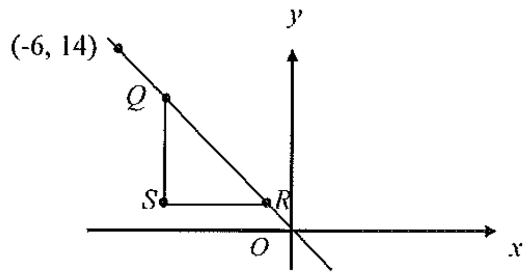
Solution: Answer: (A)



$$\text{Area} = \frac{\text{base}\times\text{height}}{2}$$

$$\Downarrow$$

$$\frac{(3+3)\cdot(5-1)}{2} = 12$$



Note: Figure above not drawn to scale.

4. In the figure above, if triangle RQS is a right triangle, which of the following could be the lengths of the sides of triangle RQS?

(A) 3, 7, and $\sqrt{49}$
(B) 6, 7, and $\sqrt{85}$
(C) 5, 6, and $\sqrt{61}$
(D) 7, 3, and $\sqrt{58}$

Solution: Answer: (D)

The line passes through point (0, 0).

$$\downarrow$$

$$\text{Slope} = -\frac{14}{6} = -\frac{7}{3}$$

Use plug-in method.

$$\downarrow$$

$$\text{Side}_{QS} = 7, \quad \text{Side}_{SR} = 3$$

$$\underbrace{\text{Side}_{QR} = \sqrt{7^2+3^2} = \sqrt{58}}_{\text{Use Pythagorean Theorem.}}$$

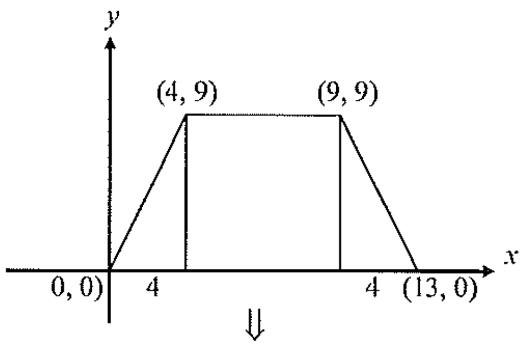
$$\Downarrow$$

Answer is (D).

5. If points (0, 0), (4, 9), and (13, 0) are the vertices of an isosceles trapezoid, which of the following is the rest vertex?

(A) (7, 9) (B) (6, -7) (C) (8, 6) (D) (9, 9)

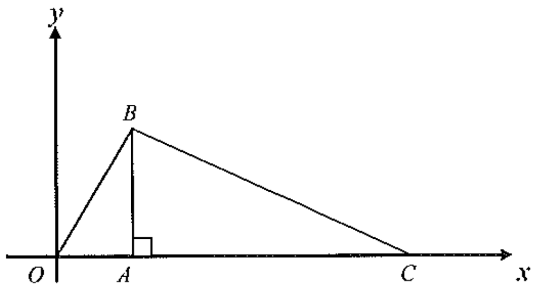
Solution: Answer: (D)



$$\Downarrow$$

x-coordinate of rest vertex $= 13 - 4 = 9$

Triangles in xy-Plane



Note: Figure above not drawn to scale.

6. In the coordinate plane, the area of triangle OBC is 30, the length of segment $\overline{AO}$ is 5, and the coordinates of C are (20, 0), what is the slope of segment $\overline{BO}$?

(A) $\frac{5}{3}$

(B) $-\frac{5}{3}$

(C) $-\frac{3}{5}$

(D) $\frac{3}{5}$

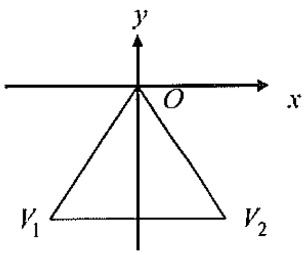
Solution: Answer: (D)

$$30 = \frac{20 \cdot AB}{2}, \quad AB = 3$$

$$\begin{cases} \text{Point } B \text{ touches the origin.} \\ \overline{BO} \text{ rises from left to right.} \end{cases}$$

$$\Downarrow$$

$$\text{Slope}_{\overline{BO}} = +\frac{3}{5}$$



7. In the xy-coordinate plane above, points V_1, O, and V_2 are the 3 vertices of a certain triangle. The coordinates of V_1 are (-2, b) and the coordinates of V_2 are (2, b). Which of the following is one possible value of b if the area of triangle V_1OV_2 is less than 7 and greater than 6 ?

(A) $-\frac{15}{4}$

(B) $-\frac{13}{4}$

(C) $-\frac{11}{4}$

(D) $-\frac{7}{2}$

Solution: Answer: (B)

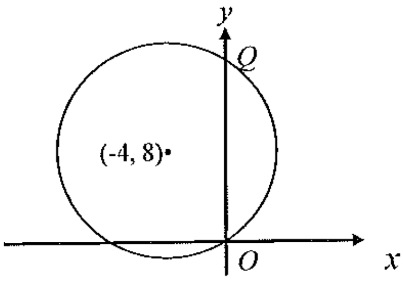
Base $2 + 2 = 4$ and height $|b|$

$$A = \frac{\text{Base} \times \text{height}}{2}, \quad 6 < \frac{4 \cdot |b|}{2} < 7$$

$$b < 0 \rightarrow \frac{6}{2} < -b < \frac{7}{2}$$

$$\Downarrow$$

$$-\frac{7}{2} < b < -\frac{6}{2}, \quad -\frac{14}{4} < b < -\frac{12}{4}$$



8. The circle in the xy-coordinate plane above has center $(-4, 8)$ and intersects the y-axis at point Q and at the origin. Which of the following are the coordinates at point Q?

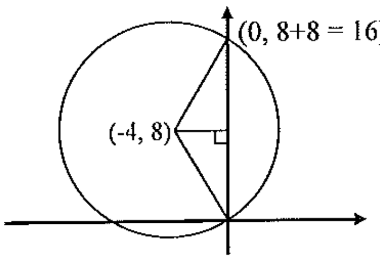
(A) (10, 0)

(B) (16, 0)

(C) (0, 16)

(D) (0, 9)

Solution: Answer: (C)



The 2 radii become the isosceles sides of the triangle.

The height is the bisector of the base of the triangle.

$$\Downarrow$$

$$Q = (0, 16)$$

1. The vertices of a triangle are (3, 1), (3, 5), and (6, 1). The area of the triangle is

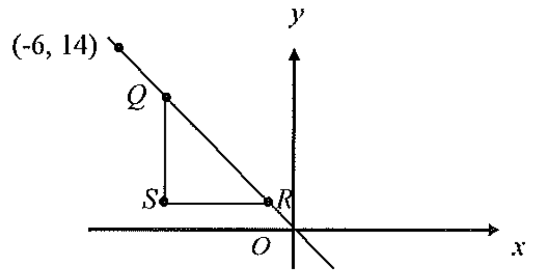
(A) 27
(B) 54
(C) 18
(D) 6

2. The vertices of a triangle are (-2, 6), (-2, 1), and (2, 1). The area of the triangle is

(A) 14
(B) 54
(C) 10
(D) 12

3. The vertices of a triangle are (-3, 1), (3, 5), and (3, 1). The area of the triangle is

(A) 12
(B) 18
(C) 22
(D) 16



Note: Figure above not drawn to scale.

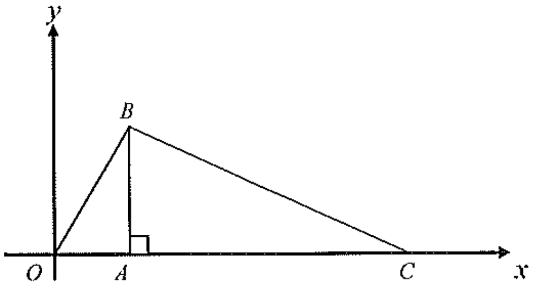
4. In the figure above, if triangle RQS is a right triangle, which of the following could be the lengths of the sides of triangle RQS?

(A) 3, 7, and $\sqrt{49}$
(B) 6, 7, and $\sqrt{85}$
(C) 5, 6, and $\sqrt{61}$
(D) 7, 3, and $\sqrt{58}$

5. If points (0, 0), (4, 9), and (13, 0) are the vertices of an isosceles trapezoid, which of the following is the rest vertex?

(A) (7, 9) (B) (6, -7) (C) (8, 6) (D) (9, 9)

Triangles in xy-Plane



Note: Figure above not drawn to scale.

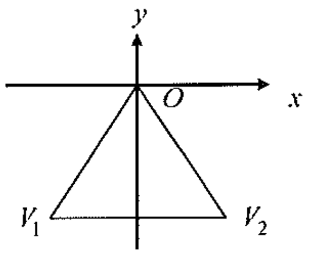
6. In the coordinate plane, the area of triangle OBC is 30, the length of segment $\overline{AO}$ is 5, and the coordinates of C are (20, 0), what is the slope of segment $\overline{BO}$?

(A) $\frac{5}{3}$

(B) $-\frac{5}{3}$

(C) $-\frac{3}{5}$

(D) $\frac{3}{5}$



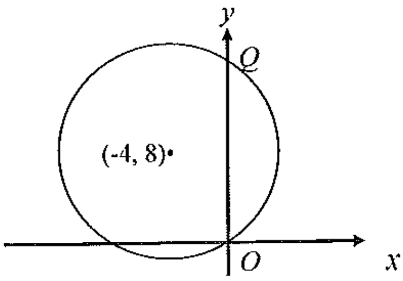
7. In the xy-coordinate plane above, points V_1, O, and V_2 are the 3 vertices of a certain triangle. The coordinates of V_1 are $(-2, b)$ and the coordinates of V_2 are $(2, b)$. Which of the following is one possible value of b if the area of triangle V_1OV_2 is less than 7 and greater than 6 ?

(A) $-\frac{15}{4}$

(B) $-\frac{13}{4}$

(C) $-\frac{11}{4}$

(D) $-\frac{7}{2}$



8. The circle in the xy-coordinate plane above has center $(-4, 8)$ and intersects the y-axis at point Q and at the origin. Which of the following are the coordinates at point Q?

(A) (10, 0)

(B) (16, 0)

(C) (0, 16)

(D) (0, 9)

1. The vertices of a triangle are (3, 1), (3, 5), and (6, 1). The area of the triangle is

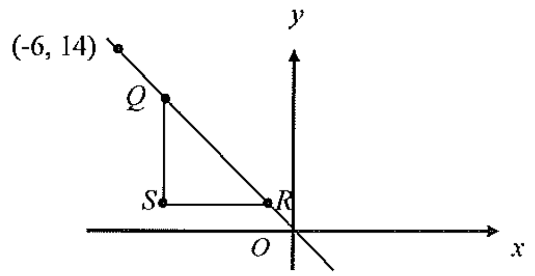
(A) 27
(B) 54
(C) 18
(D) 6

2. The vertices of a triangle are (-2, 6), (-2, 1), and (2, 1). The area of the triangle is

(A) 14
(B) 54
(C) 10
(D) 12

3. The vertices of a triangle are (-3, 1), (3, 5), and (3, 1). The area of the triangle is

(A) 12
(B) 18
(C) 22
(D) 16



Note: Figure above not drawn to scale.

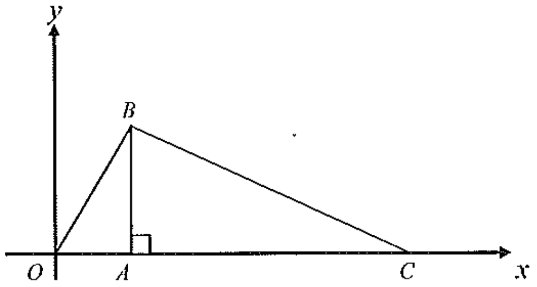
4. In the figure above, if triangle *RQS* is a right triangle, which of the following could be the lengths of the sides of triangle *RQS*?

(A) 3, 7, and $\sqrt{49}$
(B) 6, 7, and $\sqrt{85}$
(C) 5, 6, and $\sqrt{61}$
(D) 7, 3, and $\sqrt{58}$

5. If points (0, 0), (4, 9), and (13, 0) are the vertices of an isosceles trapezoid, which of the following is the rest vertex?

(A) (7, 9) (B) (6, -7) (C) (8, 6) (D) (9, 9)

Triangles in xy-Plane



Note: Figure above not drawn to scale.

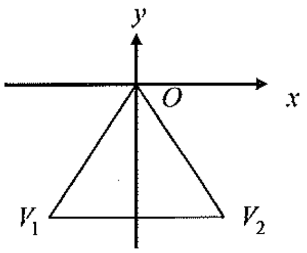
6. In the coordinate plane, the area of triangle OBC is 30, the length of segment $\overline{AO}$ is 5, and the coordinates of C are (20, 0), what is the slope of segment $\overline{BO}$?

(A) $\frac{5}{3}$

(B) $-\frac{5}{3}$

(C) $-\frac{3}{5}$

(D) $\frac{3}{5}$



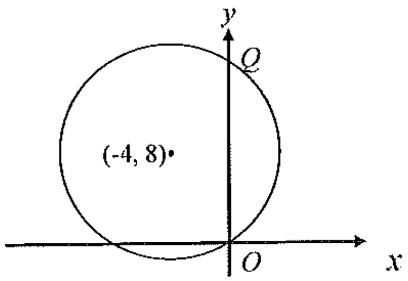
7. In the xy-coordinate plane above, points V_1, O, and V_2 are the 3 vertices of a certain triangle. The coordinates of V_1 are $(-2, b)$ and the coordinates of V_2 are $(2, b)$. Which of the following is one possible value of b if the area of triangle V_1OV_2 is less than 7 and greater than 6 ?

(A) $-\frac{15}{4}$

(B) $-\frac{13}{4}$

(C) $-\frac{11}{4}$

(D) $-\frac{7}{2}$



8. The circle in the xy-coordinate plane above has center $(-4, 8)$ and intersects the y-axis at point Q and at the origin. Which of the following are the coordinates at point Q?

(A) (10, 0)

(B) (16, 0)

(C) (0, 16)

(D) (0, 9)

1. In the xy-plane, the graph of $x = y^2 + 5$ intersects line L at $(6, t)$ and $(9, p)$. What is the least possible value of the slope of L?

(A) -1/3

(B) 2

(C) -1

(D) 1

Solution: Answer: (C)

t and p are the values of y.

$6 = t^2 + 5,\ t = \pm 1$	$9 = p^2 + 5,\ p = \pm 2$
$x_1 = 6$	$x_2 = 9$
$y_1 = \pm 1$	$y_2 = \pm 2$

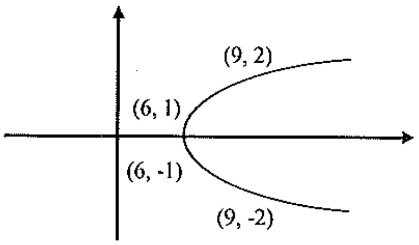


Figure 1

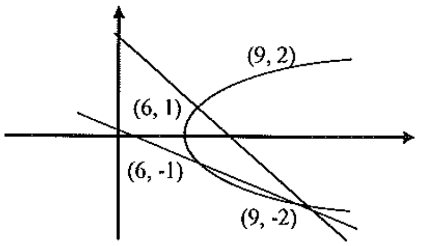
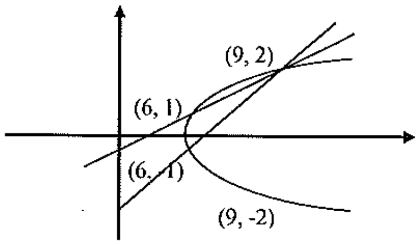


Figure 2



Lines $x = 6$ and $x = 9$ are undefined for slope.

So the 2 lines are not drawn in the figures.

Slope of Figure 1 < Slope of Figure 2

$\downarrow$

The least one is from figure 1.

Use the slope formula.

$$\frac{-2-(-1)}{9-6} = -\frac{1}{3} \text{ and } \frac{-2-1}{9-6} = -1$$

$$-1 < -\frac{1}{3}$$

$\Downarrow$

Answer is (C).

2. In the xy-plane, the graph of $x = y^2 - 4$ intersects line L at $(5, t)$ and $(0, p)$. What is the greatest possible value of the slope of L?

Solution: Answer: 1

t and p are the values of y.

$5 = t^2 - 4,\ t = \pm 3$	$0 = p^2 - 4,\ p = \pm 2$
$x_1 = 5$	$x_2 = 0$
$y_1 = \pm 3$	$y_2 = \pm 2$

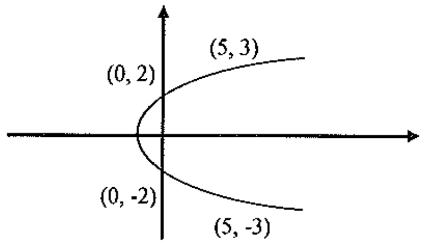


Figure 1

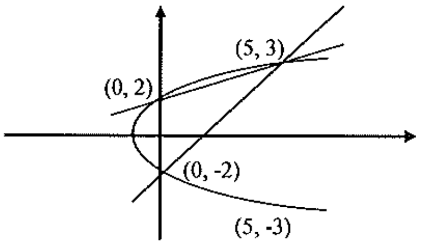
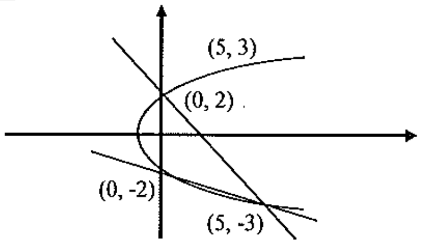


Figure 2



Lines $x = 0$ and $x = 5$ are undefined for slope.

So the 2 lines are not drawn in the figures.

Slope of Figure 1 > Slope of Figure 2

$\downarrow$

The greateat one from Figure 1

Use the slope formula.

$$\frac{3-2}{5-0} = \frac{1}{5} \text{ and } \frac{3-(-2)}{5-0} = 1$$

$\Downarrow$

$\frac{1}{5} < 1$, Answer is $\boxed{1}$.

Greatest or Least Slopes Involving Parabola

1. In the xy-plane, the graph of $x = y^2 + 5$ intersects line L at $(6, t)$ and $(9, p)$. What is the least possible value of the slope of L?

(A) -1/3

(B) 2

(C) -1

(D) 1

2. In the xy-plane, the graph of $x = y^2 - 4$ intersects line L at $(5, t)$ and $(0, p)$. What is the greatest possible value of the slope of L?

3 kinds of slope relationships of a pair of lines :

Parallel lines: $s_2 = s_1$

Perpendicular lines: $s_2 \cdot s_1 = -1$ or $s_2 = -\dfrac{1}{s_1}$

Reflection lines: $s_2 = -s_1$

1. If equation $y = -\dfrac{1}{3}x + 1$, then which of the following could be the function of its reflection?

(A) $y = 3x + 1$
(B) $y = 3x - 1$
(C) $y = \dfrac{1}{3}x + 1$
(D) $y = x + \dfrac{1}{3}$

Solution: Answer: (C)

Reflection lines $\rightarrow s_2 = -s_1$

$\downarrow$

$y = \dfrac{1}{3}x + 1$ (about y-axis)

or

$y = \dfrac{1}{3}x - 1$ (about x-axis)

"could be"

$\Downarrow$

Answer is (C).

2. If equation $y = \dfrac{3}{2}x - 1$, then which of the following could be the function of its reflection?

(A) $y = -\dfrac{2}{3}x + 1$
(B) $y = \dfrac{2}{3}x - 1$
(C) $y = -\dfrac{3}{2}x + 1$
(D) $y = x + \dfrac{2}{3}$

Solution: Answer: (C)

Reflection lines $\rightarrow s_2 = -s_1$

$\downarrow$

$y = -\dfrac{3}{2}x - 1$ (about y-axis)

or

$y = -\dfrac{3}{2}x + 1$ (about x-axis)

"could be"

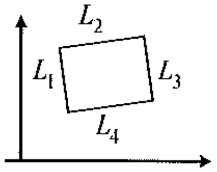
$\Downarrow$

Answer is (C).

3. A rectangle lies in the xy-plane so that its sides are not parallel to the axes. What is the sum of products of the slopes of two pairs of perpendicular sides of the rectangle?

(A) 0
(B) -2
(C) -4
(D) -1

Solution: Answer: (B)



Perpendicular lines $\rightarrow s_2 \cdot s_1 = -1$

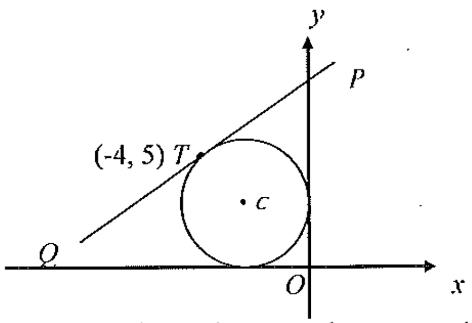
To any two pairs of perpendicular sides

For example,

$s_{L1} \cdot s_{L2} = -1$ and $s_{L3} \cdot s_{L4} = -1$

$\Downarrow$

Sum $= -1 + (-1) = -2$

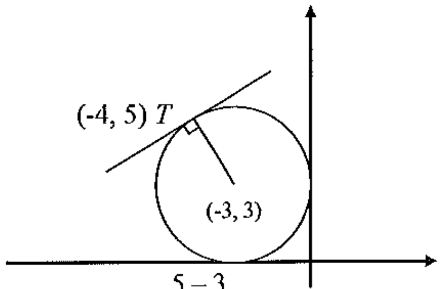


Note: Figure above not drawn to scale.

4. In the figure above, the circle with center c and diameter 6 is tangent to both the axes. The segment $\overline{PQ}$ is tangent to the circle at T. What is the slope of the segment $\overline{PQ}$?

(A) 2
(B) -2
(C) 0.5
(D) -1

Solution: Answer: (C)



$$s_1 = \frac{5-3}{-4-(-3)} = -2$$

$$\Downarrow$$

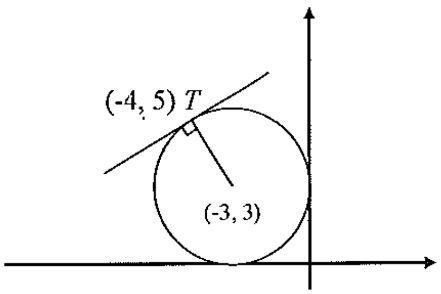
$$s_2 = -\frac{1}{s_1} = -\frac{1}{-2} = \frac{1}{2}$$

5. In the figure above, the circle with center c and area 9π is tangent to both the axes. The segment $\overline{PQ}$ is tangent to the circle at T. What is the slope of the segment $\overline{PQ}$?

(A) 2
(B) -2
(C) 0.5
(D) -1

Solution: Answer: (C)

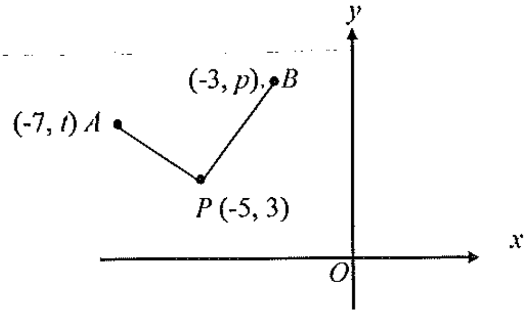
$$A = r^2\pi = 9\pi \rightarrow r = 3$$



$$s_1 = \frac{5-3}{-4-(-3)} = -2$$

$$\Downarrow$$

$$s_2 = -\frac{1}{s_1} = -\frac{1}{-2} = \frac{1}{2}$$



Note: Figure above not drawn to scale.

6. In the figure above, $\overline{AP}$ is perpendicular to $\overline{BP}$, $p = 7$, what is the value of t?

(A) 3
(B) 2
(C) 4
(D) -2

Solution: Answer: (C)

The two segments are perpendicular.

$$\downarrow$$

$$s_2 \cdot s_1 = -1$$

$$\downarrow$$

$$\frac{t-3}{-7-(-5)} \cdot \frac{7-3}{-3-(-5)} = -1$$

$$\downarrow$$

$$\frac{t-3}{-2} \cdot \frac{4}{2} = -1$$

$$\Downarrow$$

$$t = 4$$

7. In the figure above, $\overline{AP}$ is perpendicular to $\overline{BP}$, $t = 4$, what is the value of p?

(A) 1
(B) 7
(C) -3
(D) 3

Solution: Answer: (B)

The two segments are perpendicular.

$$\downarrow$$

$$s_2 \cdot s_1 = -1$$

$$\downarrow$$

$$\frac{p-3}{-3-(-5)} \cdot \frac{4-3}{-7-(-5)} = -1$$

$$\downarrow$$

$$\frac{p-3}{2} \cdot \frac{1}{-2} = -1$$

$$\Downarrow$$

$$p = 7$$

8. Which of the following is the equation of the line that passes through (5, 2) and is parallel to the line that passes through (-1, 3) and (-3, 6) ?

(A) $y=\frac{3}{2}x+9.5$

(B) $y=\frac{2}{3}x+9.5$

(C) $y=-\frac{3}{2}x+9.5$

(D) $y=-\frac{3}{2}x+19$

Solution: Answer: (C)

$$\text{Slope}_1=\frac{6-3}{-3-(-1)}=-\frac{3}{2}$$

The two lines are parallel.

$$\downarrow$$

$$\text{Slope}_2=\text{Slope}_1=-\frac{3}{2}$$

$$\downarrow$$

$$y=-\frac{3}{2}x+b$$

The line passes (5, 2).

$$\downarrow$$

$$2=-\frac{3}{2}\cdot 5+b,\ b=9.5$$

$$\Downarrow$$

$$y=-\frac{3}{2}x+9.5$$

9. Which of the following equations is the equation of a line that will never touch or cross the line $4x-7y=2$?

(A) $y=-\frac{4}{7}x-5$

(B) $7y=3x+5$

(C) $y=\frac{4}{7}x+5$

(D) $3y=4x+5$

Solution: Answer: (C)

$$4x-7y=2$$

$$\downarrow$$

$$y=\frac{4}{7}x-\frac{7}{2}$$

Check which equation has the same slope and different y-intercept.

$$\Downarrow$$

(C) $y=\frac{4}{7}x+5$ has the same slope and different y-intercept.

10. If $y=t_1x+b_1$, $y=t_2x+b_2$, and their graphs are perpendicular, what is the product of t_1 and t_2 ?

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) -1

Solution: Answer: (D)

t_1 and t_2 are the slopes of the 2 lines.

Two lines are perpendicular.

$$\Downarrow$$

$$t_1\cdot t_2=-1$$

11. The slope of a line perpendicular to the line whose equation is $\frac{x}{5}-\frac{y}{3}=1$ is

(A) $\frac{5}{3}$

(B) $-\frac{5}{3}$

(C) $\frac{3}{5}$

(D) $\pm\frac{5}{3}$

Solution: Answer: (B)

$$\frac{y}{3}=\frac{x}{5}-1,\ y=\frac{3x}{5}-3$$

$$\downarrow$$

$$S_1=\frac{3}{5}$$

$$S_2=-\frac{1}{S_1}$$

$$\Downarrow$$

$$S_2=-\frac{1}{\frac{3}{5}}=-\frac{5}{3}$$

12. What is the slope of the line that passes through (5, 2) and is perpendicular to the line that passes through (-1, 3) and (-3, 6) ?

(A) $-\frac{2}{3}$

(B) $-\frac{3}{2}$

(C) $\frac{2}{3}$

(D) $1\frac{1}{3}$

Solution: Answer: (C)

$$\begin{cases}\text{Slope}_1=\dfrac{6-3}{-3-(-1)}=-\dfrac{3}{2}\\ \text{The two lines are perpendicular.}\end{cases}$$

$$\Downarrow$$

$$\text{Slope}_2=-\frac{1}{\text{Slope}_1}=\frac{1}{-\frac{3}{2}}=\frac{2}{3}$$

13. In the xy-plane, the graph of a linear function has intercepts at $(x_0, 0)$ and $(0, y_0)$, If $x_0 = y_0 \neq 0$, which of the following is true about the slope of the graph of the function?

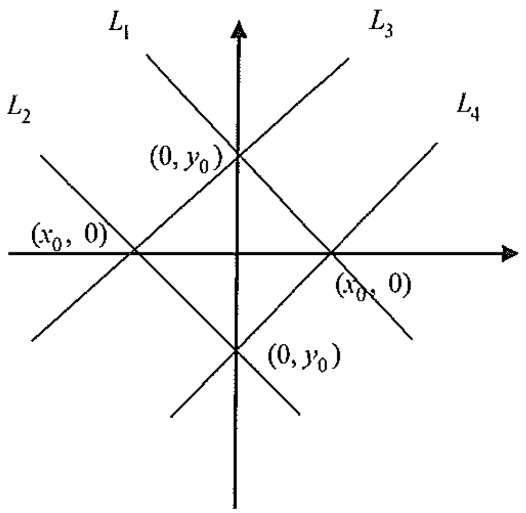
(A) The slope is positive.

(B) The slope is negative.

(C) The slope is undefined.

(D) The slope is zero.

Solution: Answer: (B)



Only x_0 and y_0 of points $(x_0, 0)$ and $(0, y_0)$ on L_1 or L_2 are equal, and the slopes of L_1 or L_2 are negative. By the direction of L_1 or L_2,

$\Downarrow$

The slope of the function is negative.

14. If line m is parallel to the line whose equation is $y - x = b$, then m must pass through which quadrants?

(A) I, II and IV

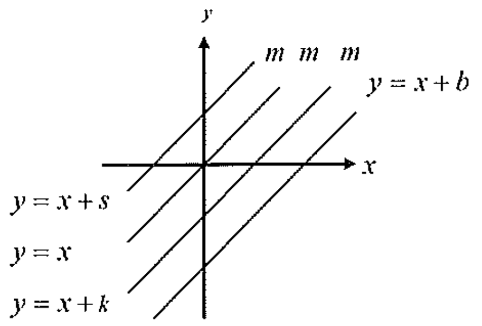
(B) II and IV

(C) II and III

(D) I and III

Solution: Answer: (D)

$$y - x = b, \quad y = x + b$$



$\Downarrow$

Line m must pass through quadrants I and III.

15. The slope of a line perpendicular to the line whose equation is $\frac{x}{5} - \frac{y}{4} = 1$ is

(A) $\frac{5}{3}$

(B) $-\frac{5}{4}$

(C) $\frac{3}{5}$

(D) $\pm\frac{5}{3}$

Solution: Answer: (B)

$$\frac{y}{4} = \frac{x}{5} - 1, \quad y = \frac{4x}{5} - 3$$

$\downarrow$

$$S_1 = \frac{4}{5}$$

$$S_2 \cdot S_1 = -1$$

$\Downarrow$

$$S_2 \cdot \frac{4}{5} = -1, \quad S_2 = -\frac{5}{4}$$

3 kinds of slope relationships of a pair of lines :

Parallel lines: $s_2 = s_1$

Perpendicular lines: $s_2 \cdot s_1 = -1$ or $s_2 = -\frac{1}{s_1}$

Reflection lines: $s_2 = -s_1$

1. If equation $y = -\frac{1}{3}x + 1$, then which of the following could be the function of its reflection?

(A) $y = 3x + 1$

(B) $y = 3x - 1$

(C) $y = \frac{1}{3}x + 1$

(D) $y = x + \frac{1}{3}$

2. If equation $y = \frac{3}{2}x - 1$, then which of the following could be the function of its reflection?

(A) $y = -\frac{2}{3}x + 1$

(B) $y = \frac{2}{3}x - 1$

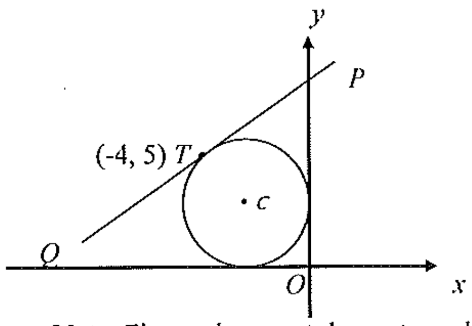
(C) $y = -\frac{3}{2}x + 1$

(D) $y = x + \frac{2}{3}$

3. A rectangle lies in the xy-plane so that its sides are not parallel to the axes. What is the sum of products of the slopes of two pairs of perpendicular sides of the rectangle?

(A) 0
(B) -2
(C) -4
(D) -1

Slopes of Parallel, Perpendicular and Reflection Lines



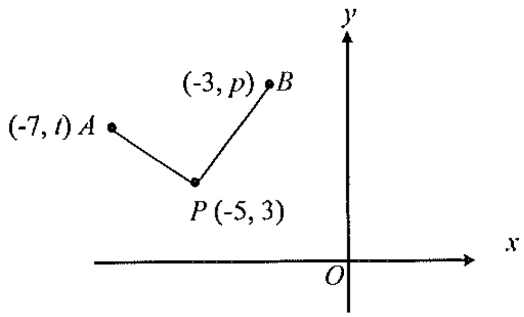
Note: Figure above not drawn to scale.

4. In the figure above, the circle with center c and diameter 6 is tangent to both the axes. The segment $\overline{PQ}$ is tangent to the circle at T. What is the slope of the segment $\overline{PQ}$?

(A) 2
(B) -2
(C) 0.5
(D) -1

5. In the figure above, the circle with center c and area 9π is tangent to both the axes. The segment $\overline{PQ}$ is tangent to the circle at T. What is the slope of the segment $\overline{PQ}$?

(A) 2
(B) -2
(C) 0.5
(D) -1



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(A) 3
(B) 2
(C) 4
(D) -2

7. In the figure above, $\overline{AP}$ is perpendicular to $\overline{BP}$, $t = 4$, what is the value of p?

(A) 1
(B) 7
(C) -3
(D) 3

8. Which of the following is the equation of the line that passes through (5, 2) and is parallel to the line that passes through (-1, 3) and (-3, 6) ?

(A) $y = \frac{3}{2}x + 9.5$

(B) $y = \frac{2}{3}x + 9.5$

(C) $y = -\frac{3}{2}x + 9.5$

(D) $y = -\frac{3}{2}x + 19$

9. Which of the following equations is the equation of a line that will never touch or cross the line $4x - 7y = 2$?

(A) $y = -\frac{4}{7}x - 5$

(B) $7y = 3x + 5$

(C) $y = \frac{4}{7}x + 5$

(D) $3y = 4x + 5$

10. If $y = t_1x + b_1$, $y = t_2x + b_2$, and their graphs are perpendicular, what is the product of t_1 and t_2 ?

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) -1

11. The slope of a line perpendicular to the line whose equation is $\frac{x}{5} - \frac{y}{3} = 1$ is

(A) $\frac{5}{3}$

(B) $-\frac{5}{3}$

(C) $\frac{3}{5}$

(D) $\pm\frac{5}{3}$

12. What is the slope of the line that passes through (5, 2) and is perpendicular to the line that passes through (-1, 3) and (-3, 6) ?

(A) $-\frac{2}{3}$

(B) $-\frac{3}{2}$

(C) $\frac{2}{3}$

(D) $1\frac{1}{3}$

13. In the xy-plane, the graph of a linear function has intercepts at $(x_0, 0)$ and $(0, y_0)$, If $x_0 = y_0 \neq 0$, which of the following is true about the slope of the graph of the function?

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(B) The slope is negative.

(C) The slope is undefined.

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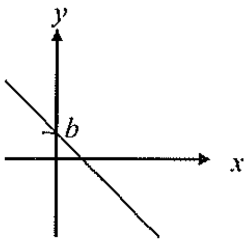
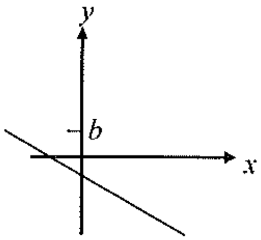
(C) $\frac{3}{5}$

(D) $\pm\frac{5}{3}$

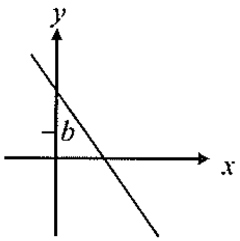


1. The figure above shows the graph of the linear function $y = sx + b$, where s and b are constants. Which of the following best represents the graph of the linear function $y = 3sx + b$?

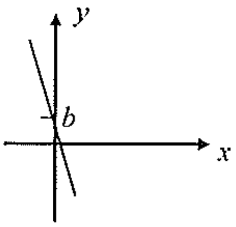
(A)



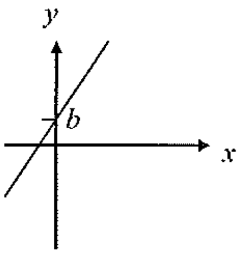
(B)



(C)



(D)



Solution: Answer: (C)

(1). $x = 0$

↓

$y = b$ (y-intercept) for $y = sx + b$ and $y = 3sx + b$.

↓

Choices (A) and (B) are eliminated.

(2). $s < 0 \rightarrow$ slope of new function should be still negative.

↓

Choice (D) is eliminated.

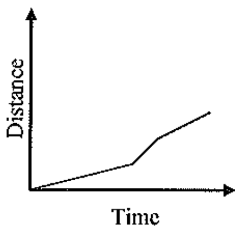
(3). $|3s| > |s| \rightarrow$ new line is closer y-axis than old line.

⇓

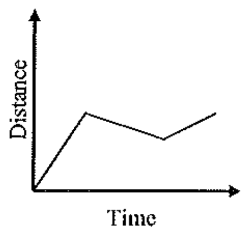
Answer = (C)

2. The vehicles of Tom's travel consist of car, bus, and bike, in that order. The bike is slower than the bus and the car is faster than the bus. Which of the following could be the figure of the distance and time during the overall time of Tom's travel?

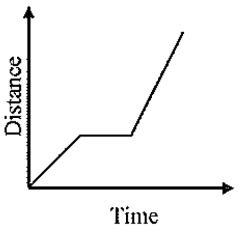
(A)



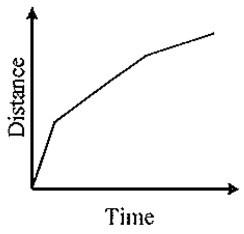
(B)



(C)



(D)



Solution: Answer: (D)

The speed order: car → bus → bike

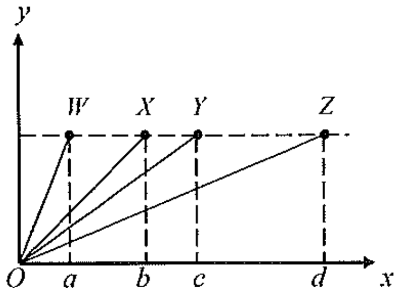
↓

The steepness order: car → bus → bike

⇓

Answer = (D)

Steepness of Slopes



3. In the figure above, which of the following segments has the second least slope?

(A) $\overline{YO}$
(B) $\overline{WO}$
(C) $\overline{XO}$
(D) $\overline{ZO}$

Solution: Answer: (D)

Each slope of the segments is positive.

$\overline{WO}$, $\overline{XO}$, $\overline{YO}$, and $\overline{ZO}$ each touch point (0, 0).

Let slope = S.

The segments have the same change of y at the points.

$\downarrow$

$S_{\overline{WO}} > S_{\overline{XO}} > S_{\overline{YO}} > S_{\overline{ZO}} > 0$ and $S_{\overline{WZ}} = 0$.

$\downarrow$

$S_{\overline{ZO}}$ is the second least slope.

$\Downarrow$

Answer = (D)

4. In the figure above, which of the following segments has the steepest slope?

(A) $\overline{YO}$
(B) $\overline{WO}$
(C) $\overline{XO}$
(D) $\overline{ZO}$

Solution: Answer: (B)

Each slope of the segments is positive.

$\overline{WO}$, $\overline{XO}$, $\overline{YO}$, and $\overline{ZO}$ each touch point (0, 0).

Let slope = S.

The segments have the same change of y at the points.

$\downarrow$

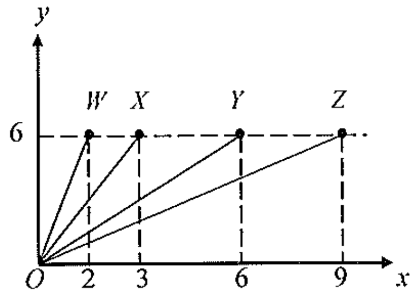
$S_{\overline{WO}} > S_{\overline{XO}} > S_{\overline{YO}} > S_{\overline{ZO}} > 0$

$\downarrow$

$S_{\overline{WO}}$ is the steepest slope (greatest slope).

$\Downarrow$

Answer is (B).



5. In the figure above, what is the median of the slopes of the following segments $\overline{WO}$, $\overline{XO}$, $\overline{YO}$, and $\overline{ZO}$?

(A) $\frac{2}{3}$
(B) 6
(C) $\frac{3}{2}$
(D) 3

Solution: Answer: (C)

$$\text{Median} = \frac{\text{Slope}_{\overline{XO}} + \text{Slope}_{\overline{YO}}}{2}$$

$\Downarrow$

$$\frac{\frac{6}{3}+\frac{6}{6}}{2} \cdot \frac{6}{6} = \frac{12+6}{12} = \frac{3}{2}$$

6. In the figure above, what is the steepest slope of the following segments $\overline{WO}$, $\overline{XO}$, $\overline{YO}$, and $\overline{ZO}$?

(A) $\frac{2}{3}$
(B) 6
(C) $\frac{3}{2}$
(D) 3

Solution: Answer: (D)

$\overline{WO}$, $\overline{XO}$, $\overline{YO}$, and $\overline{ZO}$ each touch the origin.

Let Slope = S.

The segments have the same change of y at the points.

$\downarrow$

$S_{\overline{WO}} > S_{\overline{XO}} > S_{\overline{YO}} > S_{\overline{ZO}}$

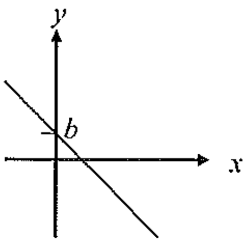
Use plug-in method.

$\downarrow$

$S_{\overline{WO}} = \frac{6}{2} = 3$ is the steepest slope.

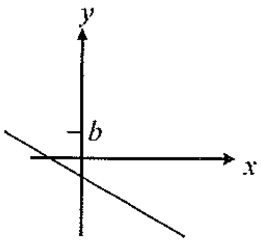
$\Downarrow$

Answer = (D)

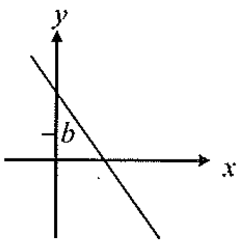


1. The figure above shows the graph of the linear function $y = sx + b$, where s and b are constants. Which of the following best represents the graph of the linear function $y = 3sx + b$?

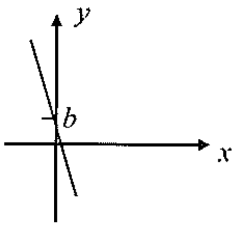
(A)



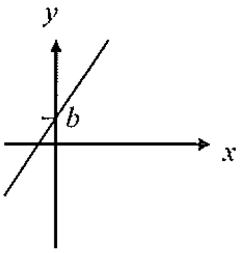
(B)



(C)

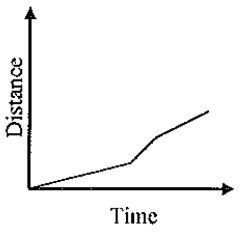


(D)

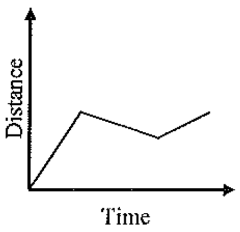


2. The vehicles of Tom's travel consist of car, bus, and bike, in that order. The bike is slower than the bus and the car is faster than the bus. Which of the following could be the figure of the distance and time during the overall time of Tom's travel?

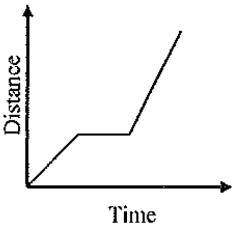
(A)



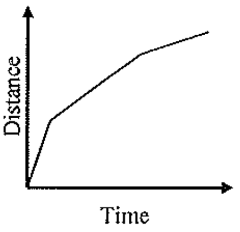
(B)



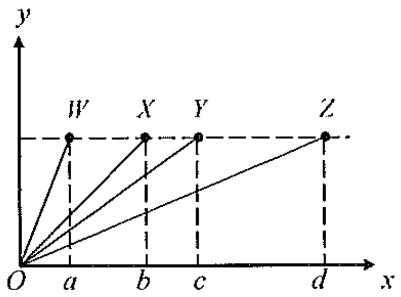
(C)



(D)



Steepness of Slopes

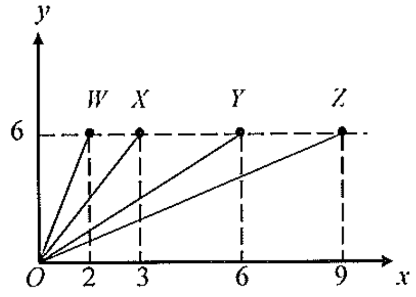


3. In the figure above, which of the following segments has the second least slope?

(A) $\overline{YO}$
(B) $\overline{WO}$
(C) $\overline{XO}$
(D) $\overline{ZO}$

4. In the figure above, which of the following segments has the steepest slope?

(A) $\overline{YO}$
(B) $\overline{WO}$
(C) $\overline{XO}$
(D) $\overline{ZO}$

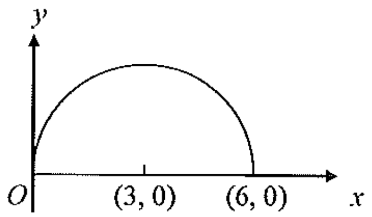


5. In the figure above, what is the median of the slopes of the following segments $\overline{WO}$, $\overline{XO}$, $\overline{YO}$, and $\overline{ZO}$?

(A) $\frac{2}{3}$
(B) 6
(C) $\frac{3}{2}$
(D) 3

6. In the figure above, what is the steepest slope of the following segments $\overline{WO}$, $\overline{XO}$, $\overline{YO}$, and $\overline{ZO}$?

(A) $\frac{2}{3}$
(B) 6
(C) $\frac{3}{2}$
(D) 3

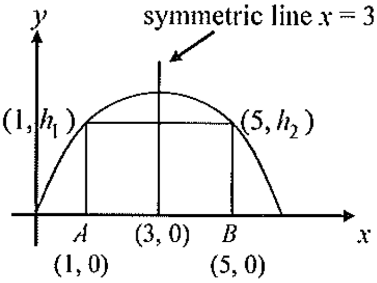


Note: Figure above not drawn to scale.

1. In the figure above, the center of the semicircle is at (3, 0). Which of the following are x-coordinates of two points on this semicircle whose y-coordinates are equal?

(A) 3 and 9
(B) 5 and 8
(C) 3 and 6
(D) 1 and 5

Solution: Solution: (D)



The distance from $x = 3$ to $x = 1$ is the same as the distance from $x = 3$ to $x = 5$.

↓

The symmetric line passes through Point (3, 0).

Move horizontally segment $\overline{AB}$ in the figure above upward until touching the two points $(1, h_1)$ and $(5, h_2)$.

A semicircle is a symmetric graph.

↓

$h_1 = h_2$.

⇓

Answer is (D).

2. In the xy-plane, A and B are distinct points, which have the same y-coordinate and lie on the parabola whose equation is $y = -x^2 + 5x + 30$. Which of the following is the x-coordinate of the midpoint of $\overline{AB}$?

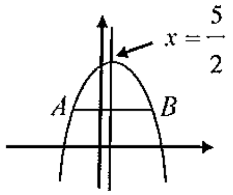
(A) $\frac{5}{2}$
(B) $-\frac{2}{5}$
(C) $\frac{2}{5}$
(D) $-\frac{5}{3}$

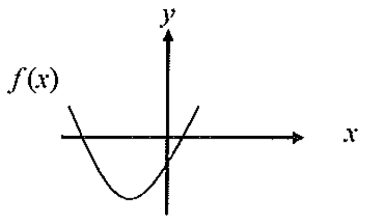
Solution: Answer: (A)

$$y = -x^2 + 5x + 30$$

⇓

$$x = -\frac{b}{2a} = -\frac{5}{-2 \cdot 1} = \frac{5}{2}$$

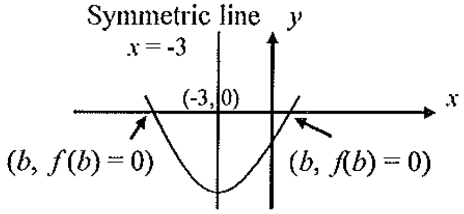




3. The figure above shows the graph of a quadratic function f whose minimum value is $f(-3)$. If $f(b) = 0$, which of the following could the value of b be?

(A) -6
(B) 1
(C) -5
(D) 0

Solution: Answer: (B)



For $f(b) = 0$, when $x = b$, two possible positions are indicated in the figure above.

(1) The minimum value of the function f is $f(-3)$.

↓

$x = -3$ is the symmetric line.

↓

The point the line $x = -3$ crosses x-axis is (-3, 0).

(2) A parabola is a symmetric graph.

↓

The distance, not coordinate, from point (-3, 0) to point $(b, 0)$ should be greater than 3.

b is a coordinate value.

↓

$b > 0$ or $b < -6$ $b = 1 > 0$.

⇓

Answer is (B).

4. In the xy-plane, M and N are distinct points, which have the same y-coordinate and lie on the parabola whose equation is $y = 2x^2 - 5x + 3$. What is the x-coordinate of midpoint $\overline{MN}$?

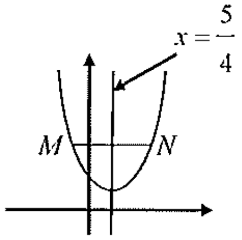
(A) $\frac{5}{2}$
(B) $-\frac{4}{5}$
(C) $\frac{2}{5}$
(D) $\frac{5}{4}$

Solution: Answer: (D)

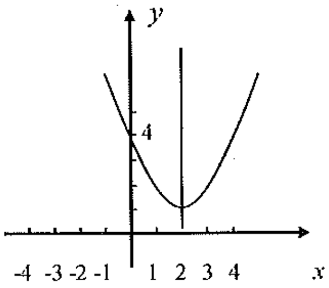
$$y = 2x^2 - 5x + 3$$

⇓

$$x = -\frac{b}{2a} = -\frac{-5}{2 \cdot 2} = \frac{5}{4}$$



Symmetric Graphs and Ordered Pairs in xy-Plane



5. The figure above shows the graph of a quadratic function g that has a minimum value at the point (2, 1). If $g(h) = g(4)$, which of the following could the value of h be?

(A) 6
(B) 3
(C) -4
(D) 0

Solution: Answer: (D)

$x = 2$ is the symmetric line.

From $x = 2$ to $x = 0$, the distance = 2.
From $x = 2$ to $x = 4$, the distance = 2.
h is a certain value of x.
A parabola is a symmetric graph.

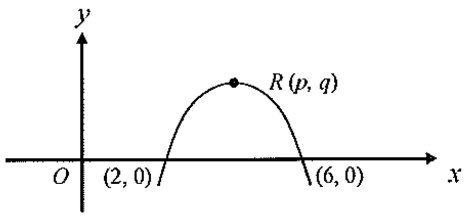
$\downarrow$

$g(0) = g(4)$

$\Downarrow$

$h = 0$

$[g(0) = g(4) = 4]$

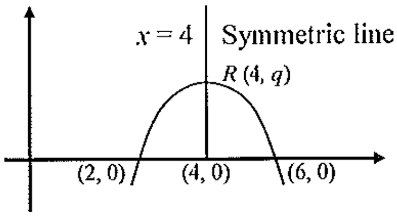


Note: Figure above not drawn to scale.

6. The quadratic function g is graphed in the xy-coordinate plane above. If $g(x) \leq q$ for all values of x, which of the following could the x and y coordinates of point R be?

(A) (5, 3)
(B) (4, 3)
(C) (3, 3)
(D) (3, 4)

Solution: Answer: (B)



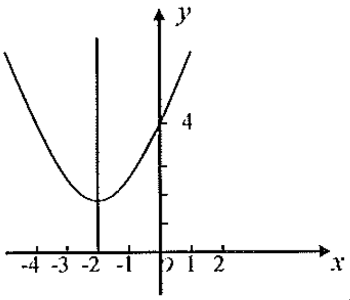
A parabola is a symmetrical graph.

$\downarrow$

The symmetric line passes through (4, 0).

$\Downarrow$

$R(p, q) = R(4, q)$
Only (B) has $x = 4$



7. The figure above shows the graph of a quadratic function g that has a minimum value at the point (-2, 2). If $g(k) = g(0)$, which of the following could the value of k be?

(A) 4
(B) 3
(C) -4
(D) 1

Solution: Answer: (C)

$x = -2$ is the symmetric line.

From $x = -2$ to $x = 0$, the distance = 2.
From $x = -2$ to $x = -4$, the distance = 2.
k is a certain value of x.
A parabola is a symmetric graph.

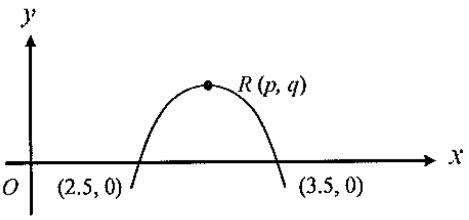
$\downarrow$

$g(-4) = g(0)$

$\Downarrow$

$k = -4$

$[g(-4) = g(0) = 4]$

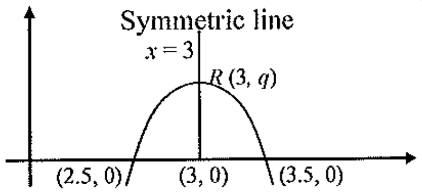


Note: Figure above not drawn to scale.

8. The quadratic function g is graphed in the xy-coordinate plane above. If $g(x) \leq q$ for all values of x, which of the following could the x and y coordinates of point R be?

(A) (2.8, 2)
(B) (2.7, 1.5)
(C) (3, 1.2)
(D) (2.8, 2)

Solution Answer: (C)



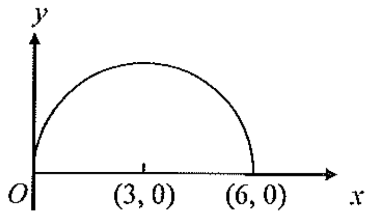
A parabola is a symmetrical graph.

$\downarrow$

The symmetric line passes through (3, 0).

$\Downarrow$

$R(p, q) = R(3, q)$
Only (C) has $x = 3$



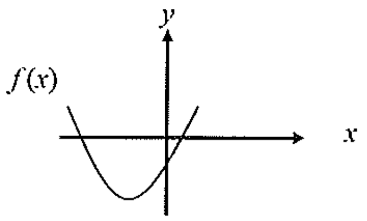
Note: Figure above not drawn to scale.

1. In the figure above, the center of the semicircle is at (3, 0). Which of the following are x-coordinates of two points on this semicircle whose y-coordinates are equal?

(A) 3 and 9
(B) 5 and 8
(C) 3 and 6
(D) 1 and 5

2. In the xy-plane, A and B are distinct points, which have the same y-coordinate and lie on the parabola whose equation is $y = -x^2 + 5x + 30$. Which of the following is the x-coordinate of the midpoint of $\overline{AB}$?

(A) $\frac{5}{2}$
(B) $-\frac{2}{5}$
(C) $\frac{2}{5}$
(D) $-\frac{5}{3}$



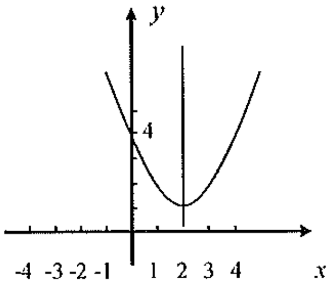
3. The figure above shows the graph of a quadratic function f whose minimum value is $f(-3)$. If $f(b) = 0$, which of the following could the value of b be?

(A) -6
(B) 1
(C) -5
(D) 0

4. In the xy-plane, M and N are distinct points, which have the same y-coordinate and lie on the parabola whose equation is $y = 2x^2 - 5x + 3$. What is the x-coordinate of midpoint $\overline{MN}$?

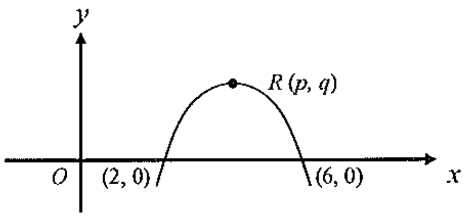
(A) $\frac{5}{2}$
(B) $-\frac{4}{5}$
(C) $\frac{2}{5}$
(D) $\frac{5}{4}$

Symmetric Graphs and Ordered Pairs in *xy*-Plane



5. The figure above shows the graph of a quadratic function g that has a minimum value at the point (2, 1). If $g(h) = g(4)$, which of the following could the value of h be?

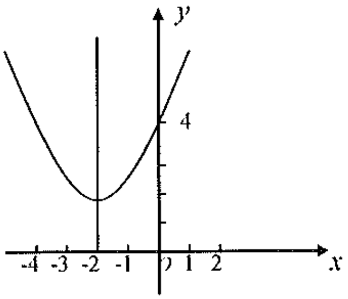
(A) 6
(B) 3
(C) -4
(D) 0



Note: Figure above not drawn to scale.

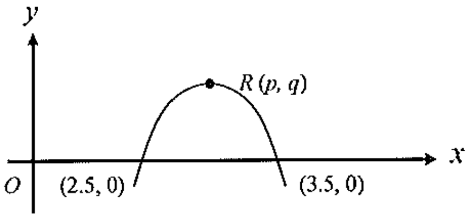
6. The quadratic function g is graphed in the xy-coordinate plane above. If $g(x) \leq q$ for all values of x, which of the following could the x and y coordinates of point R be?

(A) (5, 3)
(B) (4, 3)
(C) (3, 3)
(D) (3, 4)



7. The figure above shows the graph of a quadratic function g that has a minimum value at the point (-2, 2). If $g(k) = g(0)$, which of the following could the value of k be?

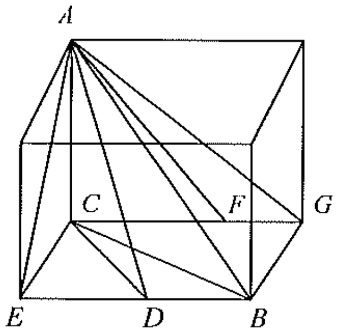
(A) 4
(B) 3
(C) -4
(D) 1



Note: Figure above not drawn to scale.

8. The quadratic function g is graphed in the xy-coordinate plane above. If $g(x) \leq q$ for all values of x, which of the following could the x and y coordinates of point R be?

(A) (2.8, 2)
(B) (2.7, 1.5)
(C) (3, 1.2)
(D) (2.8, 2)



1. In the cube shown above, *A, B, C, D, E, F,* and *G* are the points of the edges or corners. Which of the following angles has the least measure?

(A) $\angle AEC$
(B) $\angle ADC$
(C) $\angle AFC$
(D) $\angle ABC$

Solution: Answer: (D)

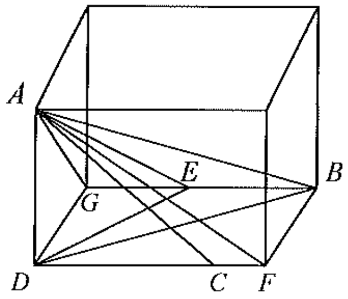
Each of the 5 angles in the 5 choices is an interior angle of 5 different right triangles. They have the same height.

So, the conclution is if a triangle has the longer base leg and the same height, then the triangle has the smaller angle measure.

Because $\angle ABC$ has the longest base leg CB, $m\angle ABC$ is the least measure among the five angles.

$\Downarrow$

Answer is (D).



2. In the cube shown above, *A, B, C, D, E, F,* and *G* are the points of the edges or corners. Which of the following angles has the greatest measure?

(A) $\angle AED$
(B) $\angle ACD$
(C) $\angle AFD$
(D) $\angle ABD$

Solution: Answer: (B)

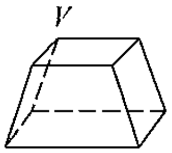
Each of the 5 angles in the 5 choices is an interior angle of 5 different right triangles. They have the same height.

So, the conclution is if a triangle has the longer base leg and the same height, then the triangle has the smaller angle measure.

Because $\angle ACD$ has the longest base leg CB, $m\angle ACD$ is the least measure among the five angles.

$\Downarrow$

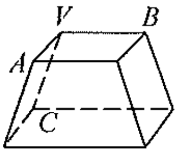
Answer is (B).



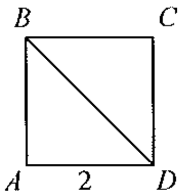
3. In the three-dimension figure above, line segments are to be drawn connecting vertex *V* with each of the other seven vertices in this figure. How many of these segments will not lie on an edge of the figure?

(A) 3
(B) 2
(C) 5
(D) 4

Solution: Answer: (D)



Segments $\overline{VA}$, $\overline{VB}$, and $\overline{VC}$ lie on an edge of the figure and they connect 4 vertices *A, V, B,* and *C*. Other 4 segments connecting the rest vertices do not lie on an edge of the figure.

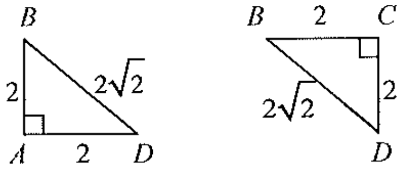


4. In the figure above, the diagonal divided square $ABCD$ into two triangles. Which of the following is the sum of the perimeters of those two triangles?

(A) $8+2\sqrt{2}$
(B) $8+4\sqrt{2}$
(C) 12
(D) 16

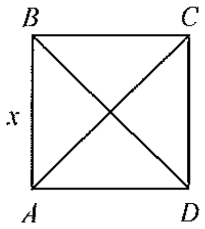
Solution: Answer: (B)

Draw a sketch shown below.



$$\Downarrow$$

$$\text{sum} = 2\times\underbrace{(2+2\sqrt{2}+2)}_{\text{One perimeter}} = 8+4\sqrt{2}$$

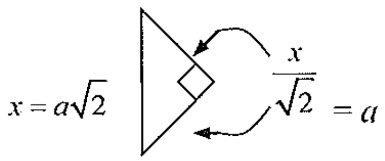


5. In the figure above, the 2 diagonals divided square $ABCD$ into 4 triangles. Which of the following is the sum of the perimeters of those 4 triangles?

(A) $4x+2\sqrt{2}x$
(B) $4x+2x$
(C) $4x+4x$
(D) $4(x+\sqrt{2}x)$

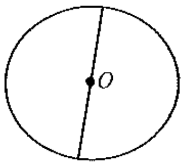
Solution: Answer: (D)

Draw a sketch shown below.



$$\Downarrow$$

$$\text{Sum} = 4\times\underbrace{(x+\frac{x}{\sqrt{2}}+\frac{x}{\sqrt{2}})}_{\text{One permiter}} = 4(x+\sqrt{2}x)$$

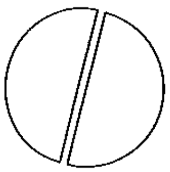


6. In the figure above, the diameter $d=3$ divided the circle into two semicircles. Which of the following is the sum of the perimeters of those two semicircles?

(A) $3\pi+3$
(B) $6\pi+3$
(C) $3\pi+6$
(D) $9\pi+3$

Solution: Answer: (C)

The sketch shown below.



$$c = d\pi = 3\pi,\quad \frac{c}{2}=\frac{3\pi}{2}$$

$$\Downarrow$$

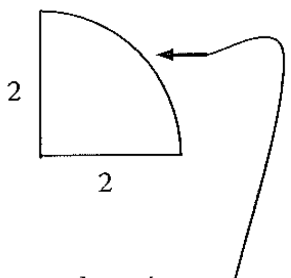
$$\text{Sum} = 2\times(\underbrace{\frac{c}{2}+d}_{\text{One perimeter}}) = 2\times(\frac{3\pi}{2}+3) = 3\pi+6$$

7. In the figure above, the diameter $d=4$ divided circle into four equal portions. Which of the following is the sum of the perimeters of those four portions?

(A) $8+4\pi$
(B) $16+4\pi$
(C) $4+4\pi$
(D) $10+4\pi$

Solution: Answer: (B)

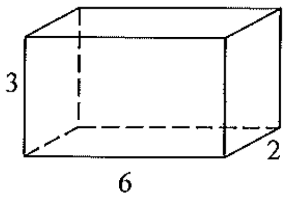
Draw a sketch shown below.



$$\frac{c}{4}=\frac{d\pi}{4}=\frac{4\pi}{4}=\pi$$

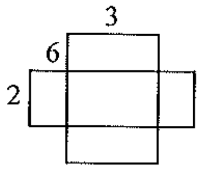
$$\Downarrow$$

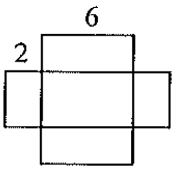
$$\text{Sum} = 4\times(\underbrace{2+2+\pi}_{\text{One perimeter}}) = 16+4\pi$$

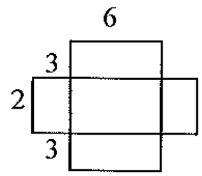


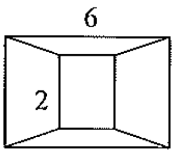
Note: The figure above not drawn to scale.

8. In the figure above, if this rectangular paper box with no top cover is cut along the four vertical edges and flattened, which of the following best represent the result?

(A) 

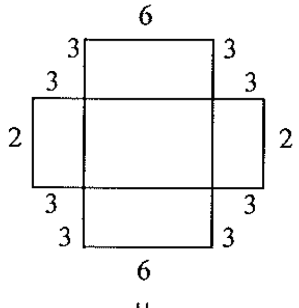
(B) 

(C) 

(D) 

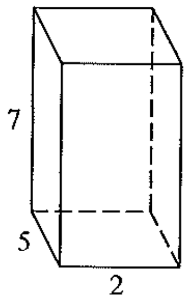
Solution: Answer: (C)

After be cutting, the figure of the box forms a plane figure roughly like below:



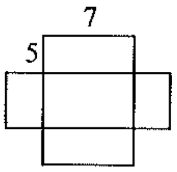
⇓

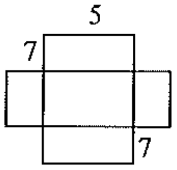
Answer is (C).

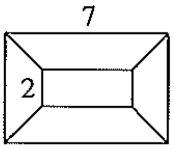


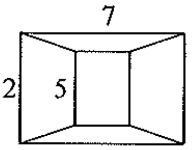
Note: The figure above not drawn to scale.

9. In the figure above, if this rectangular paper box with no bottom cover is cut along the four vertical edges and flattened, which of the following best represent the result?

(A) 

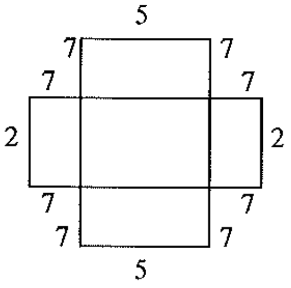
(B)2 

(C) 

(D) 2 

Solution: Answer: (B)

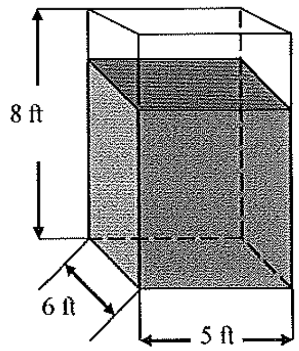
After be cutting, the figure of the box forms a plane figure roughly like below:



⇓

Answer is (B).

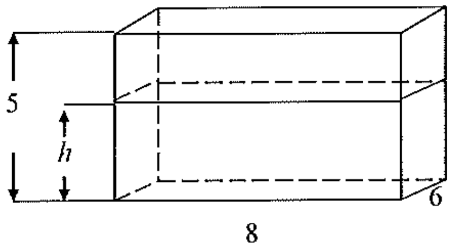
Note: Figure above not drawn to scale.



10. The figure above shows the dimensions of a rectangular container closed. There is 96 cubic feet of non-sand in the container. The container will be repositioned on level ground so that the container rests on one, which has the greatest area, of the six surfaces. If it has been repositioned, which of the following will be the entire surface area of the sand?

(A) 180
(B) 104
(C) 130
(D) 105

Solution: Answer: (A)



$$V = s_1 s_2 s_3$$

$$\downarrow$$

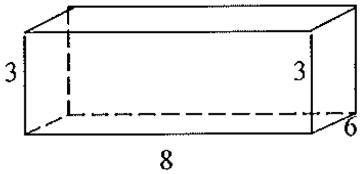
$$V_{\text{containert}} = 5 \cdot 6 \cdot 8 = 240$$

$$V_{\text{non-sand}} = 96$$

$$V_{\text{sand}} = 240 - 96 = 144$$

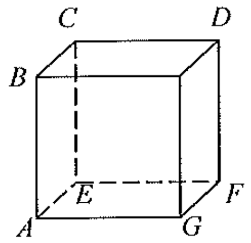
$$\downarrow$$

$$6 \cdot 8 \cdot h = 144, \quad h = 3$$



$$\Downarrow$$

$$2(3 \cdot 8 + 6 \cdot 8 + 3 \cdot 6) = 180$$



11. In the figure above, on the cube, each of the following points is the same distance from G as it from E except

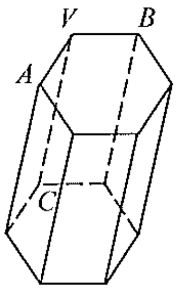
(A) F
(B) C
(C) D
(D) B

Solution: Answer: (B)

$$GC \neq EC$$

$$\Downarrow$$

Answer is (B).



12. In the three-dimension figure above, line segments are to be drawn connecting vertex V with each of the other eleven vertices in this figure. How many of these segments will not lie on an edge of the figure?

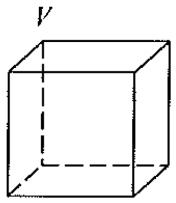
(A) 7
(B) 4
(C) 8
(D) 3

Solution: Answer: (C)

Segments $\overline{VA}, \overline{VB}$, and $\overline{VC}$ lie on an edge of the figure and they connect 4 vertices A, V, B, and C. Other 8 segments connecting the rest 8 vertices do not lie on an edge of the figure.

$$\Downarrow$$

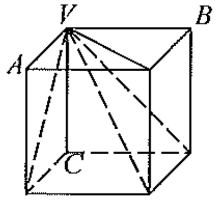
Answer is (C).



13. In the cube above, line segments are to be drawn connecting vertex V with each of the other seven vertices in the cube. How many of these segments will not lie on an edge of the cube?

(A) 3
(B) 4
(C) 5
(D) 2

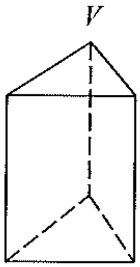
Solution: Answer: (B)



Segments $\overline{VA}, \overline{VB}$, and $\overline{VC}$ lie on an edge of the cube and they connect 4 vertices A, V, B, and C. Other 4 segments connecting the rest vertices do not lie on an edge of the cube.

$\Downarrow$

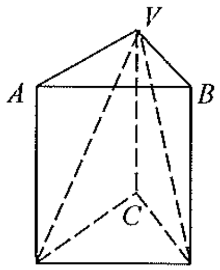
Answer is (B).



14. In the three-dimension figure above, line segments are to be drawn connecting vertex V with each of the other five vertices in this figure. How many of these segments will not lie on an edge of the figure?

(A) 3
(B) 4
(C) 5
(D) 2

Solution: Answer: (D)

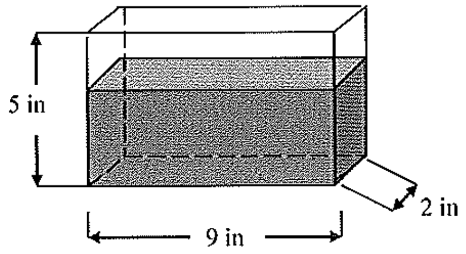


Segments $\overline{VA}, \overline{VB}$, and $\overline{VC}$ lie on an edge of the figure and they connect 4 vertices A, V, B, and C. Other 2 segments connecting the rest vertices do not lie on an edge of the figure.

$\Downarrow$

Answer is (D).

Note: Figure above not drawn to scale.



15. The figure above shows the dimensions of a rectangular container sealed up. There is 20 cubic inches of sand in the container. The container will be repositioned on level ground so that the container places on one, which has the least area, of the six surfaces. If it has been repositioned, which of the following will be ratio of the height of the non-sand to the height of the sand?

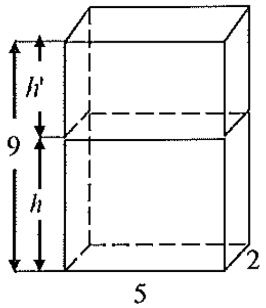
(A) $\frac{2}{7}$

(B) $\frac{5}{2}$

(C) $\frac{3}{7}$

(D) $\frac{7}{2}$

Solution: Answer: (D)



$$V = s_1 s_2 s_3$$

$$\downarrow$$

$$20 = 2 \cdot 5 \cdot h$$

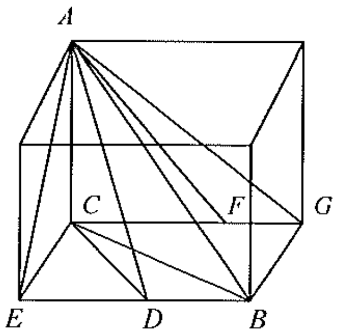
$$\downarrow$$

$$h = \boxed{2}$$

$$h' = 9 - h = 9 - 2 = \boxed{7}$$

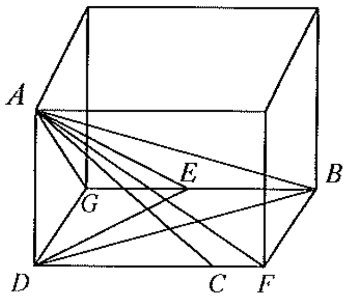
$$\Downarrow$$

$$\frac{h'}{h} = \boxed{\frac{7}{2}}$$



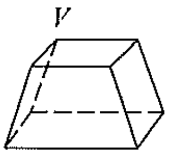
1. In the cube shown above, *A, B, C, D, E, F,* and *G* are the points of the edges or corners. Which of the following angles has the least measure?

(A) $\angle AEC$
(B) $\angle ADC$
(C) $\angle AFC$
(D) $\angle ABC$



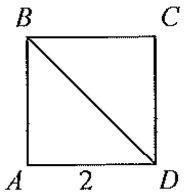
2. In the cube shown above, *A, B, C, D, E, F,* and *G* are the points of the edges or corners. Which of the following angles has the greatest measure?

(A) $\angle AED$
(B) $\angle ACD$
(C) $\angle AFD$
(D) $\angle ABD$



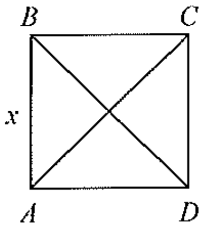
3. In the three-dimension figure above, line segments are to be drawn connecting vertex *V* with each of the other seven vertices in this figure. How many of these segments will not lie on an edge of the figure?

(A) 3
(B) 2
(C) 5
(D) 4



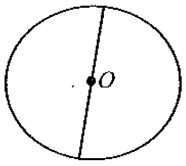
4. In the figure above, the diagonal divided square $ABCD$ into two triangles. Which of the following is the sum of the perimeters of those two triangles?

(A) $8+2\sqrt{2}$
(B) $8+4\sqrt{2}$
(C) 12
(D) 16



5. In the figure above, the 2 diagonals divided square $ABCD$ into 4 triangles. Which of the following is the sum of the perimeters of those 4 triangles?

(A) $4x+2\sqrt{2}x$
(B) $4x+2x$
(C) $4x+4x$
(D) $4(x+\sqrt{2}x)$

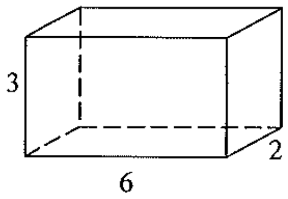


6. In the figure above, the diameter $d=3$ divided the circle into two semicircles. Which of the following is the sum of the perimeters of those two semicircles?

(A) $3\pi+3$
(B) $6\pi+3$
(C) $3\pi+6$
(D) $9\pi+3$

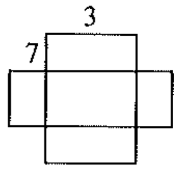
7. In the figure above, the diameter $d=4$ divided circle into four equal portions. Which of the following is the sum of the perimeters of those four portions?

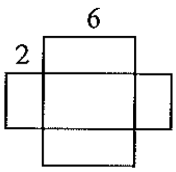
(A) $8+4\pi$
(B) $16+4\pi$
(C) $4+4\pi$
(D) $10+4\pi$

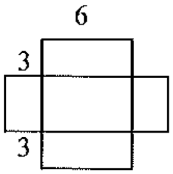


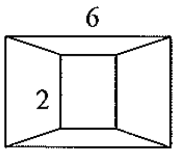
Note: The figure above not drawn to scale.

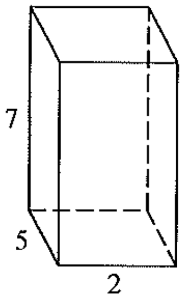
8. In the figure above, if this rectangular paper box with no top cover is cut along the four vertical edges and flattened, which of the following best represent the result?

(A) 

(B) 

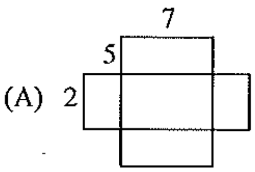
(C) 

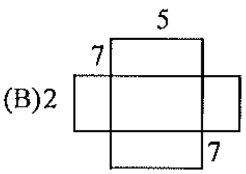
(D) 

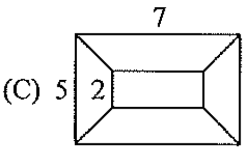


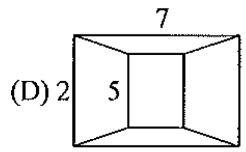
Note: The figure above not drawn to scale.

9. In the figure above, if this rectangular paper box with no bottom cover is cut along the four vertical edges and flattened, which of the following best represent the result?

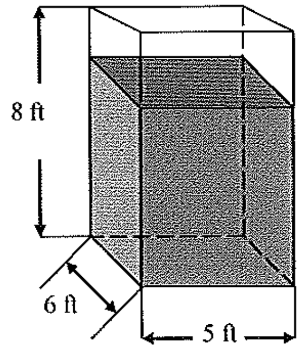
(A) 

(B) 

(C) 

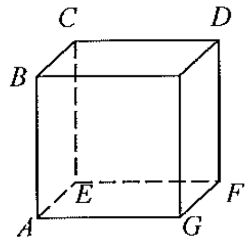
(D) 

Note: Figure above not drawn to scale.



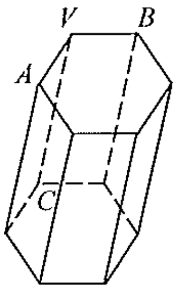
10. The figure above shows the dimensions of a rectangular container closed. There is 96 cubic feet of non-sand in the container. The container will be repositioned on level ground so that the container rests on one, which has the greatest area, of the six surfaces. If it has been repositioned, which of the following will be the entire surface area of the sand?

(A) 180
(B) 104
(C) 130
(D) 105



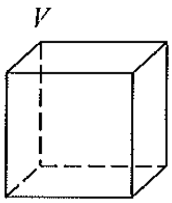
11. In the figure above, on the cube, each of the following points is the same distance from G as it from E except

(A) F
(B) C
(C) D
(D) B



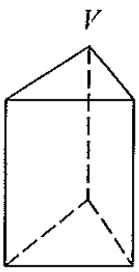
12. In the three-dimension figure above, line segments are to be drawn connecting vertex V with each of the other eleven vertices in this figure. How many of these segments will not lie on an edge of the figure?

(A) 7
(B) 4
(C) 8
(D) 3



13. In the cube above, line segments are to be drawn connecting vertex V with each of the other seven vertices in the cube. How many of these segments will not lie on an edge of the cube?

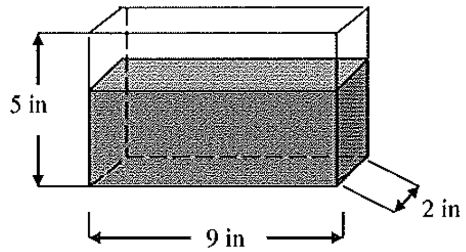
(A) 3
(B) 4
(C) 5
(D) 2



14. In the three-dimension figure above, line segments are to be drawn connecting vertex V with each of the other five vertices in this figure. How many of these segments will not lie on an edge of the figure?

(A) 3
(B) 4
(C) 5
(D) 2

Note: Figure above not drawn to scale.



15. The figure above shows the dimensions of a rectangular container sealed up. There is 20 cubic inches of sand in the container. The container will be repositioned on level ground so that the container places on one, which has the least area, of the six surfaces. If it has been repositioned, which of the following will be ratio of the height of the non-sand to the height of the sand?

(A) $\frac{2}{7}$

(B) $\frac{5}{2}$

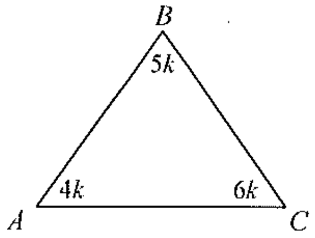
(C) $\frac{3}{7}$

(D) $\frac{7}{2}$

1. If the angles of triangle ABC are in the ratio of 4: 5: 6, and then triangle ABC is an or a

(A) obtuse triangle.
(B) acute triangle.
(C) right triangle.
(D) isosceles triangle.

Solution: Answer: (B)



$4k + 5k + 6k = 180,\ k = 12$

$6k = 72$

The maximum interior angle $< 90°$.

$\downarrow$

It is an acute triangle.

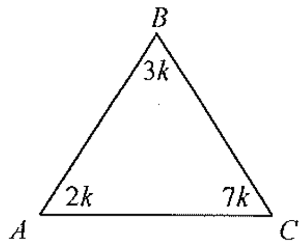
$\Downarrow$

Answer is (B).

2. If the angles of triangle ABC are in the ratio of 2: 3: 7, and then triangle ABC is an or a

(A) obtuse triangle.
(B) acute triangle.
(C) right triangle.
(D) isosceles triangle.

Solution: Answer: (A)



$2k + 3k + 7k = 180,\ k = 15$

$7k = 105$

The maximum interior angle is greater than 90°.

$\downarrow$

It is an obtuse triangle.

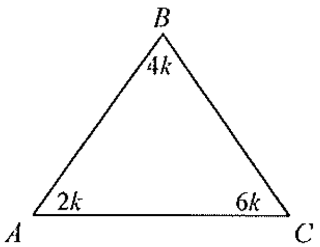
$\Downarrow$

Answer = (A)

3. If the angles of triangle ABC are in the ratio of 2: 4: 6, and then triangle ABC is an or a

(A) obtuse triangle.
(B) acute triangle.
(C) right triangle.
(D) isosceles triangle.

Solution: Answer: (C)



$2k + 4k + 6k = 180,\ k = 15$

$6k = 90$

$m\angle C = 6k = 90°$

$\downarrow$

It is a right triangle.

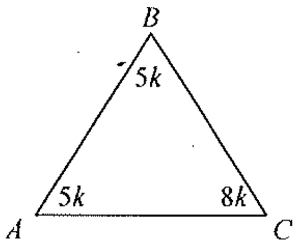
$\Downarrow$

Answer is (C).

4. If the angles of triangle ABC are in the ratio of 5: 5: 8, what is the degree value of the greatest angle of triangle ABC?

(A) 50
(B) 60
(C) 70
(D) 80

Solution: Answer: (D)



$5k + 5k + 8k = 180,\ k = 10$

$\Downarrow$

$8k = 8 \cdot 10 = 80$. Answer = (D)

Acute, Obtuse and Right Triangles

1. If the angles of triangle ABC are in the ratio of 4: 5: 6, and then triangle ABC is an or a

(A) obtuse triangle.
(B) acute triangle.
(C) right triangle.
(D) isosceles triangle.

2. If the angles of triangle ABC are in the ratio of 2: 3: 7, and then triangle ABC is an or a

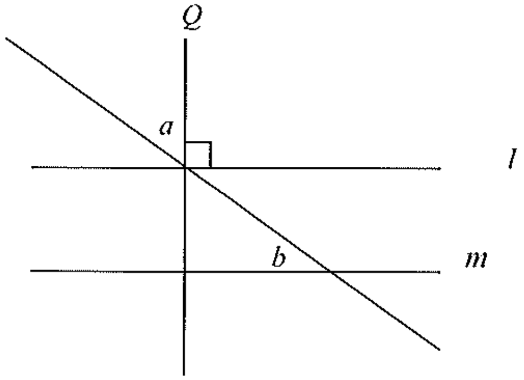
(A) obtuse triangle.
(B) acute triangle.
(C) right triangle.
(D) isosceles triangle.

3. If the angles of triangle ABC are in the ratio of 2: 4: 6, and then triangle ABC is an or a

(A) obtuse triangle.
(B) acute triangle.
(C) right triangle.
(D) isosceles triangle.

4. If the angles of triangle ABC are in the ratio of 5: 5: 8, what is the degree value of the greatest angle of triangle ABC?

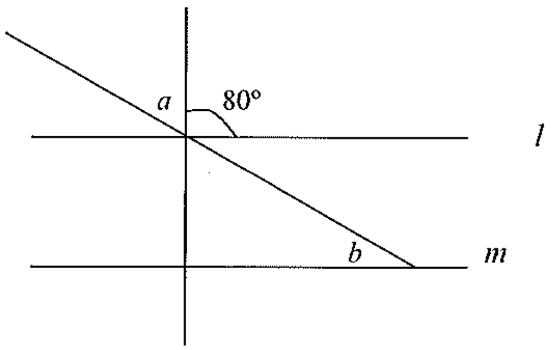
(A) 50
(B) 60
(C) 70
(D) 80



1. In the figure above, line l is parallel to line m and is perpendicular to line Q. If $a = b$, what is the value of b?

Solution: Answer: 45

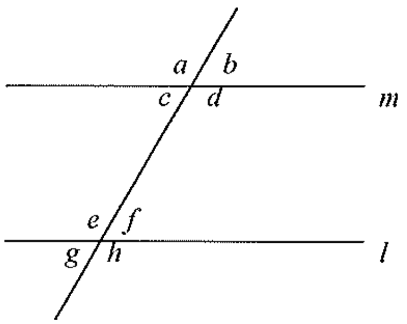
$$a + b = 90, \quad a = b$$
$$\Downarrow$$
$$b = 45$$



2. In the figure above, line l is parallel to line m. If $a = b$, what is the value of b?

Solution: Answer: 50

$$180 - 80 = 100$$
$$\downarrow$$
$$a + b = 100, \quad a = b$$
$$\Downarrow$$
$$b = 50$$

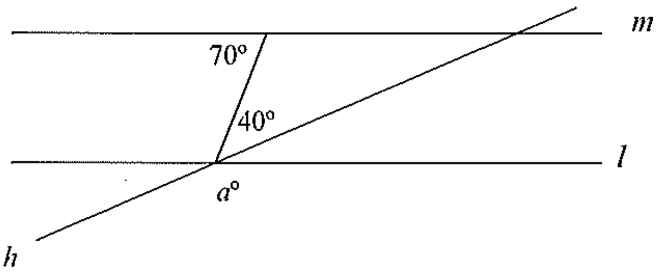


3. In the figure above, if line m is parallel to line l, then the sum of the measures of angles b and c must be equal to the sum of which of the following pairs of angles?

(A) a and b
(B) f and g
(C) e and f
(D) g and h

Solution: Answer: (B)

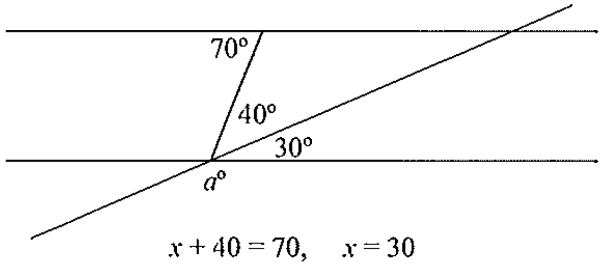
$$b = f, \quad c = g$$
$$\Downarrow$$
$$b + c = f + g$$



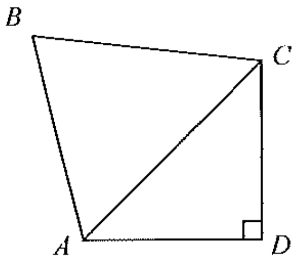
4. In the figure above, line m is parallel to line l. What is the value of a°?

(A) 140
(B) 110
(C) 150
(D) 130

Solution: Answer: (C)



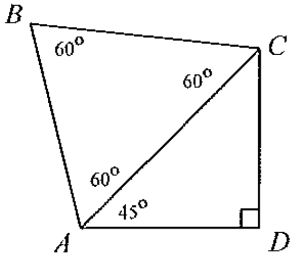
$$x + 40 = 70, \quad x = 30$$
$$\Downarrow$$
$$a^\circ = 180 - 30 = 150$$



5. In the figure above, triangle ACD is an isosceles triangle, and triangle ABC is an equilateral triangle. Which of the following is the measure of $\angle DAB$?

(A) 95°
(B) 105°
(C) 115°
(D) 75°

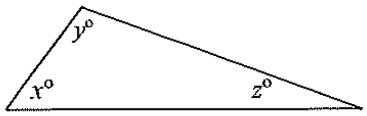
Solution: Answer: (B)



$$\Downarrow$$

$$\angle DAB = 45 + 60 = 105$$

Note: Figure below not drawn to scale.



6. In the figure above, $x < 30$ and $y = z + 3$. z is an integer. What is the least possible value of y?

Solution: Answer: 77

$$x + y + z = 180$$

$$x + y + (y - 3) = 180$$

$$2y = 183 - x,\ y = \frac{183 - x}{2}$$

$$\downarrow$$

If x is more bigger,

y will be more smaller.

$$x < 30,\ \rightarrow\ x = 29$$

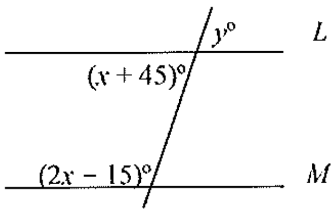
$$\downarrow$$

$$y = \frac{183 - 29}{2} = \frac{154}{2}$$

$$\Downarrow$$

$$y = 77$$

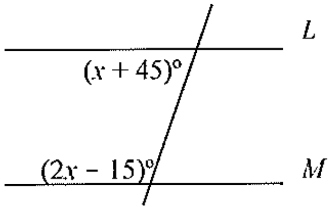
Note: Figure bellow not drawn to scale.



7. In the figure above, lines L and M are parallel to each other. What is the value of y?

(A) 95
(B) 70
(C) 65
(D) 80

Solution: Answer: (A)



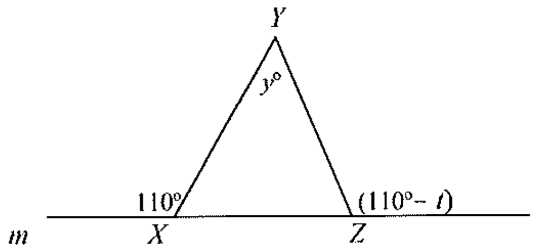
$$2x - 15 + x + 45 = 180$$

$$3x = 180 - 30$$

$$x = 50$$

$$\Downarrow$$

$$y = x + 45 = 50 + 45 = 95$$



8. In the figure above, side $\overline{XZ}$ of triangle XYZ is on line m. What is y in terms of t?

(A) $t - 40$
(B) $40 - t$
(C) $60 - t$
(D) $t - 60$

Solution: Answer: (B)

"1 Exterior = 2 non-ajacent Interiors"

(in measure)

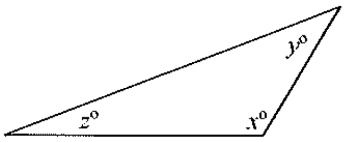
$$\downarrow$$

$$110 - t = y + 180 - 110$$

$$\Downarrow$$

$$y = 40 - t$$

Note: Figure below not drawn to scale.



9. If in the figure above, $x > 90$, $z = y - 1$, and y is an integer, what is the greatest possible value of z?

(A) 45
(B) 44
(C) 43
(D) 42

Solution: Answer: (B)

y° is an integer. $\rightarrow x^\circ$ and z° are integers.

$$y = z + 1$$

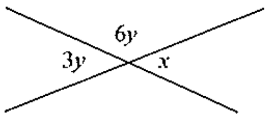
$$x + (z+1) + z = 180$$

$$z = \frac{179 - x}{2}$$

According to the equation above, when x has the least value, z has the greatest value.

$$\Downarrow$$

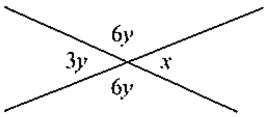
$$z = \frac{179 - 91}{2} = 44$$



10. In the figure above, what is the value of x?

(A) 30
(B) 40
(C) 50
(D) 60

Solution: Answer: (D)



$$\begin{cases} 360 = 15y + x & (1) \\ 3y = x \rightarrow y = \dfrac{x}{3} & (2) \end{cases}$$

$$(2) \rightarrow (1) \rightarrow 360 = 15 \times \frac{x}{3} + x$$

$$\Downarrow$$

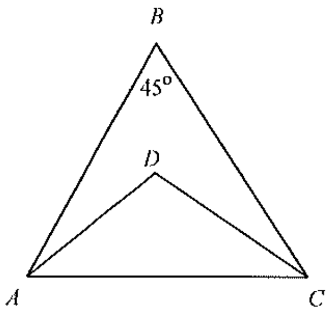
$$x = \boxed{60}$$

or

$$3y + 6y = 180,\ y = 20$$

$$\Downarrow$$

$$3y = x,\ x = 3 \times 20 = \boxed{60}$$

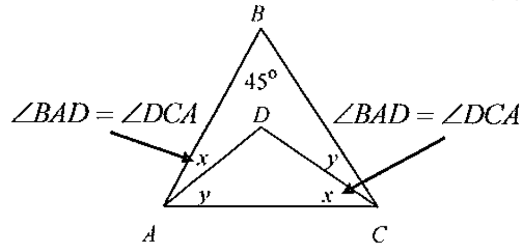


Note: Figure above not drawn to scale.

11. In the figure above, if $\overline{AB}$ is congruent to $\overline{BC}$ and $\angle BAD = \angle DCA$, what is $\angle ADC$?

(A) 112.5°
(B) 49°
(C) 78.33°
(D) 92.5°

Solution: Answer: (A)



$$180 - 45 = 135$$

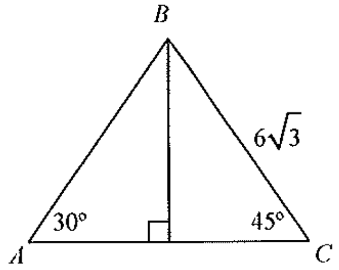
$$\angle BAC = \angle BCA = \frac{135}{2} = 67.5$$

Refer to the figure above, $x + y = 67.5$.

$$\angle ADC = 180 - (x + y) = 180 - 67.5$$

$$\Downarrow$$

$$\angle ADC = 112.5$$



Note: Figure above not drawn to scale.

12. In the figure above, which of the following is the value of $\overline{AB}$?

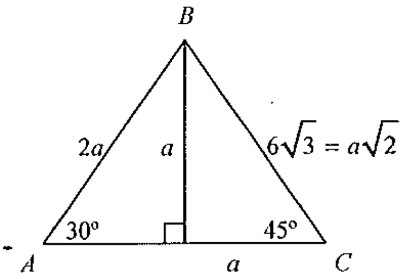
(A) $6\sqrt{5}$

(B) $3\sqrt{6}$

(C) $6\sqrt{6}$

(D) $6\sqrt{3}$

Solution: Answer: (C)



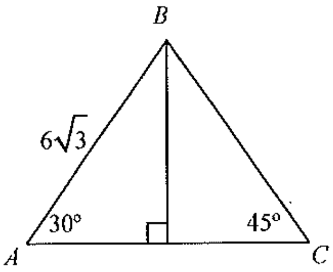
$$\frac{6\sqrt{3}}{\sqrt{2}} = a, \quad 6\sqrt{3} \cdot \frac{1}{\sqrt{2}} = a$$

$$\downarrow$$

$$\frac{6^{3}\sqrt{3} \cdot \sqrt{2}}{\cancel{2}} = a, \quad a = 3\sqrt{6}$$

$$\Downarrow$$

$$AB = 2a = 2 \times 3\sqrt{6} = \boxed{6\sqrt{6}}$$



Note: Figure above not drawn to scale.

13. In the figure above, which of the following is the value of $\overline{BC}$?

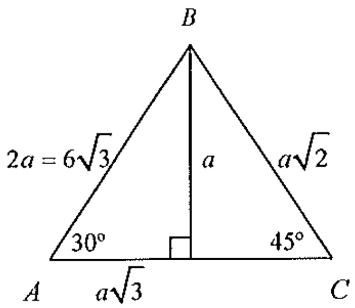
(A) $6\sqrt{3}$

(C) $6\sqrt{6}$

(B) $3\sqrt{8}$

(D) $3\sqrt{6}$

Solution: Answer: (D)



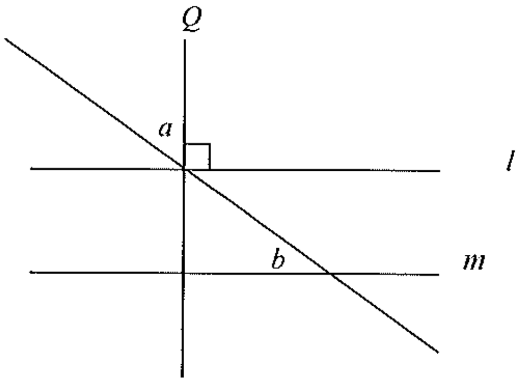
$$a = \frac{6\sqrt{3}}{2}, \quad a = 3\sqrt{3}$$

$$\downarrow$$

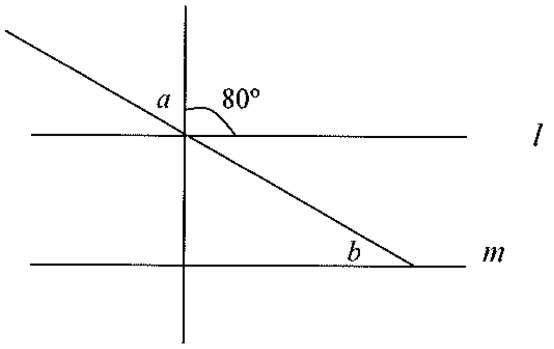
$$BC = a\sqrt{2} = 3\sqrt{3}\sqrt{2}$$

$$\Downarrow$$

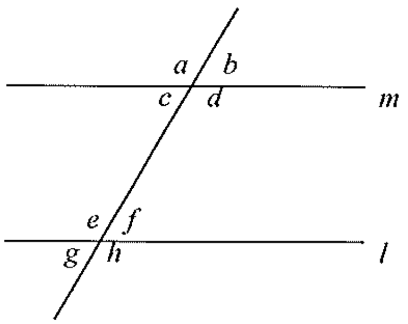
$$BC = 3\sqrt{6}$$



1. In the figure above, line l is parallel to line m and is perpendicular to line Q. If $a = b$, what is the value of b?

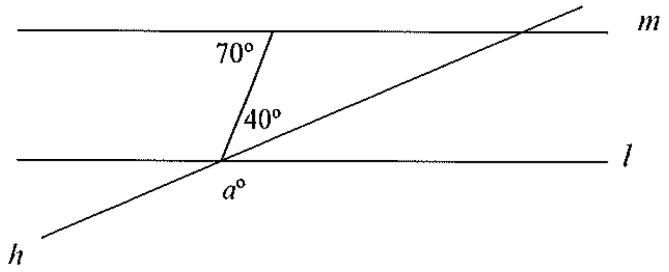


2. In the figure above, line l is parallel to line m. If $a = b$, what is the value of b?



3. In the figure above, if line m is parallel to line l, then the sum of the measures of angles b and c must be equal to the sum of which of the following pairs of angles?

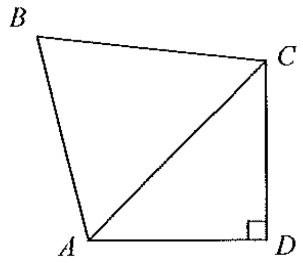
(A) a and b
(B) f and g
(C) e and f
(D) g and h



4. In the figure above, line m is parallel to line l. What is the value of $a°$?

(A) 140
(B) 110
(C) 150
(D) 130

Angles in the Plane



5. In the figure above, triangle ACD is an isosceles triangle, and triangle ABC is an equilateral triangle. Which of the following is the measure of $\angle DAB$?

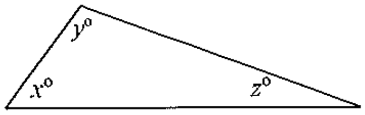
(A) 95°

(B) 105°

(C) 115°

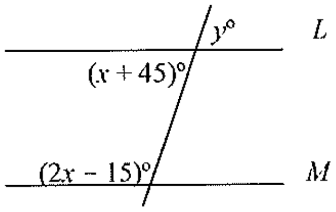
(D) 75°

Note: Figure below not drawn to scale.



6. In the figure above, $x < 30$ and $y = z + 3$. z is an integer. What is the least possible value of y?

Note: Figure bellow not drawn to scale.



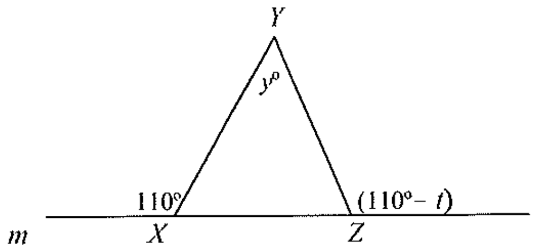
7. In the figure above, lines L and M are parallel to each other. What is the value of y?

(A) 95

(B) 70

(C) 65

(D) 80



8. In the figure above, side $\overline{XZ}$ of triangle XYZ is on line m. What is y in terms of t?

(A) $t - 40$

(B) $40 - t$

(C) $60 - t$

(D) $t - 60$

Note: Figure below not drawn to scale.

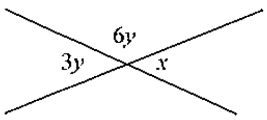
9. If in the figure above, $x > 90$, $z = y - 1$, and y is an integer, what is the greatest possible value of z?

(A) 45

(B) 44

(C) 43

(D) 42



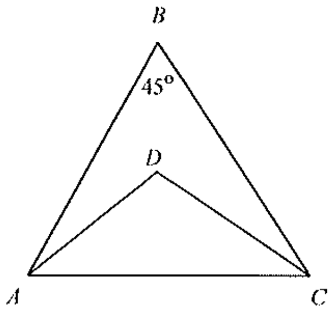
10. In the figure above, what is the value of x?

(A) 30

(B) 40

(C) 50

(D) 60



Note: Figure above not drawn to scale.

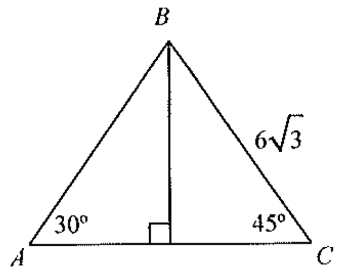
11. In the figure above, if $\overline{AB}$ is congruent to $\overline{BC}$ and $\angle BAD = \angle DCA$, what is $\angle ADC$?

(A) 112.5°

(B) 49°

(C) 78.33°

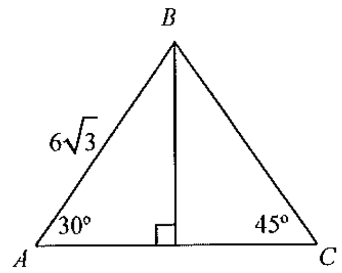
(D) 92.5°



Notc: Figure above not drawn to scale.

12. In the figure above, which of the following is the value of $\overline{AB}$?

(A) $6\sqrt{5}$
(B) $3\sqrt{6}$
(C) $6\sqrt{6}$
(D) $6\sqrt{3}$



Note: Figure above not drawn to scale.

13. In the figure above, which of the following is the value of $\overline{BC}$?

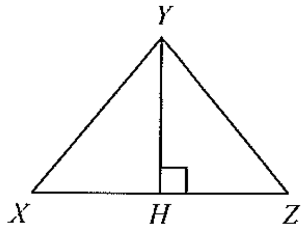
(A) $6\sqrt{3}$
(C) $6\sqrt{6}$
(B) $3\sqrt{8}$
(D) $3\sqrt{6}$

Legend:

When you see words "equilateral triangle", your reaction should have three things, not one or two:

(1) $\text{Side}_1 = \text{Side}_2 = \text{Side}_3$, (2) 60°, 60°, 60°

(3) $\text{Area} = \dfrac{S^2\sqrt{3}}{4}$



1. Triangle XYZ above is an equilateral triangle. If the length of side, S, is 10, what is the area of triangle XYZ?

(A) $\dfrac{5\sqrt{3}}{2}$
(B) 30
(C) 40
(D) $25\sqrt{3}$

Solution: Answer: (D)

$$\text{Area} = \frac{S^2\sqrt{3}}{4}$$

$$\Downarrow$$

$$\text{Area} = \frac{10^2\sqrt{3}}{4} = 25\sqrt{3}$$

2. Triangle XYZ above is an equilateral triangle. If the length of side, S, is 4, what is the area of XYZ?

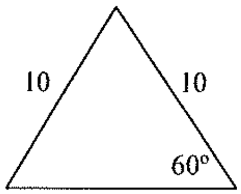
(A) $\dfrac{5\sqrt{3}}{2}$
(B) 30
(C) $4\sqrt{3}$
(D) $25\sqrt{3}$

Solution: Answer: (C)

$$\text{Area} = \frac{S^2\sqrt{3}}{4}$$

$$\Downarrow$$

$$\text{Area} = \frac{4^2\sqrt{3}}{4} = 4\sqrt{3}$$



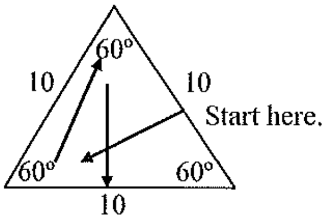
Note: Figure above not drawn to scale.

3. What is the perimeter of the triangle above?

(A) 60
(B) 30
(C) 40
(D) 28

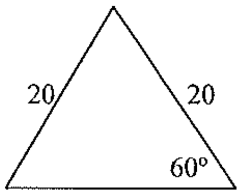
Solution: Answer: (B)

(1) $\text{Side}_1 = \text{Side}_2 = \text{Side}_3$, (2) 60°, 60°, 60°



$$\Downarrow$$

P = 10 + 10 + 10 = 30



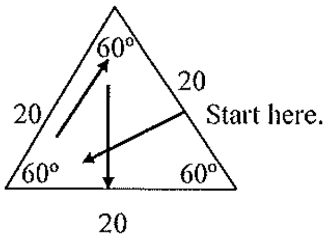
Note: Figure above not drawn to scale.

4. What is the perimeter of the triangle above?

(A) 60
(B) 30
(C) 40
(D) 28

Solution: Answer: (A)

(1) $\text{Side}_1 = \text{Side}_2 = \text{Side}_3$, (2) 60°, 60°, 60°



$$\Downarrow$$

P = 20 + 20 + 20 = 60

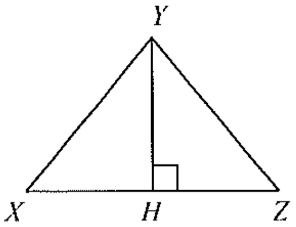
Areas and Perimeters of Equilateral Triangles

Legend:

When you see words "equilateral triangle", your reaction should have three things, not one or two:

(1) $\text{Side}_1 = \text{Side}_2 = \text{Side}_3$, (2) 60°, 60°, 60°

(3) Area $= \dfrac{S^2\sqrt{3}}{4}$

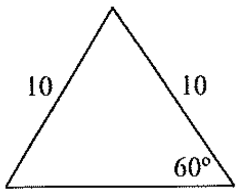


1. Triangle *XYZ* above is an equilateral triangle. If the length of side, *S*, is 10, what is the area of triangle *XYZ*?

(A) $\dfrac{5\sqrt{3}}{2}$
(B) 30
(C) 40
(D) $25\sqrt{3}$

2. Triangle *XYZ* above is an equilateral triangle. If the length of side, *S*, is 4, what is the area of *XYZ*?

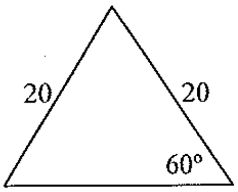
(A) $\dfrac{5\sqrt{3}}{2}$
(B) 30
(C) $4\sqrt{3}$
(D) $25\sqrt{3}$



Note: Figure above not drawn to scale.

3. What is the perimeter of the triangle above?

(A) 60
(B) 30
(C) 40
(D) 28



Note: Figure above not drawn to scale.

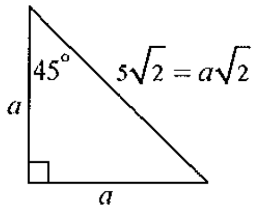
4. What is the perimeter of the triangle above?

(A) 60
(B) 30
(C) 40
(D) 28

1. A right triangle has one side of length $5\sqrt{2}$. If the lengths of the other two sides are equal, what is the area of the triangle?

(A) 25
(B) 18
(C) 12.5
(D) 10

Solution: Answer: (C)



This triangle is a 45° right triangle.

$$\downarrow$$

$$a = 5$$

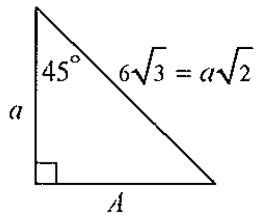
$$\Downarrow$$

$$A = \frac{a \times a}{2} = \frac{5 \times 5}{2} = 12.5$$

2. A right triangle has one side of length $6\sqrt{3}$. If the lengths of the other two sides are equal, what is the area of the triangle?

(A) 27
(B) 16
(C) 26
(D) 30

Solution: Answer: (A)



This triangle is a 45° right triangle.

$$\downarrow$$

$$a = \frac{6\sqrt{3}}{\sqrt{2}} = 6\sqrt{3} \cdot \frac{\sqrt{2}}{2} = 3\sqrt{6}$$

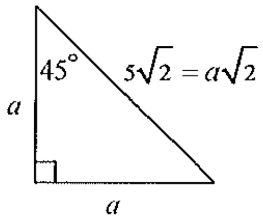
$$\Downarrow$$

$$\text{Area} = \frac{3\sqrt{6} \times 3\sqrt{6}}{2} = \frac{9 \times 6}{2} = 27$$

3. A right triangle has one side of length $5\sqrt{2}$. If the lengths of the other two sides are equal, what is the perimeter of the triangle?

(A) 15
(B) $5(2+\sqrt{2})$
(C) 20
(D) $10+\sqrt{2}$

Solution: Answer: (B)



This triangle is a 45° right triangle.

$$\downarrow$$

$$a = 5$$

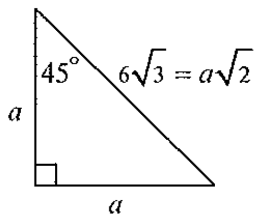
$$\Downarrow$$

$$P = 5 + 5 + 5\sqrt{2} = 5(2+\sqrt{2})$$

4. Hypotenuse of a right triangle is $6\sqrt{3}$. It is an isosceles triangle. What is the perimeter of the triangle?

(A) $12+\sqrt{2}$
(B) 18
(C) 20
(D) $6(\sqrt{6}+\sqrt{3})$

Solution: Answer: (D)



This triangle is a 45° right triangle.

$$\downarrow$$

$$a = 6\frac{\sqrt{3}}{\sqrt{2}} = 6\sqrt{3} \cdot \frac{\sqrt{2}}{2} = 3\sqrt{6}$$

$$\Downarrow$$

$$\text{Perimeter} = 3\sqrt{6} + 3\sqrt{6} + 6\sqrt{3} = 6(\sqrt{6}+\sqrt{3})$$

Areas and Perimeters of Isosceles Right Triangles

1. A right triangle has one side of length $5\sqrt{2}$. If the lengths of the other two sides are equal, what is the area of the triangle?

(A) 25
(B) 18
(C) 12.5
(D) 10

2. A right triangle has one side of length $6\sqrt{3}$. If the lengths of the other two sides are equal, what is the area of the triangle?

(A) 27
(B) 16
(C) 26
(D) 30

3. A right triangle has one side of length $5\sqrt{2}$. If the lengths of the other two sides are equal, what is the perimeter of the triangle?

(A) 15
(B) $5(2+\sqrt{2})$
(C) 20
(D) $10+\sqrt{2}$

4. Hypotenuse of a right triangle is $6\sqrt{3}$. It is an isosceles triangle. What is the perimeter of the triangle?

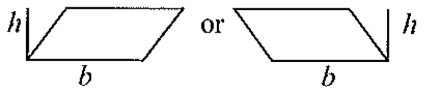
(A) $12+\sqrt{2}$
(B) 18
(C) 20
(D) $6(\sqrt{6}+\sqrt{3})$

1. In a parallelogram whose area is 75, the base is represented by $x + 5$ and the altitude is $x - 5$. What is the base of the parallelogram?

(A) 12
(B) 10
(C) 15
(D) 14

Solution: Answer: (C)

Altitude = height



$$\text{Area}_{\text{Parallelogram}} = \text{Base} \times \text{Height} = b \times h$$

$$75 = (x+5)(x-5),\ 75 = x^2 - 5^2$$

$$100 = x^2,\ x = 10$$

$$\Downarrow$$

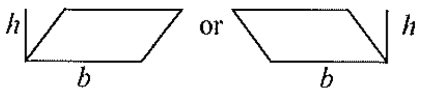
$$b = x + 5 = 10 + 5 = 15$$

2. In a parallelogram whose area is 72, the base is represented by $x + 3$ and the altitude is $x - 3$. What is the altitude of the parallelogram?

(A) 6
(B) 17
(C) 9
(D) 8

Solution: Answer: (A)

Altitude = height



$$\text{Area}_{\text{Parallelogram}} = \text{Base} \times \text{Height} = b \times h$$

$$72 = (x+3)(x-3),\ 72 = x^2 - 3^2$$

$$81 = x^2,\ x = 9$$

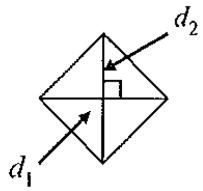
$$\Downarrow$$

$$h = x - 3 = 9 - 3 = 6$$

3. If the diagonals of a rhombus are represented by $6x$ and $8x$, what is the area of rhombus?

(A) $24x$
(B) $48x$
(C) $24x^2$
(D) $48x^2$

Solution: Answer: (C)



$$\text{Area}_{\text{Rhombus}} = \frac{\text{Diagonal}_1 \times \text{Diagonal}_2}{2} = \frac{d_1 \cdot d_2}{2}$$

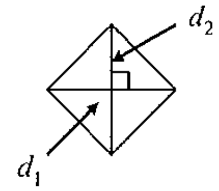
$$\Downarrow$$

$$\frac{6x \cdot 8x}{2} = 24x^2$$

4. If the diagonals of a rhombus are represented by $8x$ and $12x$, what is the area of rhombus?

(A) $24x$
(B) $48x$
(C) $24x^2$
(D) $48x^2$

Solution: Answer: (D)



$$\text{Area}_{\text{Rhombus}} = \frac{\text{Diagonal}_1 \times \text{Diagonal}_2}{2} = \frac{d_1 \cdot d_2}{2}$$

$$\Downarrow$$

$$\frac{8x \cdot 12x}{2} = 48x^2$$

Areas and Perimeters of Parallelograms

1. In a parallelogram whose area is 75, the base is represented by $x + 5$ and the altitude is $x - 5$. What is the base of the parallelogram?

(A) 12
(B) 10
(C) 15
(D) 14

2. In a parallelogram whose area is 72, the base is represented by $x + 3$ and the altitude is $x - 3$. What is the altitude of the parallelogram?

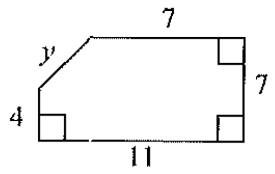
(A) 6
(B) 17
(C) 9
(D) 8

3. If the diagonals of a rhombus are represented by $6x$ and $8x$, what is the area of rhombus?

(A) $24x$
(B) $48x$
(C) $24x^2$
(D) $48x^2$

4. If the diagonals of a rhombus are represented by $8x$ and $12x$, what is the area of rhombus?

(A) $24x$
(B) $48x$
(C) $24x^2$
(D) $48x^2$

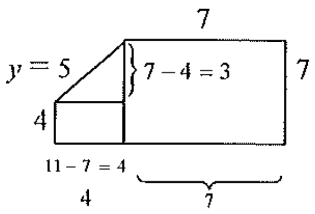


Note: Figure above not drawn to scale.

1. What is the perimeter of the figure shown above?

(A) 33
(B) 32
(C) 35
(D) 34

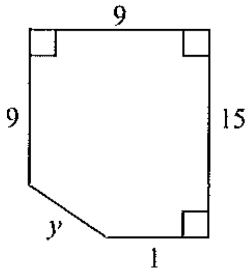
Solution: Answer: (D)



By $3-4-5$ right triangle $\rightarrow y = 5$

$\Downarrow$

$P = 5 + 7 + 7 + 7 + 4 + 4 = 34$

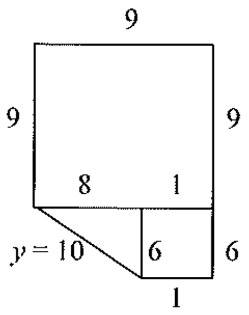


Note: Figure above not drawn to scale.

2. What is the perimeter of the figure shown above?

(A) 40
(B) 44
(C) 35
(D) 38

Solution: Answer: (B)



By $6-8-10$ right triangle $\rightarrow y = 10$

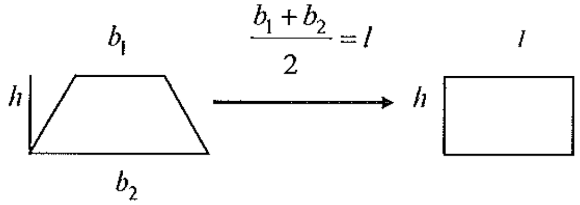
$\Downarrow$

$P = 10 + 9 + 9 + 9 + 6 + 1 = 44$

3. The area of a trapezoid whose bases are 12 and 10, and whose altitude is 5 is

(A) 33
(B) 44
(C) 55
(D) 66

Solution: Answer: (C)



$$l = \frac{b_1+b_2}{2} = \frac{12+10}{2} = 11$$

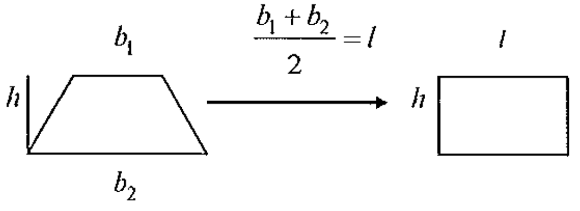
$\Downarrow$

$$A = h \cdot l = 5 \cdot 11 = 55$$

4. The area of a trapezoid whose bases are 13 and 9, and whose altitude is 3 is

(A) 33
(B) 44
(C) 55
(D) 66

Solution: Answer: (A)

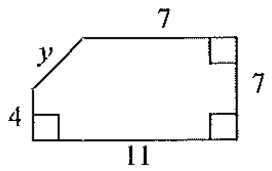


$$l = \frac{b_1+b_2}{2} = \frac{13+9}{2} = 11$$

$\Downarrow$

$$A = h \cdot l = 3 \cdot 11 = 33$$

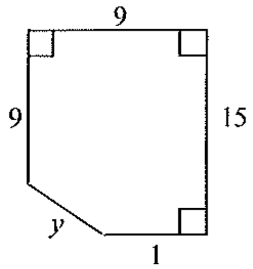
Areas and Perimeters of Polygons



Note: Figure above not drawn to scale.

1. What is the perimeter of the figure shown above?

(A) 33
(B) 32
(C) 35
(D) 34



Note: Figure above not drawn to scale.

2. What is the perimeter of the figure shown above?

(A) 40
(B) 44
(C) 35
(D) 38

3. The area of a trapezoid whose bases are 12 and 10, and whose altitude is 5 is

(A) 33
(B) 44
(C) 55
(D) 66

4. The area of a trapezoid whose bases are 13 and 9, and whose altitude is 3 is

(A) 33
(B) 44
(C) 55
(D) 66

1. The length of a rectangle is $9L$ and the width is $6W$. What is the perimeter?

(A) $18L + 12W$
(B) $15LW$
(C) $18L + 6W$
(D) $9L^2 + 6W^2$

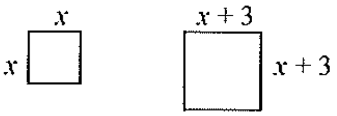
Solution: Answer: (A)

$$P = 2l + 2w$$
$$\Downarrow$$
$$18L + 12W$$

2. When 3 feet lengthen each side of a given square, 21 square feet increase the area of the square. What is the length, in feet, of a side of the original square?

(A) 6
(B) 5
(C) 4
(D) 3

Solution: Answer: (C)



$$(x+3)^2 - x^2 = 21$$
$$\downarrow$$
$$x^2 + 6x + 9 - x^2 = 21$$
$$\Downarrow$$
$$x = 2$$

3. If perimeter of a rectangle is 60 feet, the greatest area of the rectangle must be

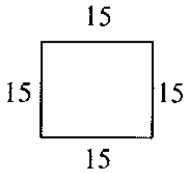
(A) 200 ft^2
(B) 400 ft^2
(C) 300 ft^2
(D) 225 ft^2

Solution: Answer: (D)

A square is a special rectangle.

When a non-square rectangle is transmuted into a square with the same perimeter, the square has the greater area than the area of the non-square rectangle.

$$s = \frac{60}{4} = 15$$
$$\downarrow$$



$$\Downarrow$$
$$A = 15^2 = 225$$

4. A rectangular living room is 90 feet length and 50 feet width. How many square yards of carpeting are needed to cover the floor?

(A) 4,500
(B) 1,500
(C) 800
(D) 500

Solution: Answer: (D)

feet $\rightarrow$ yards
$$\downarrow$$
$$\frac{90}{3}, \frac{50}{3}$$
$$\Downarrow$$
$$\text{Area}_{\text{yards}} = \frac{90}{3} \times \frac{50}{3} = 500$$

5. If perimeter of a rectangle is 80 feet, the greatest area of the rectangle must be

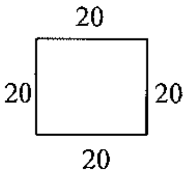
(A) 200 ft^2
(B) 400 ft^2
(C) 300 ft^2
(D) 225 ft^2

Solution: Answer: (B)

A square is a special rectangle.

When a non-square rectangle is transmuted into a square with the same perimeter, the square has the greater area than the area of the non-square rectangle.

$$s = \frac{80}{4} = 20$$
$$\downarrow$$



$$\Downarrow$$
$$A = 20^2 = 400$$

6. By what percent is the area increased if the length and width of a rectangle are each tripled?

(A) 300%
(B) 600%
(C) 500%
(D) 800%

Solution: Answer: (D)

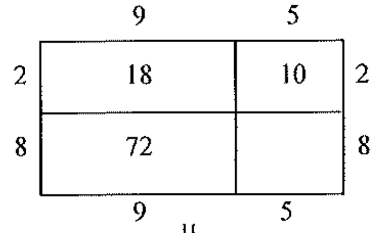
$$A = l \times w$$
$$A_{\text{tripled}} = 3l \times 3w = 9lw$$
$$\text{Percent}_{\text{increased}} = \frac{A_{\text{tripled}} - A}{A} \cdot 100\%$$
$$\Downarrow$$
$$\frac{9lw - lw}{lw} \cdot 100\% = 800\%$$

R	S
Q	T

7. In the figure above, the lengths and widths of rectangles Q, R, S, and T are integers. If the areas of rectangles Q, R, and S are 72, 18, and 10, respectively, what is the area of the greatest rectangle?

(A) 140
(B) 120
(C) 130
(D) 150

Solution: Answer: (A)

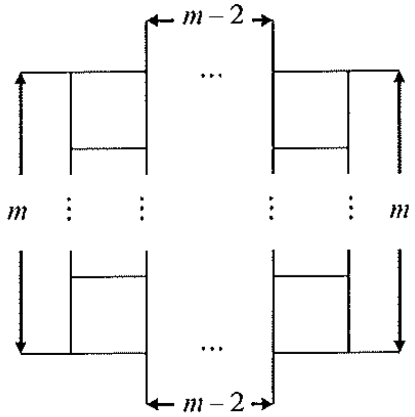


$$A = (9+5)\times(2+8) = 140$$

8. A square board is divided into m columns of m squares each. If n of these squares lie along the boundary of the board, which of the following is a possible value of n?

(A) 6
(B) 13
(C) 50
(D) 56

Solution: Answer: (D)



$$n = 2(m-2)+2m = 2m-4+2m$$
$$4m-4 = 4(m-1)$$

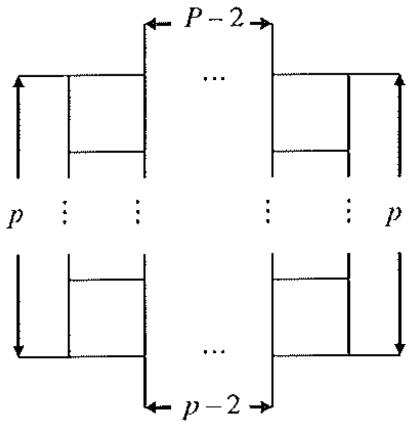
Try $m = 1, 2, 3,$ and, 10, 15,

$$n = 4(15-1) = 4\cdot 14 = \boxed{56}$$

9. A square board is divided into p columns of p squares each. If q of these squares lie along the boundary of the board, which of the following is a possible value of q?

(A) 4
(B) 62
(C) 50
(D) 13

Solution: Answer: (A)



$$q = 2(p-2)+2p = 2p-4+2p$$
$$4p-4 = 4(p-1)$$

When $p = 2$,

$$q = 2(2-2)+2\times 2 = \boxed{4}$$

1. The length of a rectangle is $9L$ and the width is $6W$. What is the perimeter?

(A) $18L + 12W$
(B) $15LW$
(C) $18L + 6W$
(D) $9L^2 + 6W^2$

2. When 3 feet lengthen each side of a given square, 21 square feet increase the area of the square. What is the length, in feet, of a side of the original square?

(A) 6
(B) 5
(C) 4
(D) 3

3. If perimeter of a rectangle is 60 feet, the greatest area of the rectangle must be

(A) 200 ft^2
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4. A rectangular living room is 90 feet length and 50 feet width. How many square yards of carpeting are needed to cover the floor?

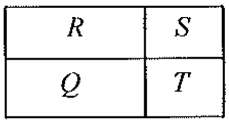
(A) 4,500
(B) 1,500
(C) 800
(D) 500

5. If perimeter of a rectangle is 80 feet, the greatest area of the rectangle must be

(A) 200 ft^2
(B) 400 ft^2
(C) 300 ft^2
(D) 225 ft^2

6. By what percent is the area increased if the length and width of a rectangle are each tripled?

(A) 300%
(B) 600%
(C) 500%
(D) 800%



7. In the figure above, the lengths and widths of rectangles Q, R, S, and T are integers. If the areas of rectangles Q, R, and S are 72, 18, and 10, respectively, what is the area of the greatest rectangle?

(A) 140
(B) 120
(C) 130
(D) 150

8. A square board is divided into m columns of m squares each. If n of these squares lie along the boundary of the board, which of the following is a possible value of n?

(A) 6
(B) 13
(C) 50
(D) 56

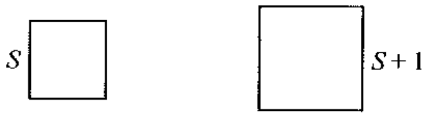
9. A square board is divided into p columns of p squares each. If q of these squares lie along the boundary of the board, which of the following is a possible value of q?

(A) 4
(B) 62
(C) 50
(D) 13

1. A first square has a side of length S units while the side of a second square is one unit greater than the length of side of the first square. What is an expression for the increase of the areas of the two squares?

(A) $2S^2 + 2S + 1$
(B) $2S^2 - 2S + 1$
(C) $2S - 1$
(D) $2S + 1$

Solution: Answer: (D)

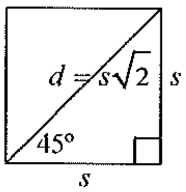


$$(S+1)^2 - S^2 = S^2 + 2S + 1 - S^2$$
$$\Downarrow$$
Difference = 2S + 1

2. What is the perimeter of a square whose diagonal is d?

(A) $4d$
(B) $2\sqrt{2}\cdot d$
(C) $3d^2$
(D) $2d^2$

Solution: Answer: (B)



$$\downarrow$$
$$s = \frac{d}{\sqrt{2}} = d\cdot\frac{1}{\sqrt{2}} = d\frac{\sqrt{2}}{2}$$
$$\downarrow$$
$$P_{\text{Square}} = 4s$$
$$\Downarrow$$
$$\not{4}^{2}\cdot\frac{d\sqrt{2}}{\not{2}} = 2\sqrt{2}d$$

3. A square is equal in area to a rectangle whose base is b and altitude is a. What is the perimeter of the square?

(A) $2\sqrt{ab}$
(B) ab
(C) $4\sqrt{ab}$
(D) $(ab)^2$

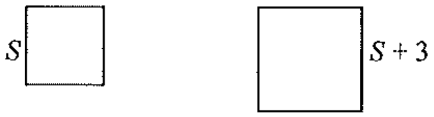
Solution: Answer: (C)

$$ab = s^2,\ s = \sqrt{ab}$$
$$\Downarrow$$
$$P_{\text{Square}} = 4s = 4\sqrt{ab}$$

4. A first square has a side of length S units while the side of a second square is 3 units greater than the length of side of the first square. What is an expression for the sum of the areas of the two squares?

(A) $S^2 + 6S + 9$
(B) $2S^2 - 6S + 9$
(C) $2S + 9$
(D) $2S^2 + 6S + 9$

Solution: Answer: (D)



$$(S+3)^2 + S^2 = S^2 + 6S + 9 + S^2$$
$$\Downarrow$$
Sum = $2S^2 + 6S + 9$

Areas and Perimeters of Squares

1. A first square has a side of length S units while the side of a second square is one unit greater than the length of side of the first square. What is an expression for the increase of the areas of the two squares?

(A) $2S^2 + 2S + 1$
(B) $2S^2 - 2S + 1$
(C) $2S - 1$
(D) $2S + 1$

2. What is the perimeter of a square whose diagonal is d?

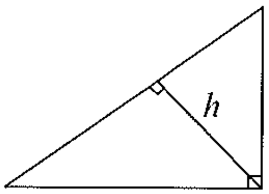
(A) $4d$
(B) $2\sqrt{2} \cdot d$
(C) $3d^2$
(D) $2d^2$

3. A square is equal in area to a rectangle whose base is b and altitude is a. What is the perimeter of the square?

(A) $2\sqrt{ab}$
(B) ab
(C) $4\sqrt{ab}$
(D) $(ab)^2$

4. A first square has a side of length S units while the side of a second square is 3 units greater than the length of side of the first square. What is an expression for the sum of the areas of the two squares?

(A) $S^2 + 6S + 9$
(B) $2S^2 - 6S + 9$
(C) $2S + 9$
(D) $2S^2 + 6S + 9$



Note: Figure above not drawn to scale.

1. In the figure above, if the sides of the right triangle are 6, 8, and 10, what is the altitude h drawn to the hypotenuse?

(A) 2.4
(B) 4.8
(C) 3.2
(D) 6.4

Solution: Answer: (B)

Any side of a triangle can be a base.

The corresponding heights of the bases in an triangle may be different. But the area is unique for the same triangle.

When 6 is its base and 8 is its height,

$$\text{Area} = \frac{6 \cdot 8}{2} = 24.$$

When 10 is its base and h is its height, the area is still equal to 24.

$$\Downarrow$$

$$24 = \frac{10 \cdot h}{2},\ h = \boxed{4.8}$$

2. In the figure above, if the two legs of the right triangle are 3 and 4, what is the altitude h drawn to the hypotenuse?

(A) 2.4
(B) 3.6
(C) 3.2
(D) 6.4

Solution: Answer: (A)

Any side of a triangle can be a base.

The corresponding heights of the bases in an triangle may be different. But the area is unique for the same triangle.

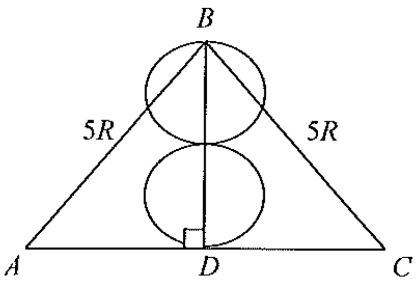
When 4 is its base and 3 is its height,

$$\text{Area} = \frac{3 \cdot 4}{2} = 6.$$

When 5 is its base and h is its height, the area is still equal to 6.

$$\Downarrow$$

$$6 = \frac{5 \cdot h}{2},\ h = 2.4$$

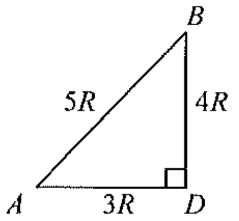


Note: Figure above not drawn to scale.

3. In the figure above, if the radius R for each circle is 4 and the lengths of two sides of triangle ABC are $5R$, what is the perimeter of triangle ABD?

(A) 72
(B) 36
(C) 48
(D) 24

Solution: Answer: (C)

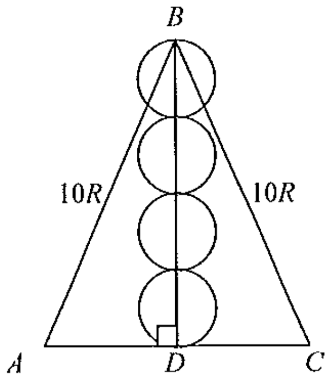


$$R = 4$$

$$\Downarrow$$

$$P = 3R + 4R + 5R = 12R = 12 \cdot 4 = 48$$

Areas and Perimeters of Triangles



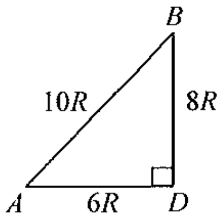
Note: Figure above not drawn to scale.

4. In the figure above, if the radius R for each circle is 2 and the lengths of two sides of triangle ABC are $10R$, what is the perimeter of triangle ABD?

(A) 72
(B) 36
(C) 48
(D) 24

Solution: Answer: (C)

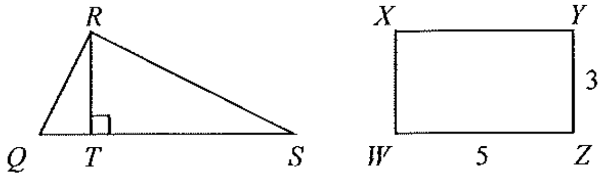
One of the triangles is a 6-8-10 right triangle.



$R = 2$

$\Downarrow$

$P = 6R + 8R + 10R = 24R = 24 \cdot 2 = 48$



Note: Figure above not drawn to scale.

5. Triangle QRS and rectangle $WXYZ$ have eqaul areas. The length $\overline{RT}$ is 6. What is the length of $\overline{QS}$?

(A) $\frac{5}{2}$
(B) 10
(C) 9
(D) 5

Solution: Answer: (D)

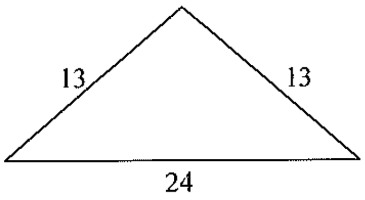
$A_{\text{Tri}} = A_{\text{Rec}}$

$\downarrow$

$\frac{6 \times QS}{2} = 3 \times 5$

$\Downarrow$

$QS = 5$

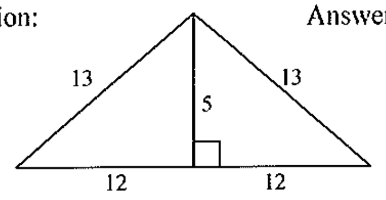


Note: Figure above not drawn to scale.

6. In the figure above, what is the area of a triangle whose sides are 13, 13, and 24 ?

(A) 48
(B) 60
(C) 24
(D) 36

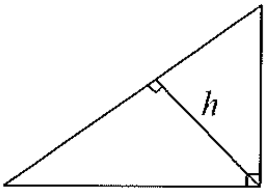
Solution: Answer: (B)



5-12-13 right triangle

$\Downarrow$

$A_{\text{Tri}} = \frac{24 \times 5}{2} = 60$



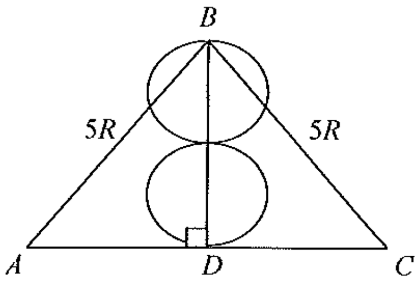
Note: Figure above not drawn to scale.

1. In the figure above, if the sides of the right triangle are 6, 8, and 10, what is the altitude h drawn to the hypotenuse?

(A) 2.4
(B) 4.8
(C) 3.2
(D) 6.4

2. In the figure above, if the two legs of the right triangle are 3 and 4, what is the altitude h drawn to the hypotenuse?

(A) 2.4
(B) 3.6
(C) 3.2
(D) 6.4

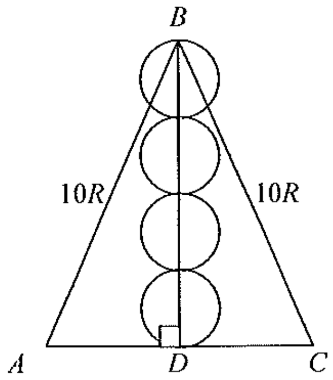


Note: Figure above not drawn to scale.

3. In the figure above, if the radius R for each circle is 4 and the lengths of two sides of triangle ABC are $5R$, what is the perimeter of triangle ABD?

(A) 72
(B) 36
(C) 48
(D) 24

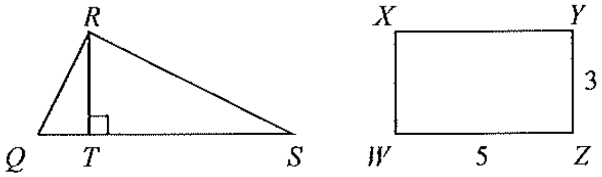
Areas and Perimeters of Triangles



Note: Figure above not drawn to scale.

4. In the figure above, if the radius R for each circle is 2 and the lengths of two sides of triangle ABC are $10R$, what is the perimeter of triangle ABD?

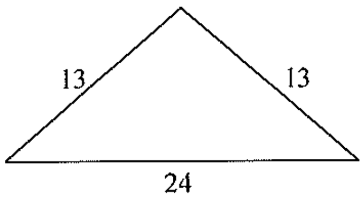
(A) 72
(B) 36
(C) 48
(D) 24



Note: Figure above not drawn to scale.

5. Triangle QRS and rectangle $WXYZ$ have eqaul areas. The length $\overline{RT}$ is 6. What is the length of $\overline{QS}$?

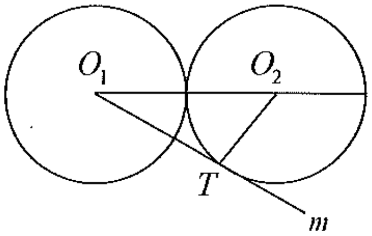
(A) $\frac{5}{2}$
(B) 10
(C) 9
(D) 5



Note: Figure above not drawn to scale.

6. In the figure above, what is the area of a triangle whose sides are 13, 13, and 24 ?

(A) 48
(B) 60
(C) 24
(D) 36

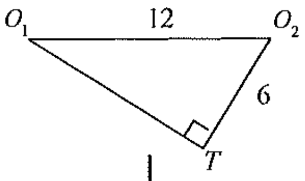


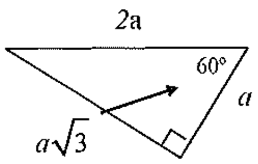
Note: Figure above not drawn to scale.

1. In the figure above, the circle with center O_1 and the circle with center O_2 are tangent. If the circles each have radius 6, and if line m is tangent to the circle with center O_2 at point T, what is the measure of $\angle O_2$?

(A) 30
(B) 62
(C) 60
(D) 61

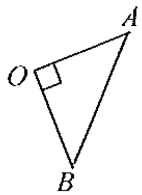
Solution: Answer: (C)





$\Downarrow$

Answer is (C).



2. In the figure above, O is the center of a circle with radius 12. If A and B are on the circle and $OA=OB=12$, what is the area of the triangle?

Solution: Answer: 72

$$\text{Area}_{\text{triangle}}=\frac{\text{leg}_1\times\text{leg}_2}{2}=\frac{\text{radius}\times\text{radius}}{2}$$

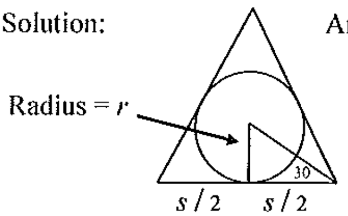
$\Downarrow$

$$\text{Area}_{\text{triangle}}=\frac{12\times 12}{2}=72$$

3. Which of the following is the area of the circle that is inscribed in an equilateral triangle whose sides each are s?

(A) $\frac{s}{2\sqrt{3}}$
(B) $\frac{s^2}{12}\pi$
(C) $\frac{s}{2}$
(D) $\frac{s}{4}$

Solution: Answer: (B)



$$\frac{s}{2}=r\sqrt{3},\quad r=\frac{s}{2\sqrt{3}}$$

$$A_{\text{C}}=r^2\pi$$

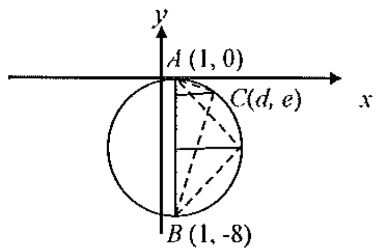
$\Downarrow$

$$A_{\text{C}}=\left(\frac{s}{2\sqrt{3}}\right)^2\pi=\frac{s^2}{12}\pi$$

4. In the xy-plane, point $A(1, 0)$, $B(1, -8)$, and $C(d, e)$ are the vertices of a right triangle. If $\overline{AB}$ is the hypotenuse of the triangle, which of the following cannot be the area of the triangle?

(A) 16
(B) 6.2
(C) 15.59
(D) 16.1

Solution: Answer: (D)



Because $\overline{AB}$ is the hypotenuse of triangle ABC, triangle ABC can be inscribed in the circle with diameter $\overline{AB}$, as shown in the figure above.

$$\text{Area}_{ABC}=\frac{1}{2}bh=\frac{1}{2}\cdot|-8|h$$

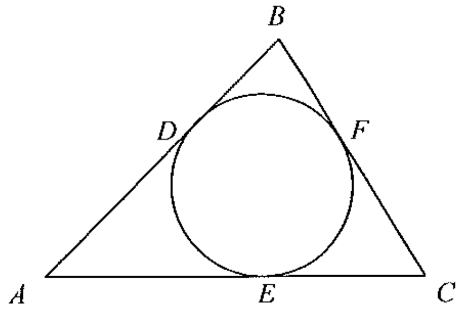
$$0<h\le 4$$

$\downarrow$

$$0<\text{Area}_{ABC}\le 16$$

$\Downarrow$

Answer is (D).



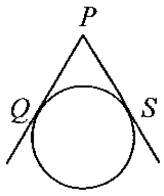
Note: Figure above not drawn to scale.

5. In the figure above, if circle is inscribed in triangle ABC, $AD = 8$ and $CF = 2$ what is the length of the side AC?

(A) 16
(B) 12
(C) 14
(D) 10

Solution: Answer: (D)

Type: A circle is tangent by segments.



The circle is tangent by the segments PQ and PS.

$\downarrow$

$PQ = PS$.

$AE = AD = 8$ and $EC = CF = 2$

$\Downarrow$

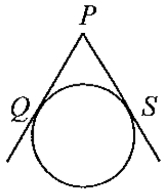
$AC = AE + EC = 8 + 2 = 10$

6. In the figure above, if circle is inscribed in triangle ABC, $BF = 3$ and $AE = 9$ what is the length of the side AB?

(A) 16
(B) 12
(C) 14
(D) 10

Solution: Answer: (B)

Type: A circle is tangent by segments.



The circle is tangent by the segments PQ and PS.

$\downarrow$

$PQ = PS$

$\downarrow$

$DA = AE = 9$ and $BD = BF = 3$

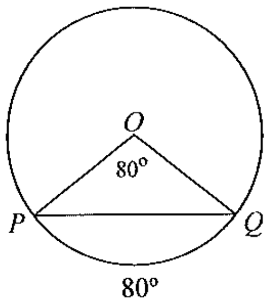
$\Downarrow$

$AB = DA + BD = 9 + 3 = 12$

7. In a circle whose center is O, arc PQ contains 80°. What is the number of degrees in angle PQO?

(A) 80
(B) 70
(C) 45
(D) 50

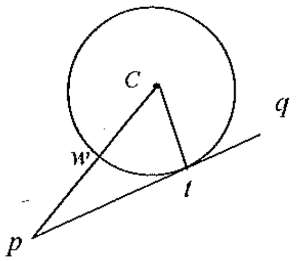
Solution: Answer: (D)



$PO = QO =$ Radius.

$\Downarrow$

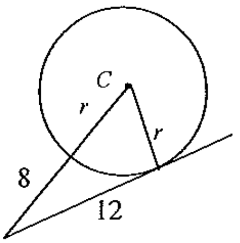
$$m\angle PQO = m\angle QPO = \frac{180 - 80}{2} = 50$$



Note: Figure above not drawn to scale.

8. In the figure above, the segment pq is tangent to circle C at point t. If $pt = 12$ and $pw = 8$, what is the radius of circle C?

Solution: Answer: 5



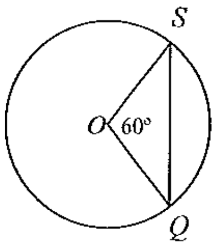
$(8 + r)^2 = 12^2 + r^2$

$\downarrow$

$64 + 2 \cdot 8 \cdot r + \cancel{r^2} = 144 + \cancel{r^2}$

$\Downarrow$

$r = 5$



Note: Figure above not drawn to scale.

9. In the figure above, O is the center of the circle with diameter 12. What is the length of the segment $\overline{SQ}$?

(A) 6
(B) 12
(C) 18
(D) 24

Solution: Answer: (A)

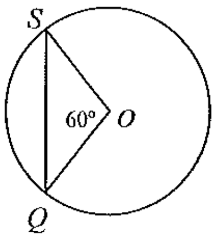
$\angle SOQ = 60^\circ$ and $r = SO = QO$

$\rightarrow \angle OSQ = \angle OQS = 60^\circ$

$\rightarrow SOQ$ = equilateral triangle

$\Downarrow$

$$SQ = SO = QO = r = \frac{D}{2} = \frac{12}{2} = 6$$



Note: Figure above not drawn to scale.

10. In the figure above, O is the center of the circle with circumference 12. What is the area of triangle SOQ?

(A) $\frac{6\sqrt{2}}{\pi}$
(B) $\frac{8\sqrt{2}}{\pi}$
(C) $\frac{9\sqrt{3}}{\pi^2}$
(D) $\frac{9\sqrt{2}}{\pi^2}$

Solution: Answer: (C)

$\angle SOQ = 60^\circ$ and $r = SO = QO$

$\downarrow$

$\angle OSQ = \angle OQS = 60^\circ$

$\downarrow$

SOQ = equilateral triangle

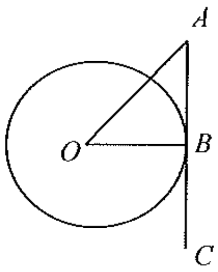
$$C = 2r\pi,\ r = \frac{C}{2\pi} = \frac{12}{2\pi} = \frac{6}{\pi}$$

$$SQ = SO = QO = r = \frac{6}{\pi}$$

Use the formula of area of equilateral triangle.

$\Downarrow$

$$\text{Area} = \frac{s^2\sqrt{3}}{4} = \frac{\left(\frac{6}{\pi}\right)^2\sqrt{3}}{4} = \frac{9\sqrt{3}}{\pi^2}$$



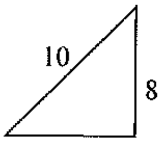
Note: Figure above not drawn to scale.

11. In the figure above, $\overline{AC}$ is tangent to circle O at point B. $\overline{BO}$ is radius of the circle. $AB = 8$, $AO = 10$. What is circumference of the circle?

(A) $2\sqrt{2}$
(B) 1
(C) 2
(D) 12π

Solution: Answer: (D)

AOB is a 6-8-10 right triangle.



$BO = 6$

$\Downarrow$

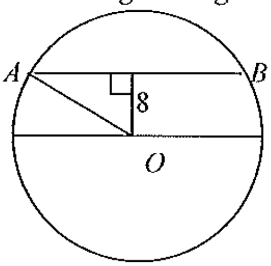
$$C = 2r\pi = 2\times 6\times \pi = 12\pi$$

12. The shortest distance from the center of a circle to a chord AB is 8. If the diameter of the center is 20, what is the length of the chord?

(A) 10
(B) 12
(C) 6
(D) 13

Solution: Answer: (B)

6- 8-10 right triangle



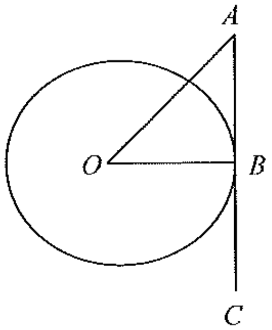
$$\text{Radius} = \frac{\text{Diameter}}{2} = \frac{20}{2} = 10$$

$\downarrow$

$OA = 10$ = radius

$\Downarrow$

$$\frac{AB}{2} = 6,\ AB = 12$$



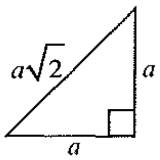
Note: Figure above not drawn to scale.

13. In the figure above, $\overline{AC}$ is tangent to the circle O at point B. BO is the radius of this circle. $BO = AB = \sqrt{2}$. What is AO?

(A) $2\sqrt{2}$
(B) 1
(C) 2
(D) $\sqrt{3}$

Solution: Answer: (C)

$\triangle AOB$ is a right triangle with $45°$.



$$\Downarrow$$

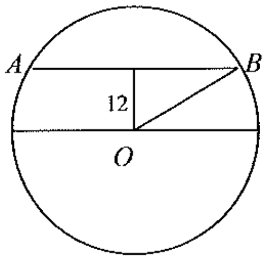
$$AO = \sqrt{2}\ a = \sqrt{2} \times \sqrt{2} = 2$$

14. The shortest distance from the center of a circle to a chord AB is 12. If the length of the chord is 10, what is the length of radius of the circle?

(A) 10
(B) 12
(C) 6
(D) 13

Solution: Answer: (D)

Type: 5- 12-13 right triangle



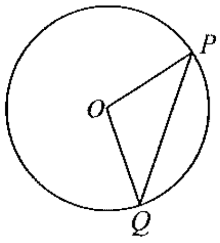
$$\frac{AB}{2} = \frac{10}{2} = 5$$

$$\Downarrow$$

Radius $= OB = 13$

or

$$OB = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$$



Note: Figure above not drawn to scale.

15. Point O is the center of the circle in the figure above. $m\angle POQ = 40°$. What is the degree measure of the angle PQO?

(A) 50
(B) 70
(C) 60
(D) 65

Solution: Answer: (B)

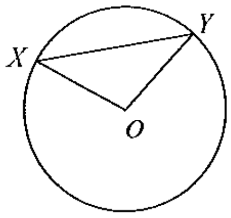
$$OP = OQ = \text{radius}$$

$$\downarrow$$

$$m\angle PQO = m\angle QPO$$

$$\Downarrow$$

$$m\angle PQO = \frac{180 - 40}{2} = 70°$$



Note: Figure above not drawn to scale.

16. Point O is the center of the circle in the figure above. If $m\angle XOY = 50°$, what is the measure of $\angle OXY$?

(A) 50
(B) 70
(C) 60
(D) 65

Solution: Answer: (D)

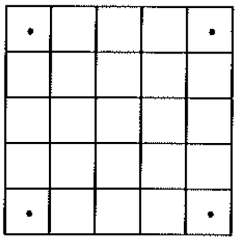
$$OX = OY = \text{radius}$$

$$\downarrow$$

$$m\angle OXY = m\angle OYX$$

$$\Downarrow$$

$$m\angle OXY = \frac{180 - 50}{2} = 65$$



Note: Figure above not drawn to scale.

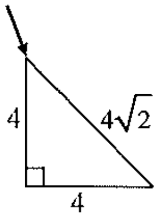
17. In the figure above, a square with side length 10 is divided into 25 squares. What is the area of the circle (not shown here) that passes through the 4 points, which are the centers of the 4 corner squares?

(A) 8π
(B) $2\sqrt{2}\pi$
(C) 12π
(D) 32π

Solution: Answer: (D)

$\frac{10}{5} = 2 \Rightarrow$ Length of each grid = 2; Length of each half grid = 1

Circle Center



$\Downarrow$

$$A = r^2\pi = \left(4\sqrt{2}\right)^2 \pi = 32\pi$$

18. If the radius r of a circle is increased by $2r$, the circumference of this circle is increased by

(A) $r\pi$
(B) $2r\pi$
(C) $3r\pi$
(D) $4r\pi$

Solution: Answer: (D)

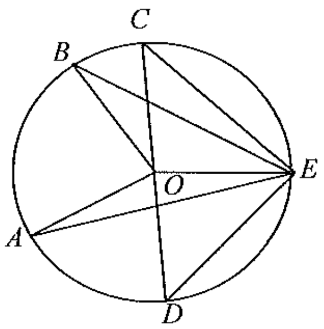
$$C_r = 2r\pi$$

$\downarrow$

$$C_{r+2r} = 2(r+2r)\pi = 6r\pi$$

$\Downarrow$

$$C_{r+2r} - C_r = 6r\pi - 2r\pi = 4r\pi$$



19. In the figure above, $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, $\overline{OD}$, and $\overline{OE}$ each are a radius of the circle with centering O. Which of the following triangles has the least area?

(A) DOE
(B) COE
(C) BOE
(D) AOE

Solution: Answer: (D)

Choose radius as the base for each triangle.

$$\text{Area}_{\text{TRi}} = \frac{\text{base} \times \text{height}}{2}$$

Triangle AOE has the same base and the least height.

$\Downarrow$

Answer is (D).

20. If the hour hand of a watch is 6 units long, how many units does the hand tip move between 8:30am and 12:30pm?

(A) 2π
(B) 4π
(C) π
(D) 3π

Solution: Answer: (B)

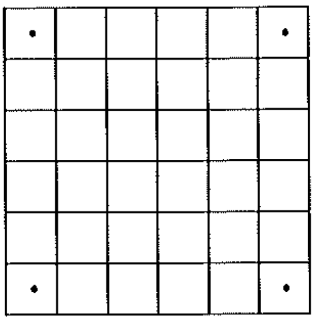
$$C = 2r\pi = 2 \times 6\pi = 12\pi$$

$\downarrow$

$$\text{Ratio} = \frac{4}{12} = \frac{1}{3}$$

$\Downarrow$

$$12\pi \times \frac{1}{3} = 4\pi$$



Note: Figure above not drawn to scale.

21. In the figure above, a square with side length 6 is divided into 36 squares. What is ratio of the area of the circle (not shown here) that passes through the 4 points, which are the centers of the 4 corner squares, to circumference of the circle?

(A) $\frac{5\sqrt{2}}{4}$

(B) $4\sqrt{2}$

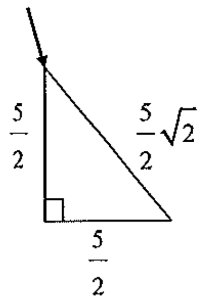
(C) $5\sqrt{2}$

(D) $\frac{25}{\sqrt{2}}$

Solution: Answer: (A)

$\frac{6}{6}=1$ → Length of each grid = 1; Length of each half grid = $\frac{1}{2}$

Circle Center



$\downarrow$

$A=r^2\pi, \quad C=2r\pi$

$\Downarrow$

$$\frac{A}{C}=\frac{r\not{\pi}}{2\not{\pi}}=\frac{r}{2}=\frac{\frac{5}{2}\sqrt{2}}{2}=\frac{5\sqrt{2}}{4}$$

22. The area of the isosceles right triangle in a circle with radius $\frac{AB}{2}=6\sqrt{2}$, where AB is the hypotenuse of the triangle, is

(A) 144

(B) 98

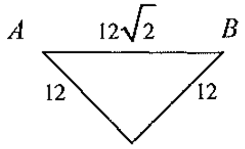
(C) 72

(D) 86

Solution: Answer: (C)

Diameter $=2\times 6\sqrt{2}=12\sqrt{2}$

It is a 45° right triangle. So *Leg* = 12



$\Downarrow$

$$\text{Area}=\frac{b\times h}{2}=\frac{\text{leg}\times\text{leg}}{2}=\frac{12\times 12}{2}=72$$

23. If the radius of a circle is doubled, then the

(A) circumference and area are both doubled.

(B) circumference is tripled and area is doubled.

(C) circumference is doubled and area is tripled.

(D) circumference is doubled and area is 4 times of the original area.

Solution: Answer: (D)

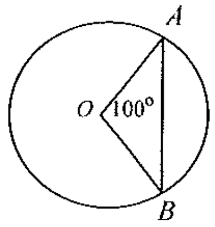
$C_r=2r\pi$

$A_r=r^2\pi$

$C_{2r}=2(2r)\pi=4r\pi=2(2r\pi)=2C_r$

$\Downarrow$

$A_{2r}=(2r)^2\pi=4r^2\pi=4A_r$



Note: Figure above not drawn to scale.

24. In the figure above, what is the number of degrees in angle ABO?

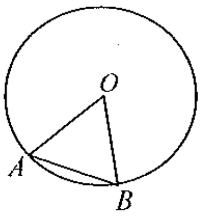
(A) 40
(B) 70
(C) 30
(D) 10

Solution: Answer: (A)

Radius = $AO = BO$

$\Downarrow$

$$m\angle BAO = \angle ABO = \frac{180-100}{2} = 40^\circ$$



Note: Figure above not drawn to scale.

25. In the figure above, radius = 60, what is the perimeter of triangle AOB?

(A) 240
(B) 90
(C) 120
(D) 180

Solution: Answer: (D)

Radius = $AO = BO$ and $\angle AOB = 60^\circ$

$\downarrow$

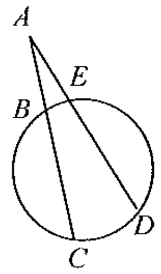
$$m\angle BAO = \angle ABO = \frac{180-60}{2} = 60^\circ$$

$\downarrow$

$AB = AO = BO = 60$

$\Downarrow$

Perimeter = $3 \times 60 = 180$



26. In the figure above, the number of degrees in angle CAD is

(A) $\frac{CD+BE}{2}$

(B) $\frac{CD}{2}-\frac{BE}{2}$

(C) $\frac{CD}{2}+BE$

(D) $\frac{BE-CD}{2}$

Solution: Answer: (B)

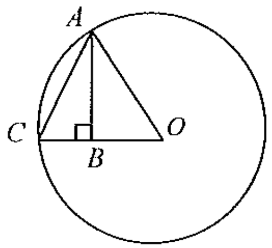
Rule : $\left[\begin{array}{l} x = \frac{CD-BE}{2} \\ CD > BE \end{array}\right.$

$\downarrow$

$x = m\angle CAD$

$\Downarrow$

$$m\angle CAD = \frac{CD-BE}{2} = \left(\frac{CD}{2}-\frac{BE}{2}\right)^\circ$$



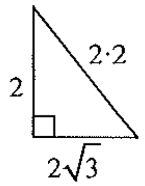
Note: Figure above not drawn to scale.

27. In the figure above shows a circle whose center is at O. If $AO = 4$ and $AB = 2$, what is the perimeter of triangle ABO?

(A) $2(3+\sqrt{2})$
(B) $2(3+\sqrt{3})$
(C) $6+\sqrt{3}$
(D) $2(3+2\sqrt{3})$

Solution: Answer: (B)

ABO is a 30°-60°-right triangle.



$\Downarrow$

Perimeter = $2+4+2\sqrt{3} = 2(3+\sqrt{3})$

28. A particle moves in a circular path at 2π feet per minute. If it travels an arc of 40° in 20 seconds, what is the diameter of the circular path?

(A) 6π
(B) 45π
(C) 1
(D) 6

Solution: Answer: (D)

20 seconds, 40° → $\underbrace{60 \text{ seconds}}_{\text{one minute}}$, 120°

degrees ↑, feet ↑

↓

Direct proportion → Use the equation.

$$\frac{x}{\square} = \frac{\square_x}{\square} \rightarrow \frac{C}{360} = \frac{2\pi}{120}$$

↓

$$C = \frac{2\pi \times 360}{120} = 6\pi, \qquad C = D\pi$$

⇓

$$D = \frac{C}{\pi} = \frac{6\pi}{\pi} = 6$$

29. A particle moves in a circular path at π yards per minute. If it travels an arc of 60° in 20 seconds, what is the radius of the circular path?

(A) 6π
(B) 45π
(C) 1
(D) 6

Solution: Answer: (C)

20 seconds, 60° → $\underbrace{60 \text{ seconds}}_{\text{one minute}}$, 180°

degrees ↑, yards ↑

↓

Direct proportion → Use the equation.

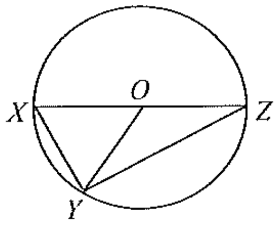
$$\frac{x}{\square} = \frac{\square_x}{\square} \rightarrow \frac{C}{360} = \frac{\pi}{180}$$

↓

$$C = \frac{\pi \times 360}{180} = 2\pi, \qquad C = 2r\pi$$

⇓

$$r = \frac{C}{2\pi} = \frac{2\pi}{2\pi} = 1$$



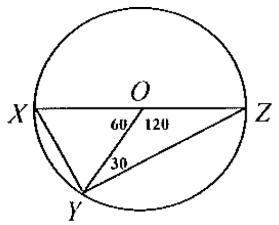
Note: Figure above not drawn to scale

30. In the figure above, triangle XYZ is inscribed in the circle that has center O. If $XY = OX$, which of the following is the measure, in degree, of angle OYZ?

(A) 60
(B) 45
(C) 70
(D) 30

Solution: Answer: (D)

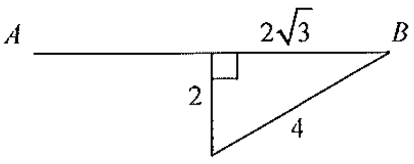
The radii form an equilateral triangle OYX and an isosceles triangle YOZ.



31. A line segment joining 2 points A and B on circumference of a circle is 2 yards from the center of the circle at its closest point. If the circle has a radius of four yards, what is the length of the segment?

(A) $2\sqrt{3}$
(B) $2\sqrt{2}$
(C) $4\sqrt{3}$
(D) $3\sqrt{2}$

Solution: Answer: (C)



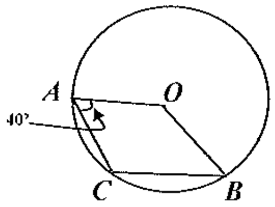
30°-right triangle

↓

$a,\ 2a,\ a\sqrt{3}$, Now $a = 2$

⇓

$$AB = 2 \times 2\sqrt{3} = 4\sqrt{3}$$



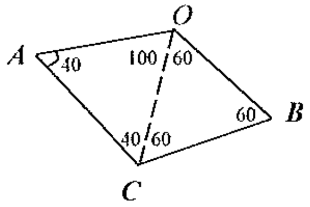
Note: Figure above not drawn to scale.

32. In the figure above, points A, B, and C lie on the circumference of the circle centered at O. If $m\angle OBC = 60^\circ$, then $m\angle AOB = ?$

(A) 160
(B) 120
(C) 130
(D) 140

Solution: Answer: (A)

Draw a segment $\overline{OC}$ in the figure below.



(1). $OC = OB$ = Radius

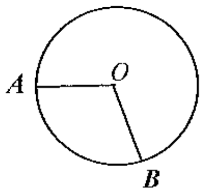
$m\angle BOC = 60^\circ \rightarrow m\angle OCB = 60^\circ$

(2). $OC = OA$ = Radius

$m\angle OCA = 40^\circ \rightarrow m\angle AOC = 100^\circ$

$$\Downarrow$$

$$m\angle AOB = 100 + 60 = 160^\circ$$



Note: Figure above not drawn to scale.

33. In the figure below, points A and B lie on the circumference of the circle centered at O. If $m\angle OAB = 20^\circ$, then $m\angle OBA = ?$

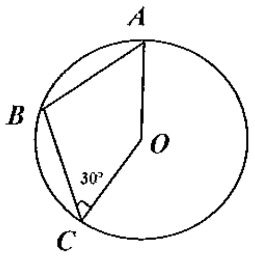
(A) 150
(B) 20
(C) 280
(D) 260

Solution: Answer: (B)

$OA = OB$ = Radius

$$\Downarrow$$

$$m\angle OAB = m\angle OBA = 20$$



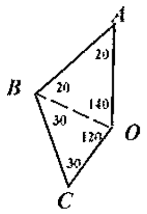
Note: Figure above not drawn to scale.

34. In the figure above, points A, B, and C lie on the circumference of the circle centered at O. If $\angle OAB = 20^\circ$, then $m\angle AOC = ?$

(A) 150
(B) 20
(C) 280
(D) 260

Solution: Answer: (D)

Draw a segment $\overline{OB}$ in the figure below.



(1). $OA = OB$ = Radius

$m\angle OAB = 20^\circ \rightarrow m\angle BOA = 140^\circ$

(2). $OB = OC$ = Radius

$m\angle OBC = 30^\circ \rightarrow m\angle BOC = 120^\circ$

$$\Downarrow$$

$$m\angle AOC = 140 + 120 = 260^\circ$$

35. The circumference of a circle whose area is equals

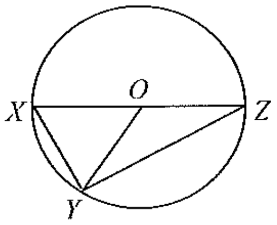
(A) 3π
(B) 4π
(C) 2π
(D) 8π

Solution: Answer: (D)

$$A = r^2\pi = 16\pi,\ r = 4$$

$$\Downarrow$$

$$C = 2r\pi = 2 \cdot 4\pi = 8\pi$$



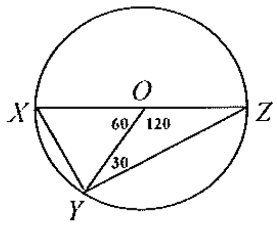
Note: Figure above not drawn to scale

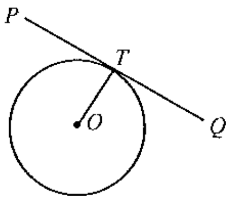
36. In the figure above, triangle XYZ is inscribed in the circle that has center O. If $XY = OX$, which of the following is the measure, in degree, of angle OYZ?

(A) 60
(B) 45
(C) 70
(D) 30

Solution: Answer: (D)

The radii form an equilateral triangle OYX and an isosceles triangle YOZ.





37. In the figure above, O is the center of the circle and $\overline{PQ}$ is tangent to the circle at point T. How many possible values are there for the measure of $\angle PTO$?

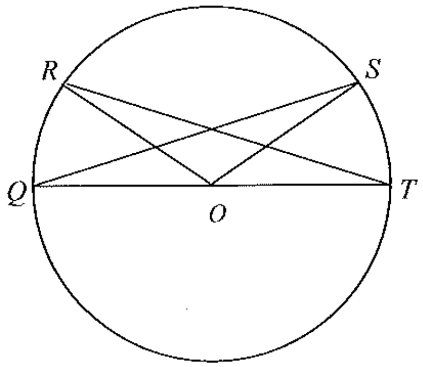
(A) three
(B) four
(C) five
(D) one

Solution: Answer: (D)

"$\overline{PQ}$ is tangent to the circle at point T"

$\Downarrow$

The measure of $\angle PTO$ is a right angle, and the measure is only one.

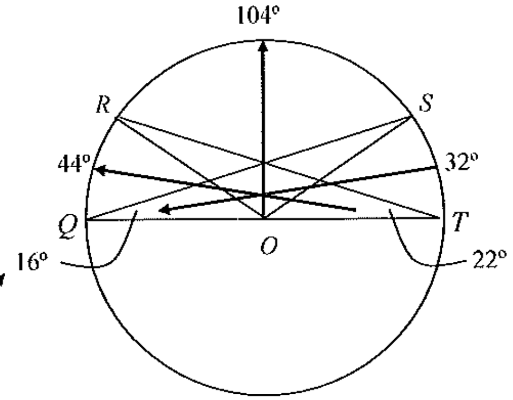


Note: Figure above not drawn to scale.

38. In the figure above, points Q, R, S, and T are all on the circle. O is center of the circle. If $\overline{QT}$ is the diameter of the circle, the measure of $\angle QTR$ is 22°, the measure of arc RS is 104°, which of the following is the degree measure of $\angle TQS$?

(A) 61° (B) 104° (C) 120° (D) 16°

Solution: Answer: (D)



$$\begin{cases} \text{Measure of arc } QR = 2\angle QTR = 2\times 22 = 44 \\ \text{Measure of arc } RS = \angle ROS = 104 \end{cases}$$

$\downarrow$

$$m\angle TQS = \frac{180 - (\text{Measure of } QR + \text{Measure of } RS)}{2}$$

$\Downarrow$

$$\frac{180-(44+104)}{2} = 16^\circ$$

39. Which of the following is the circumference of the circle that is inscribed in an equilateral triangle whose sides each are s?

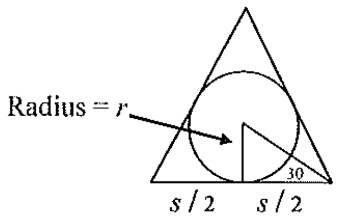
(A) $\frac{\sqrt{2}}{2}\pi s$

(B) $\frac{3}{\sqrt{3}}\pi s$

(C) $\frac{\sqrt{3}}{3}\pi s$

(D) $\frac{\sqrt{3}}{3s}\pi$

Solution: Answer: (C)

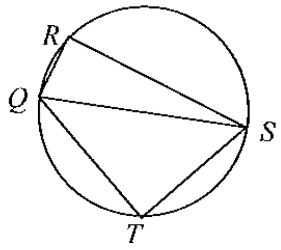


$$\frac{s}{2} = r\sqrt{3}, \qquad r = \frac{s}{2\sqrt{3}}$$

$$C_{\odot} = 2r\pi$$

$$\Downarrow$$

$$C_{\odot} = \cancel{2}\cdot\frac{s}{\cancel{2}\sqrt{3}}\pi = \frac{\sqrt{3}}{3}\pi s$$



Note: Figure above not drawn to scale.

40. In the figure above, if $QR = 5, QS = 13,\ ST = 12,$ and $\overline{QS}$ is a diameter of the circle, which of the following is the sum of areas of triangles QST and QSR?

(A) 74

(B) 60

(C) 56

(D) 65

Solution: Answer: (B)

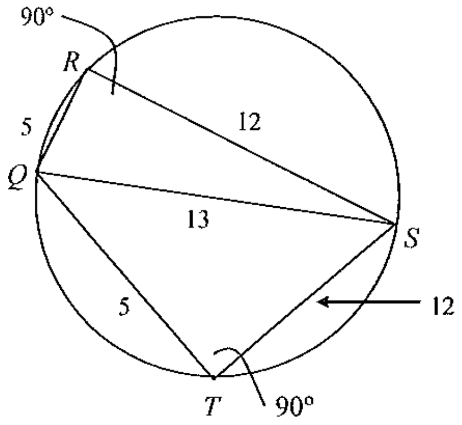
$$\underbrace{\angle R = \angle T}_{\text{They face half circumference of circle, respectively.}} = 90^{\circ}$$

$$\frac{QS}{2} = \frac{180}{2} = 90$$

$$QR = 5, QS = 13,\ ST = 12$$

$$\downarrow$$

OST and QSR both are 5-12-13 right triangle.



$$\text{Area}_{\text{Tri-}QST} + \text{Area}_{\text{Tri-}QSR} = 2\text{Area}_{\text{Tri-}QST}$$

$$\Downarrow$$

$$2\times\frac{5\times 12}{2} = 60$$

41. A circle with radius r is inscribed in a given square and another circle is circumscribed about the same square. What is the sum of the circumferences of the circumscribed circle and the inscribed circle?

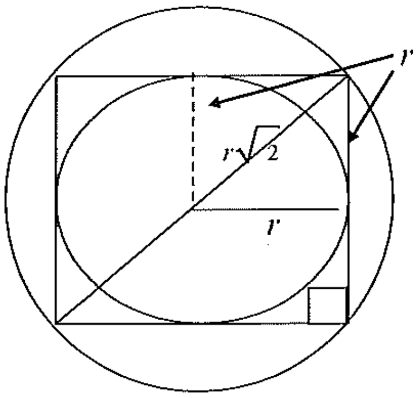
(A) $2r\pi + \sqrt{2}$

(B) $2r\pi - \sqrt{2}$

(C) $2r\pi \times \sqrt{2}$

(D) $2r\pi(1+\sqrt{2})$

Solution: Answer: (D)

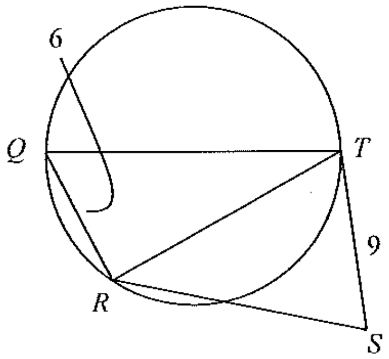


$$\downarrow$$

$$C_1 = 2r\pi, \quad C_2 = 2\left(r\sqrt{2}\right)\pi$$

$$\Downarrow$$

$$2r\pi + 2\left(r\sqrt{2}\right)\pi = 2r\pi(1+\sqrt{2})$$



Note: Figure above not drawn to scale.

42. In the figure above, $\overline{ST}$ is tangent to the circle at point T, $\overline{QT}$ is the diameter of the circle, and $RS = RT$. If the perimeter of triangle RST is 25, which of the following is the circumference of the circle?

(A) 20π

(B) 100π

(C) 12π

(D) 10π

Solution: Answer: (D)

$$RT = \frac{25-9}{2} = 8$$

$$\begin{cases} \angle R \text{ is opposite to arc } QT. \\ \text{arc } QT = 180^\circ \end{cases}$$

$$\downarrow$$

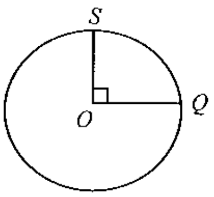
$$\angle R = \frac{QT}{2} = \frac{180^\circ}{2} = 90^\circ$$

$$\downarrow$$

QRT is a 6-8-10 right triangle.

$$\Downarrow$$

$$P = d\pi = 10\pi$$



43. In the figure above, O is the center of the circle with radius 12. What is the area of the sector SOQ?

(A) $12^2\pi$
(B) 24π
(C) 36π
(D) 12π

Solution: Answer: (C)

For a circle, the ratio obtained from its degree measure can be used to get its area or arc length.

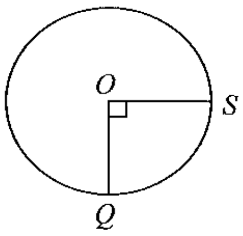
$$\begin{cases} \text{Ratio} = \dfrac{90}{360} = \dfrac{1}{4} \text{ or } = \dfrac{270}{360} = \dfrac{3}{4} \\ \text{Area} = r^2\pi \end{cases}$$

$$\downarrow$$

$$\text{Area}_S = \frac{1}{4}A = \frac{1}{4}r^2\pi$$

$$\Downarrow$$

$$\frac{1}{4}\cdot 12^2\pi = 36\pi$$



44. In the figure above, O is the center of the circle with diameter 12. What is the area of the sector SOQ?

(A) 18π
(B) 36π
(C) 16π
(D) 9π

Solution: Answer: (D)

For a circle, the ratio obtained from its degree measure can be used to get its area or arc length.

$$\text{Ratio} = \frac{90}{360} = \frac{1}{4}, \quad r = \frac{D}{2} = \frac{12}{2} = 6$$

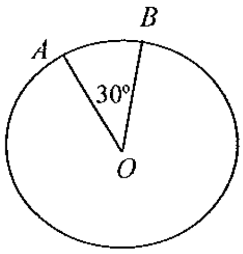
$$\downarrow$$

$$\text{Area}_O = r^2\pi = 6^2\pi = 36\pi$$

$$\text{Area}_{O\text{-Sestor}} = \frac{1}{4}\text{Area}_O$$

$$\Downarrow$$

$$\frac{1}{4}36\pi = 9\pi$$



Note: Figure above not drawn to scale.

45. In the figure above, O is the center of the circle. The area of its sector AOB with the angle 30° is 12π. What is the radius of the circle?

(A) $12^2\pi$
(B) 18
(C) 12
(D) 12^2

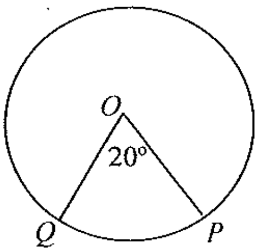
Solution: Answer: (C)

$$\text{Ratio} = \frac{30}{360} = \frac{1}{12}$$

$$\begin{cases} \text{Area} = r^2\pi = 12\pi \\ \text{Area}_{\text{sector}} = \dfrac{1}{12}r^2\pi \end{cases}$$

$$\Downarrow$$

$$12\pi = \frac{1}{12}r^2\pi, \; r^2 = 12^2, \; r = \boxed{12}$$



Note: Figure above not drawn to scale.

46. In the figure above, O is the center of the circle. The area of its sector QOP with the angle 20° is 18π. What is the radius of the circle?

(A) 18
(B) 12π
(C) 12
(D) 12^2

Solution: Answer: (A)

$$\text{Ratio} = \frac{20}{360} = \frac{1}{18}$$

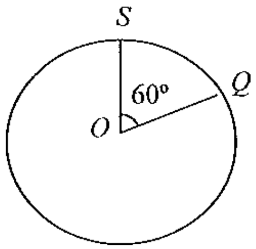
$$\begin{cases} \text{Area} = r^2\pi = 18\pi \\ \text{Area}_{\text{sector}} = \dfrac{1}{18}A = \dfrac{1}{18}r^2\pi \end{cases}$$

$$\downarrow$$

$$18\pi = \frac{1}{18}r^2\pi$$

$$\Downarrow$$

$$r^2 = 18^2, \; r = \boxed{18}$$



Note: Figure above not drawn to scale.

47. In the figure above, O is the center of the circle of radius 12. What is the length of arc SQ?

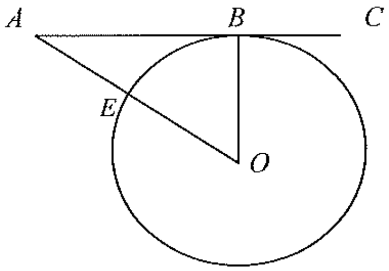
(A) 6π
(B) 8π
(C) 36π
(D) 4π

Solution: Answer: (D)

$$\begin{cases} \text{Ratio} = \dfrac{60}{360} = \dfrac{1}{6} \\ C = 2r\pi = 2\times 12\times \pi = 24\pi \end{cases}$$

$$\Downarrow$$

$$SQ = \frac{1}{6}C = \frac{1}{6}\times 24\pi = 4\pi$$



Note: Figure above not drawn to scale.

48. In the figure above, $\overline{AC}$ is tangent to the circle O at point B. $\overline{OE}$ is radius of the circle. $OB = 6$ and $m\angle BOA = 30^\circ$. What is length of arc BE?

(A) 12π
(B) π
(C) 4π
(D) $\dfrac{1}{2}\pi$

Solution: Answer: (B)

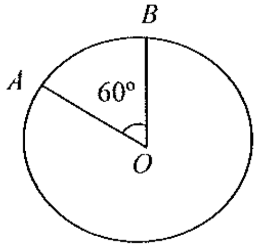
AOB is a right triangle.

$$\text{Ratio} = \frac{30}{360} = \frac{1}{12}$$

$$BE = \frac{1}{12}C = \frac{1}{12}\times 2r\pi$$

$$\Downarrow$$

$$\frac{1}{\cancel{12}}\times \cancel{2}\times 6\times \pi = \pi$$



Note: Figure above not drawn to scale.

49. In the figure above, O is the center of the circle with circumference 12. What is the length of arc AB?

(A) 10
(B) 4
(C) 6
(D) 8

Solution: Answer: (A)

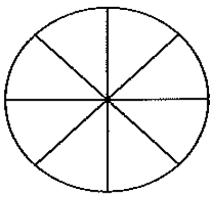
$$\text{Ratio} = \frac{300}{360} = \frac{5}{6} \text{ or } = \frac{60}{360} = \frac{1}{6},\ C = 12$$

$$\Downarrow$$

$$\text{Arc}_{AB} = \frac{1}{6}C = \frac{1}{6}\times 12 = 2$$

or

$$\text{Arc}_{AB} = \frac{5}{6}C = \frac{5}{6}\times 12 = \boxed{10}$$



50. In the figure above, the circle has a circumference of 8π and is divided into 8 congruent regions. What is the area of one of these regions?

(A) 8π
(B) $16+2\pi$
(C) 2π
(D) 4π

Solution: Answer: (C)

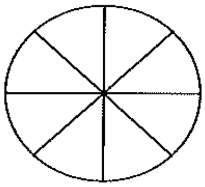
$$C = 2r\pi,\ r = \frac{C}{2\pi} = \frac{8\pi}{2\pi} = 4$$

$$\downarrow$$

$$\begin{cases} A = r^2\pi = 4^2\pi = 16\pi \\ \text{Ratio} = \dfrac{1}{8} \end{cases}$$

$$\Downarrow$$

$$\frac{1}{8}A = \frac{1}{8}\times 16\pi = 2\pi$$



Note: Figure above not drawn to scale.

51. In the figure above, the circle has an area of 64π and is divided into 8 congruent regions. What is the perimeter of one of these regions?

(A) 8π
(B) $16+2\pi$
(C) $8+2\pi$
(D) 4π

Solution: Answer: (B)

$$\text{Area} = r^2\pi,\ r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{64\pi}{\pi}} = 8$$

$$\downarrow$$

$$\begin{cases} C = 2r\pi = 2\times 8\times\pi = 16\pi \\ \text{Ratio} = \dfrac{1}{8} \end{cases}$$

$$\downarrow$$

$$\frac{1}{8}C = \frac{1}{8}\times 16\pi = 2\pi$$

$$\Downarrow$$

$$\text{Perimeter} = r + r + \frac{1}{8}C = 16 + 2\pi$$

52. In the figure above, the circle has a diameter of 8 and is divided into 8 congruent regions. What is the area of one of these regions?

(A) 2π
(B) 8π
(C) $8+2\pi$
(D) 4π

Solution: Answer: (A)

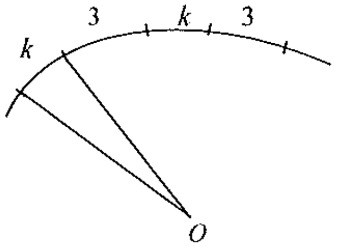
$$r = \frac{D}{2} = \frac{8}{2} = 4$$

$$\downarrow$$

$$\begin{cases} \text{Area} = r^2\pi = 4^2\pi = 16\pi \\ \text{Ratio} = \dfrac{1}{8} \end{cases}$$

$$\Downarrow$$

$$\frac{1}{8}\text{Area} = \frac{1}{8}\times 16\pi = 2\pi$$



Note: Figure above not drawn to scale.

53. The figure above shows part of a circle with center O. The diameter of the circle is $120/\pi$. If arcs of length 3 and length k continue to alternate around the entire circle so that there are 10 arcs for each of the two kinds of length, what is the degree measure of each of the arcs of length k?

(A) 20
(B) 30
(C) 27
(D) 60

Solution: Answer: (C)

For a circle, the ratio obtained from its arc length can be used to get its area or degree measure.

$$C = D\pi = \frac{120}{\pi}\cdot\pi = 120$$

$$10\times 3 + 10k = 120,\ k = 9$$

$$\underbrace{\text{Ratio} = \frac{9}{120}}_{\text{This ratio is from arc.}}$$

$$\Downarrow$$

$$360^\circ \times \frac{9}{120} = 27^\circ$$

54. The figure above shows part of a circle. The circumference of the circle is 90. If arcs of length 3 and length k continue to alternate around the entire circle so that there are 20 arcs for each of the two kinds of length, what is the degree measure of each of the arcs of length k?

(A) 60
(B) 3
(C) 20
(D) 6

Solution: Answer: (D)

For a circle, the ratio obtained from its arc length can be used to get its area or degree measure.

$$20\times 3 + 20k = 90,\ k = \frac{3}{2}$$

$$\underbrace{\text{Ratio} = \frac{\frac{3}{2}}{90} = \frac{3}{180} = \frac{1}{60}}_{\text{This ratio is from arc.}}$$

$$\Downarrow$$

$$36\cancel{0}^\circ \times \frac{1}{6\cancel{0}} = 6^\circ$$

55. A circle is inscribed in a given square and another circle is circumscribed about the same square. What is the ratio of the diameter of the inscribed circle to the diameter of the circumscribed circle?

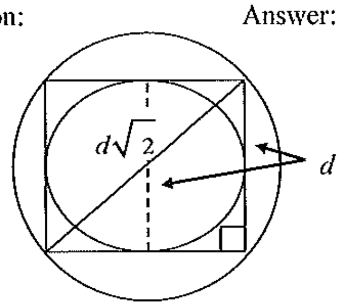
(A) $\frac{\sqrt{2}}{1}$

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{1}{2}$

(D) $\frac{2}{\sqrt{2}}$

Solution: Answer: (B)



$$\Downarrow$$

$$\text{Ratio} = \frac{d}{d\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

56. A circle is circumscribed about a given square and another circle is inscribed in the same square. What is the ratio of the radius of the circumscribed circle to the radius of the inscribed circle?

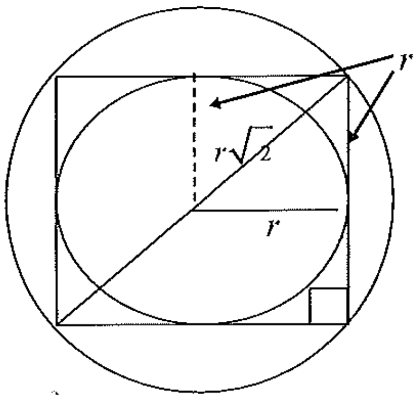
(A) $\frac{\sqrt{2}}{1}$

(B) 2

(C) $\frac{1}{2}$

(D) $\frac{\sqrt{2}}{2}$

Solution: Answer: (A)



$$\Downarrow$$

$$\text{Ratio} = \frac{r\sqrt{2}}{r} = \frac{\sqrt{2}}{1}$$

57. A circle is circumscribed about a given square and another circle is inscribed in the same square. What is the ratio of the radius of the inscribed circle to the radius of the circumscribed circle?

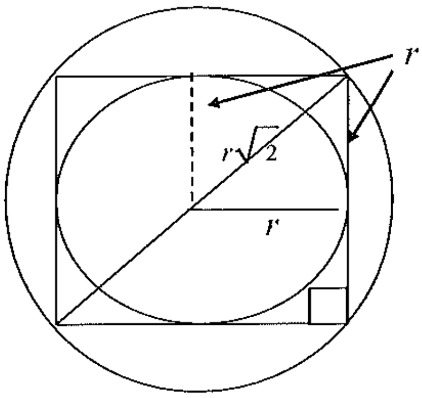
(A) $\sqrt{2}:1$

(B) $2:1$

(C) $\sqrt{2}:2$

(D) $2:\sqrt{2}$

Solution: Answer: (C)



$$\Downarrow$$

$$\text{Ratio} = \frac{r}{r\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

58. If a circle is inscribed in the greater square and the smaller square is inscribed in the circle, what is the ratio of the diagonal of the smaller square to the diagonal of the greater square?

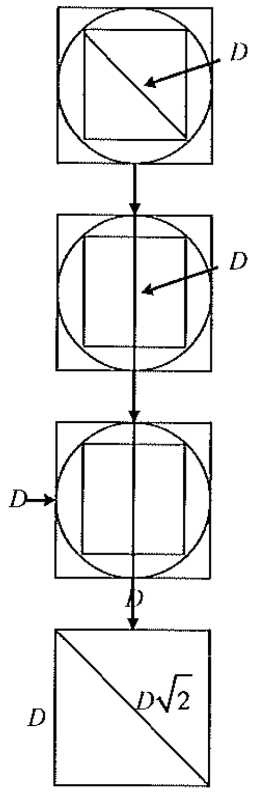
(A) $\frac{2}{\sqrt{2}}$

(B) $4\sqrt{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $2\sqrt{2}$

Solution: Answer: (C)



$\begin{cases} \text{Diameter}_{\text{greater circle}} = \text{Side}_{\text{greater square}} \\ \text{Diameter}_{\text{smaller circle}} = \text{Diagonal}_{\text{smaller square}} \\ \text{Refer to the figures above.} \end{cases}$

$\Downarrow$

$$\text{Ratio} = \frac{D}{D\sqrt{2}} = \frac{1}{\sqrt{2}}$$

59. If a rectangle is inside a circle with radius k, what is the possible maximum area of the rectangle?

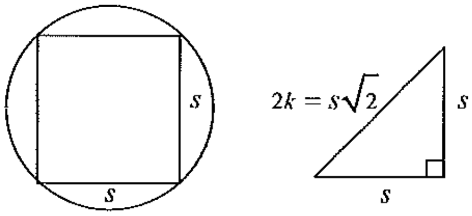
(A) $\sqrt{2}k^2$

(B) $2k^{\frac{1}{2}}$

(C) $\frac{\sqrt{2}}{2}k^2$

(D) $2k^2$

Solution: Answer: (D)



A square is a special rectangle.

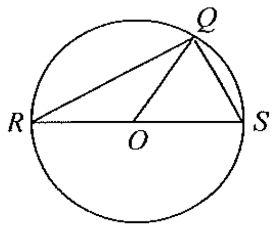
The greatest square inside a circle can make the greatest area inside the circle.

$\downarrow$

$$s = \frac{2k}{\sqrt{2}} = \frac{\cancel{2}k\sqrt{2}}{\cancel{2}} = k\sqrt{2}$$

$\Downarrow$

$$A = s^2 = \left(k\sqrt{2}\right)^2 = 2k^2$$



Note: Figure above not drawn to scale

60. In the figure above, triangle QRS is inscribed in the circle that has center O. If $QS = QO$, which of the following is the measure, in degree, of angle QRS ?

(A) 30

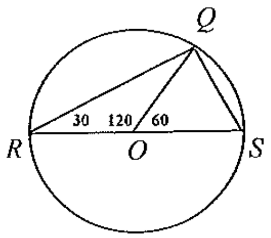
(B) 45

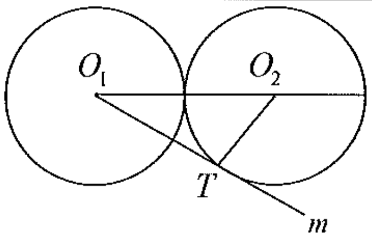
(C) 60

(D) 75

Solution: Answer: (A)

The radii form an equilateral triangle QOS and an isosceles triangle ROQ.

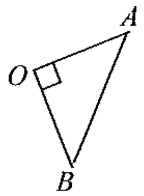




Note: Figure above not drawn to scale.

1. In the figure above, the circle with center O_1 and the circle with center O_2 are tangent. If the circles each have radius 6, and if line m is tangent to the circle with center O_2 at point T, what is the measure of $\angle O_2$?

(A) 30
(B) 62
(C) 60
(D) 61



2. In the figure above, O is the center of a circle with radius 12. If A and B are on the circle and $OA=OB=12$, what is the area of the triangle?

3. Which of the following is the area of the circle that is inscribed in an equilateral triangle whose sides each are s?

(A) $\frac{s}{2\sqrt{3}}$

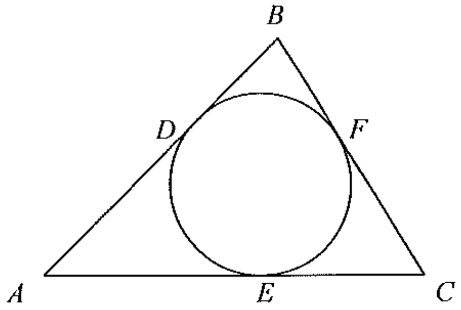
(B) $\frac{s^2}{12}\pi$

(C) $\frac{s}{2}$

(D) $\frac{s}{4}$

4. In the xy-plane, point $A(1, 0)$, $B(1, -8)$, and $C(d, e)$ are the vertices of a right triangle. If $\overline{AB}$ is the hypotenuse of the triangle, which of the following cannot be the area of the triangle?

(A) 16
(B) 6.2
(C) 15.59
(D) 16.1



Note: Figure above not drawn to scale.

5. In the figure above, if circle is inscribed in triangle ABC, $AD = 8$ and $CF = 2$ what is the length of the side AC?

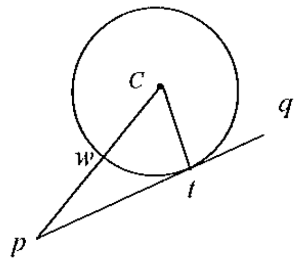
(A) 16
(B) 12
(C) 14
(D) 10

6. In the figure above, if circle is inscribed in triangle ABC, $BF = 3$ and $AE = 9$ what is the length of the side AB?

(A) 16
(B) 12
(C) 14
(D) 10

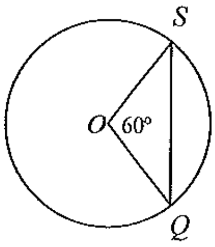
7. In a circle whose center is O, arc PQ contains 80°. What is the number of degrees in angle PQO?

(A) 80
(B) 70
(C) 45
(D) 50



Note: Figure above not drawn to scale.

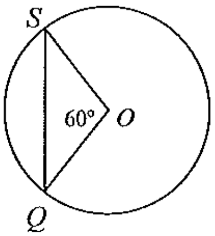
8. In the figure above, the segment pq is tangent to circle C at point t. If $pt = 12$ and $pw = 8$, what is the radius of circle C?



Note: Figure above not drawn to scale.

9. In the figure above, O is the center of the circle with diameter 12. What is the length of the segment $\overline{SQ}$?

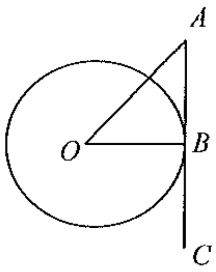
(A) 6
(B) 12
(C) 18
(D) 24



Note: Figure above not drawn to scale.

10. In the figure above, O is the center of the circle with circumference 12. What is the area of triangle SOQ?

(A) $\frac{6\sqrt{2}}{\pi}$
(B) $\frac{8\sqrt{2}}{\pi}$
(C) $\frac{9\sqrt{3}}{\pi^2}$
(D) $\frac{9\sqrt{2}}{\pi^2}$



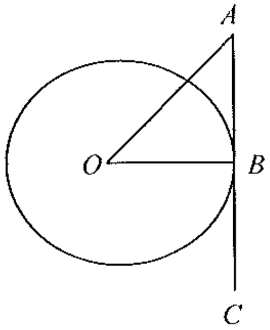
Note: Figure above not drawn to scale.

11. In the figure above, $\overline{AC}$ is tangent to circle O at point B. $\overline{BO}$ is radius of the circle. $AB = 8$, $AO = 10$. What is circumference of the circle?

(A) $2\sqrt{2}$
(B) 1
(C) 2
(D) 12π

12. The shortest distance from the center of a circle to a chord AB is 8. If the diameter of the center is 20, what is the length of the chord?

(A) 10
(B) 12
(C) 6
(D) 13



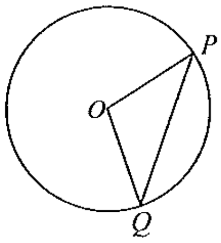
Note: Figure above not drawn to scale.

13. In the figure above, $\overline{AC}$ is tangent to the circle O at point B. BO is the radius of this circle. $BO = AB = \sqrt{2}$. What is AO?

(A) $2\sqrt{2}$
(B) 1
(C) 2
(D) $\sqrt{3}$

14. The shortest distance from the center of a circle to a chord AB is 12. If the length of the chord is 10, what is the length of radius of the circle?

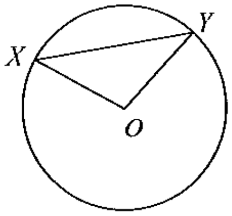
(A) 10
(B) 12
(C) 6
(D) 13



Note: Figure above not drawn to scale.

15. Point O is the center of the circle in the figure above. $m\angle POQ = 40°$. What is the degree measure of the angle PQO?

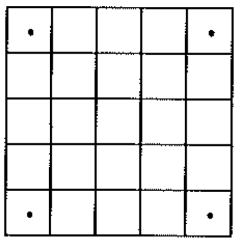
(A) 50
(B) 70
(C) 60
(D) 65



Note: Figure above not drawn to scale.

16. Point O is the center of the circle in the figure above. If $m\angle XOY = 50°$, what is the measure of $\angle OXY$?

(A) 50
(B) 70
(C) 60
(D) 65



Note: Figure above not drawn to scale.

17. In the figure above, a square with side length 10 is divided into 25 squares. What is the area of the circle (not shown here) that passes through the 4 points, which are the centers of the 4 corner squares?

(A) 8π

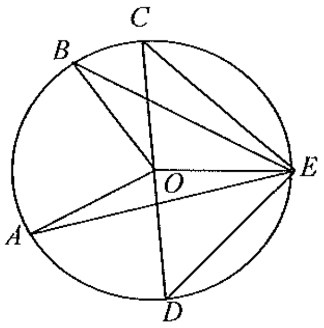
(B) $2\sqrt{2}\pi$

(C) 12π

(D) 32π

18. If the radius r of a circle is increased by $2r$, the circumference of this circle is increased by

(A) $r\pi$
(B) $2r\pi$
(C) $3r\pi$
(D) $4r\pi$



19. In the figure above, $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, $\overline{OD}$, and $\overline{OE}$ each are a radius of the circle with centering O.Which of the following triangles has the least area?

(A) DOE

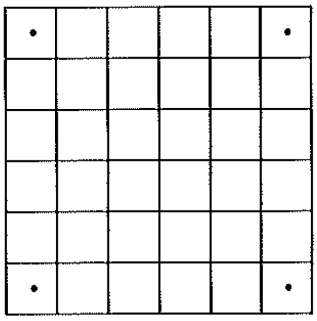
(B) COE

(C) BOE

(D) AOE

20. If the hour hand of a watch is 6 units long, how many units does the hand tip move between 8:30am and 12:30pm?

(A) 2π
(B) 4π
(C) π
(D) 3π



Note: Figure above not drawn to scale.

21. In the figure above, a square with side length 6 is divided into 36 squares. What is ratio of the area of the circle (not shown here) that passes through the 4 points, which are the centers of the 4 corner squares, to circumference of the circle?

(A) $\frac{5\sqrt{2}}{4}$

(B) $4\sqrt{2}$

(C) $5\sqrt{2}$

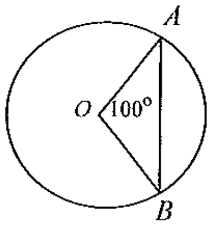
(D) $\frac{25}{\sqrt{2}}$

22. The area of the isosceles right triangle in a circle with radius $\frac{AB}{2}=6\sqrt{2}$, where AB is the hypotenuse of the triangle, is

(A) 144
(B) 98
(C) 72
(D) 86

23. If the radius of a circle is doubled, then the

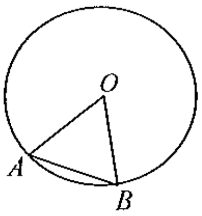
(A) circumference and area are both doubled.
(B) circumference is tripled and area is doubled.
(C) circumference is doubled and area is tripled.
(D) circumference is doubled and area is 4 times of the original area.



Note: Figure above not drawn to scale.

24. In the figure above, what is the number of degrees in angle *ABO*?

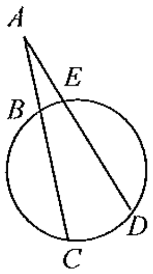
(A) 40
(B) 70
(C) 30
(D) 10



Note: Figure above not drawn to scale.

25. In the figure above, radius = 60, what is the perimeter of triangle *AOB*?

(A) 240
(B) 90
(C) 120
(D) 180



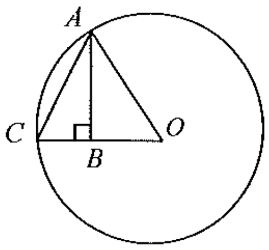
26. In the figure above, the number of degrees in angle *CAD* is

(A) $\frac{CD + BE}{2}$

(B) $\frac{CD}{2} - \frac{BE}{2}$

(C) $\frac{CD}{2} + BE$

(D) $\frac{BE - CD}{2}$



Note: Figure above not drawn to scale.

27. In the figure above shows a circle whose center is at *O*. If $AO = 4$ and $AB = 2$, what is the perimeter of triangle *ABO*?

(A) $2(3+\sqrt{2})$

(B) $2(3+\sqrt{3})$

(C) $6+\sqrt{3}$

(D) $2(3+2\sqrt{3})$

28. A particle moves in a circular path at 2π feet per minute. If it travels an arc of 40° in 20 seconds, what is the diameter of the circular path?

(A) 6π

(B) 45π

(C) 1

(D) 6

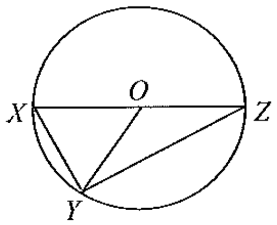
29. A particle moves in a circular path at π yards per minute. If it travels an arc of 60° in 20 seconds, what is the radius of the circular path?

(A) 6π

(B) 45π

(C) 1

(D) 6



Note: Figure above not drawn to scale

30. In the figure above, triangle XYZ is inscribed in the circle that has center O. If $XY = OX$, which of the following is the measure, in degree, of angle OYZ?

(A) 60

(B) 45

(C) 70

(D) 30

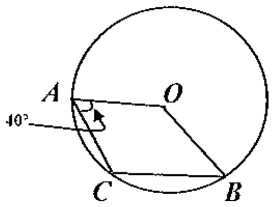
31. A line segment joining 2 points A and B on circumference of a circle is 2 yards from the center of the circle at its closest point. If the circle has a radius of four yards, what is the length of the segment?

(A) $2\sqrt{3}$

(B) $2\sqrt{2}$

(C) $4\sqrt{3}$

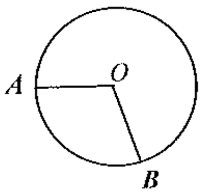
(D) $3\sqrt{2}$



Note: Figure above not drawn to scale.

32. In the figure above, points A, B, and C lie on the circumference of the circle centered at O. If $m\angle OBC = 60^\circ$, then $m\angle AOB = ?$

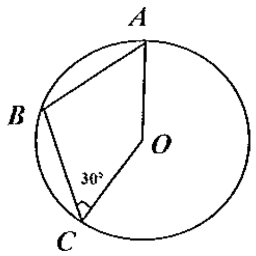
(A) 160
(B) 120
(C) 130
(D) 140



Note: Figure above not drawn to scale.

33. In the figure below, points A and B lie on the circumference of the circle centered at O. If $m\angle OAB = 20^\circ$, then $m\angle OBA = ?$

(A) 150
(B) 20
(C) 280
(D) 260



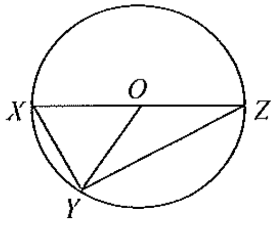
Note: Figure above not drawn to scale.

34. In the figure above, points A, B, and C lie on the circumference of the circle centered at O. If $\angle OAB = 20^\circ$, then $m\angle AOC = ?$

(A) 150
(B) 20
(C) 280
(D) 260

35. The circumference of a circle whose area is equals

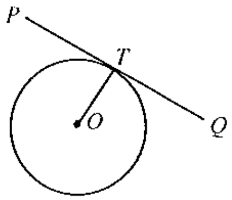
(A) 3π
(B) 4π
(C) 2π
(D) 8π



Note: Figure above not drawn to scale

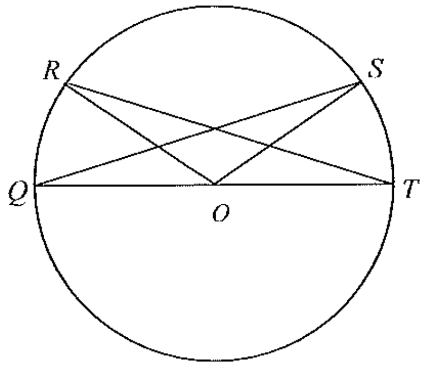
36. In the figure above, triangle XYZ is inscribed in the circle that has center O. If $XY = OX$, which of the following is the measure, in degree, of angle OYZ?

(A) 60
(B) 45
(C) 70
(D) 30



37. In the figure above, O is the center of the circle and $\overline{PQ}$ is tangent to the circle at point T. How many possible values are there for the measure of $\angle PTO$?

(A) three
(B) four
(C) five
(D) one



Note: Figure above not drawn to scale.

38. In the figure above, points Q, R, S, and T are all on the circle. O is center of the circle. If $\overline{QT}$ is the diameter of the circle, the measure of $\angle QTR$ is 22°, the measure of arc RS is 104°, which of the following is the degree measure of $\angle TQS$?

(A) 61° (B) 104° (C) 120° (D) 16°

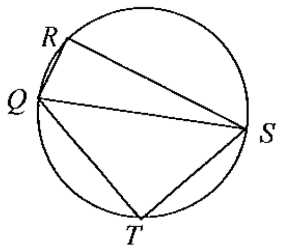
39. Which of the following is the circumference of the circle that is inscribed in an equilateral triangle whose sides each are s?

(A) $\frac{\sqrt{2}}{2}\pi s$

(B) $\frac{3}{\sqrt{3}}\pi s$

(C) $\frac{\sqrt{3}}{3}\pi s$

(D) $\frac{\sqrt{3}}{3s}\pi$



Note: Figure above not drawn to scale.

40. In the figure above, if $QR = 5, QS = 13$, $ST = 12$, and $\overline{QS}$ is a diameter of the circle, which of the following is the sum of areas of triangles QST and QSR?

(A) 74

(B) 60

(C) 56

(D) 65

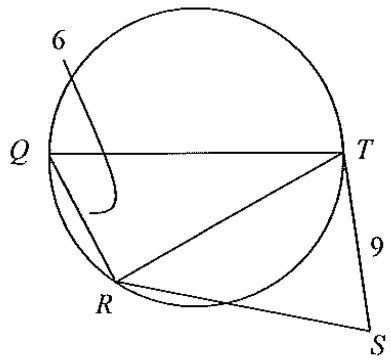
41. A circle with radius r is inscribed in a given square and another circle is circumscribed about the same square. What is the sum of the circumferences of the circumscribed circle and the inscribed circle?

(A) $2r\pi + \sqrt{2}$

(B) $2r\pi - \sqrt{2}$

(C) $2r\pi \times \sqrt{2}$

(D) $2r\pi(1 + \sqrt{2})$



Note: Figure above not drawn to scale.

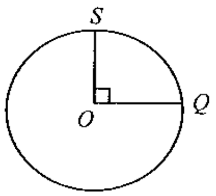
42. In the figure above, $\overline{ST}$ is tangent to the circle at point T, $\overline{QT}$ is the diameter of the circle, and $RS = RT$. If the perimeter of triangle RST is 25, which of the following is the circumference of the circle?

(A) 20π

(B) 100π

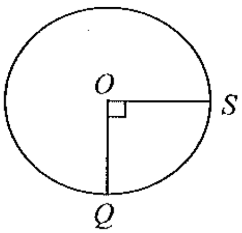
(C) 12π

(D) 10π



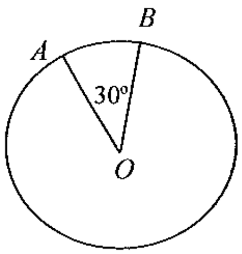
43. In the figure above, O is the center of the circle with radius 12. What is the area of the sector SOQ?

(A) $12^2\pi$
(B) 24π
(C) 36π
(D) 12π



44. In the figure above, O is the center of the circle with diameter 12. What is the area of the sector SOQ?

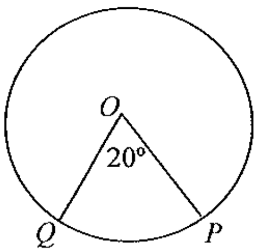
(A) 18π
(B) 36π
(C) 16π
(D) 9π



Note: Figure above not drawn to scale.

45. In the figure above, O is the center of the circle. The area of its sector AOB with the angle 30° is 12π. What is the radius of the circle?

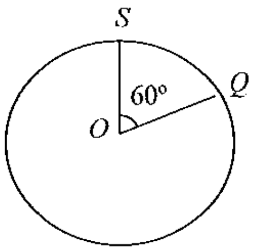
(A) $12^2\pi$
(B) 18
(C) 12
(D) 12^2



Note: Figure above not drawn to scale.

46. In the figure above, O is the center of the circle. The area of its sector QOP with the angle 20° is 18π. What is the radius of the circle?

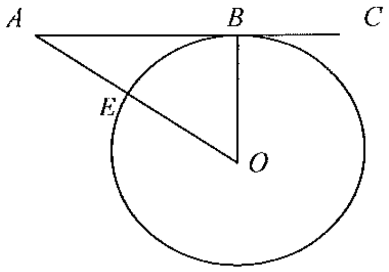
(A) 18
(B) 12π
(C) 12
(D) 12^2



Note: Figure above not drawn to scale.

47. In the figure above, O is the center of the circle of radius 12. What is the length of arc SQ?

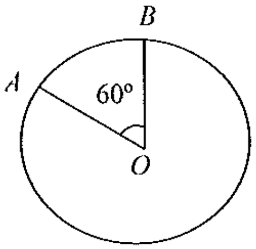
(A) 6π
(B) 8π
(C) 36π
(D) 4π



Note: Figure above not drawn to scale.

48. In the figure above, $\overline{AC}$ is tangent to the circle O at point B. $\overline{OE}$ is radius of the circle. $OB = 6$ and $m\angle BOA = 30^\circ$. What is length of arc BE?

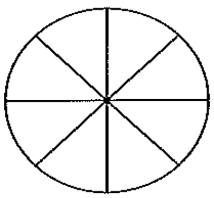
(A) 12π
(B) π
(C) 4π
(D) $\frac{1}{2}\pi$



Note: Figure above not drawn to scale.

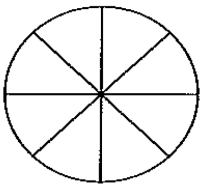
49. In the figure above, O is the center of the circle with circumference 12. What is the length of arc AB?

(A) 10
(B) 4
(C) 6
(D) 8



50. In the figure above, the circle has a circumference of 8π and is divided into 8 congruent regions. What is the area of one of these regions?

(A) 8π
(B) $16 + 2\pi$
(C) 2π
(D) 4π



Note: Figure above not drawn to scale.

51. In the figure above, the circle has an area of 64π and is divided into 8 congruent regions. What is the perimeter of one of these regions?

(A) 8π

(B) $16 + 2\pi$

(C) $8 + 2\pi$

(D) 4π

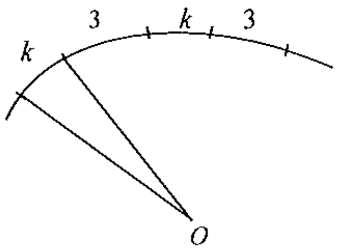
52. In the figure above, the circle has a diameter of 8 and is divided into 8 congruent regions. What is the area of one of these regions?

(A) 2π

(B) 8π

(C) $8 + 2\pi$

(D) 4π



Note: Figure above not drawn to scale.

53. The figure above shows part of a circle with center O. The diameter of the circle is $120/\pi$. If arcs of length 3 and length k continue to alternate around the entire circle so that there are 10 arcs for each of the two kinds of length, what is the degree measure of each of the arcs of length k?

(A) 20
(B) 30
(C) 27
(D) 60

54. The figure above shows part of a circle. The circumference of the circle is 90. If arcs of length 3 and length k continue to alternate around the entire circle so that there are 20 arcs for each of the two kinds of length, what is the degree measure of each of the arcs of length k?

(A) 60
(B) 3
(C) 20
(D) 6

55. A circle is inscribed in a given square and another circle is circumscribed about the same square. What is the ratio of the diameter of the inscribed circle to the diameter of the circumscribed circle?

(A) $\frac{\sqrt{2}}{1}$

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{1}{2}$

(D) $\frac{2}{\sqrt{2}}$

56. A circle is circumscribed about a given square and another circle is inscribed in the same square. What is the ratio of the radius of the circumscribed circle to the radius of the inscribed circle?

(A) $\frac{\sqrt{2}}{1}$

(B) 2

(C) $\frac{1}{2}$

(D) $\frac{\sqrt{2}}{2}$

57. A circle is circumscribed about a given square and another circle is inscribed in the same square. What is the ratio of the radius of the inscribed circle to the radius of the circumscribed circle?

(A) $\sqrt{2}:1$

(B) $2:1$

(C) $\sqrt{2}:2$

(D) $2:\sqrt{2}$

58. If a circle is inscribed in the greater square and the smaller square is inscribed in the circle, what is the ratio of the diagonal of the smaller square to the diagonal of the greater square?

(A) $\frac{2}{\sqrt{2}}$

(B) $4\sqrt{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $2\sqrt{2}$

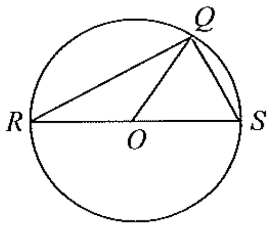
59. If a rectangle is inside a circle with radius k, what is the possible maximum area of the rectangle?

(A) $\sqrt{2}k^2$

(B) $2k^{\frac{1}{2}}$

(C) $\frac{\sqrt{2}}{2}k^2$

(D) $2k^2$



Note: Figure above not drawn to scale

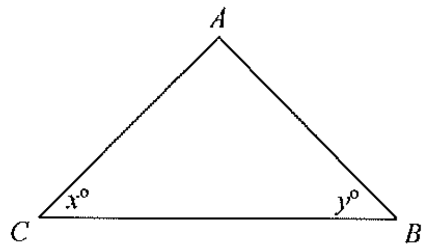
60. In the figure above, triangle QRS is inscribed in the circle that has center O. If $QS = QO$, which of the following is the measure, in degree, of angle QRS ?

(A) 30

(B) 45

(C) 60

(D) 75



Note: Figure above not drawn to scale.

1. In the figure above, if $y^\circ > x^\circ$, which of the following must be true?

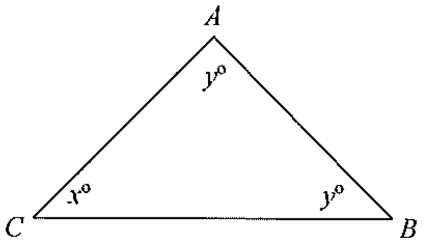
(A) $AC > BC$
(B) $AB > AC$
(C) $AC > AB$
(D) $AB = BC$

Solution: Answer: (C)

In any triangle, the greater side subtends the greater angle, and vice versa.

$\Downarrow$

$AC > AB$



Note: Figure above not drawn to scale.

2. In the figure above, if $y^\circ < x^\circ$, which of the following must be true?

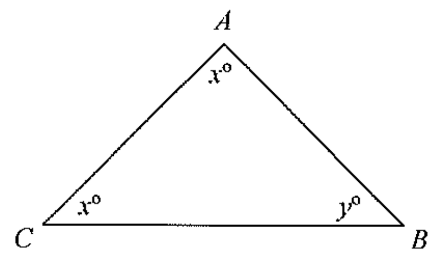
(A) $AC > AB$
(B) $AB < BC$
(C) $AB = BC$
(D) $AB > BC$

Solution: Answer: (D)

In any triangle, the greater side subtends the greater angle, and vice versa.

$\Downarrow$

$AB > BC$



Note: Figure above not drawn to scale.

3. In the figure above, if $y^\circ < x^\circ$, which of the following must be true?

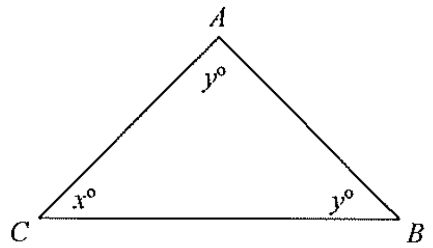
(A) $AC > BC$
(B) $AB = BC$
(C) $AC = AB$
(D) $AC = BC$

Solution: Answer: (B)

In any triangle, the greater side subtends the greater angle, and vice versa.

$\Downarrow$

$AB = BC$



Note: Figure above not drawn to scale.

4. In the figure above, if $y^\circ > x^\circ$, which of the following must be true?

(A) $BC > AB$
(B) $AB > CB$
(C) $AB = BC$
(D) $AB > BC$

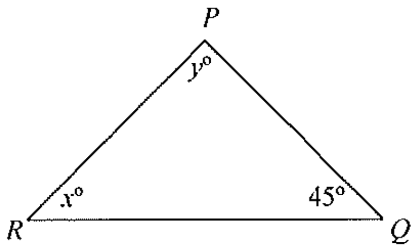
Solution: Answer: (A)

In any triangle, the greater side subtends the greater angle, and vice versa.

$\Downarrow$

$BC > AB$

Greater Side Subtending Greater Angle



Note: Figure above not drawn to scale.

5. In the figure above, $PR > PQ$. Which of the following must be true?

(A) $x^\circ > 45^\circ$
(B) $x^\circ < 45^\circ$
(C) $x^\circ > y^\circ$
(D) $y^\circ < 45^\circ$

Solution: Answer: (B)

In any triangle, the greater side subtends the greater angle, and vice versa.

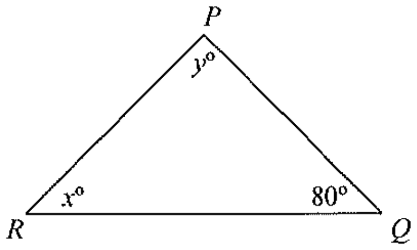
$\downarrow$

$PR > PQ$

Angle of 45° is opposite PR.
x° is opposite PQ.

$\Downarrow$

$x^\circ < 45^\circ$



Note: Figure above not drawn to scale.

6. In the figure above, $PR < PQ$. Which of the following must be true?

(A) $x^\circ > 81^\circ$
(B) $x^\circ = y^\circ$
(C) $x^\circ < y^\circ$
(D) $y^\circ < 20^\circ$

Solution: Answer: (D)

In any triangle, the greater side subtends the greater angle, and vice versa.

$\downarrow$

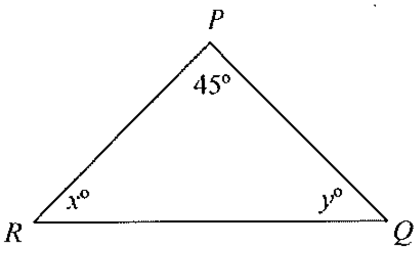
$PR < PQ$
$x > 80^\circ$

$\downarrow$

$(80^\circ + x^\circ) > 160^\circ$

$\Downarrow$

The degrees of the third angle $y^\circ < 20^\circ$



Note: Figure above not drawn to scale.

7. In the figure above, $QR < PQ$. Which of the following must be true?

(A) $y^\circ < 90^\circ$
(B) $x^\circ = y^\circ$
(C) $x^\circ > y^\circ$
(D) $y^\circ < 45^\circ$

Solution: Answer: (A)

In any triangle, the greater side subtends the greater angle, and vice versa.

$\downarrow$

$QR < PQ$
$x > 45^\circ$

$\downarrow$

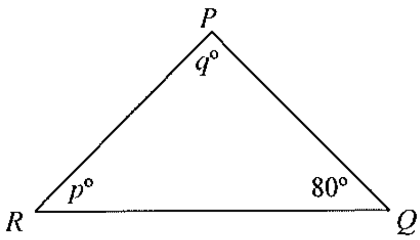
$(45^\circ + x^\circ) > 90^\circ$

$\downarrow$

The degrees of the third angle

$\Downarrow$

$y^\circ < 90^\circ$



Note: Figure above not drawn to scale.

8. In the figure above, $PR = PQ$. Which of the following must be true?

(A) $p^\circ > 80^\circ$
(B) $q^\circ < 20^\circ$
(C)) $q^\circ = 20^\circ$
(D) $p^\circ < 20^\circ$

Solution: Answer: (C)

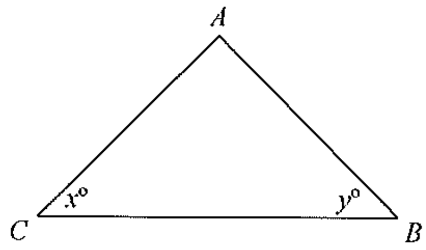
In any triangle, the greater side subtends the greater angle, and vice versa.

$\downarrow$

$p^\circ = 80^\circ$

$\Downarrow$

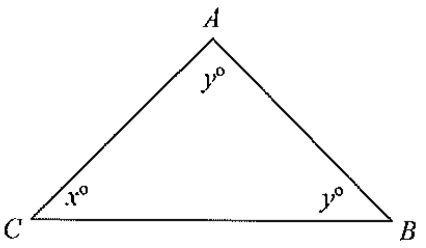
$q^\circ = 20^\circ$



Note: Figure above not drawn to scale.

1. In the figure above, if $y^\circ > x^\circ$, which of the following must be true?

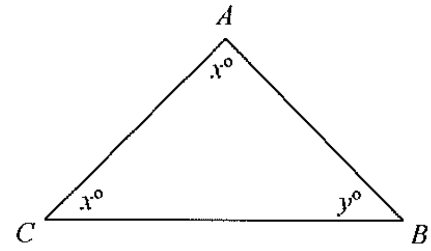
(A) $AC > BC$
(B) $AB > AC$
(C) $AC > AB$
(D) $AB = BC$



Note: Figure above not drawn to scale.

2. In the figure above, if $y^\circ < x^\circ$, which of the following must be true?

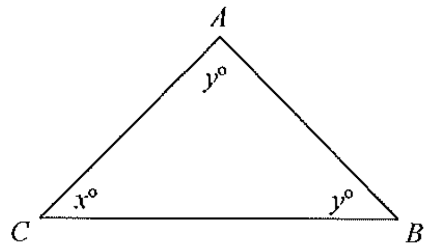
(A) $AC > AB$
(B) $AB < BC$
(C) $AB = BC$
(D) $AB > BC$



Note: Figure above not drawn to scale.

3. In the figure above, if $y^\circ < x^\circ$, which of the following must be true?

(A) $AC > BC$
(B) $AB = BC$
(C) $AC = AB$
(D) $AC = BC$

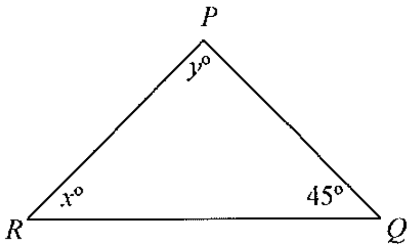


Note: Figure above not drawn to scale.

4. In the figure above, if $y^\circ > x^\circ$, which of the following must be true?

(A) $BC > AB$
(B) $AB > CB$
(C) $AB = BC$
(D) $AB > BC$

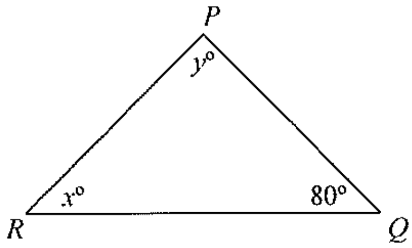
Greater Side Subtending Greater Angle



Note: Figure above not drawn to scale.

5. In the figure above, $PR > PQ$. Which of the following must be true?

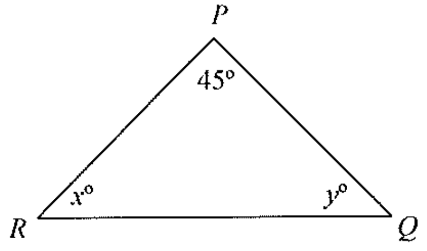
(A) $x^\circ > 45^\circ$
(B) $x^\circ < 45^\circ$
(C) $x^\circ > y^\circ$
(D) $y^\circ < 45^\circ$



Note: Figure above not drawn to scale.

6. In the figure above, $PR < PQ$. Which of the following must be true?

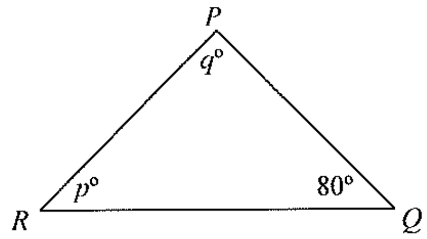
(A) $x^\circ > 81^\circ$
(B) $x^\circ = y^\circ$
(C) $x^\circ < y^\circ$
(D) $y^\circ < 20^\circ$



Note: Figure above not drawn to scale.

7. In the figure above, $QR < PQ$. Which of the following must be true?

(A) $y^\circ < 90^\circ$
(B) $x^\circ = y^\circ$
(C) $x^\circ > y^\circ$
(D) $y^\circ < 45^\circ$



Note: Figure above not drawn to scale.

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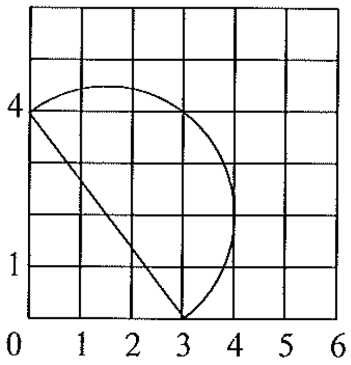
(A) $p^\circ > 80^\circ$
(B) $q^\circ < 20^\circ$
(C)) $q^\circ = 20^\circ$
(D) $p^\circ < 20^\circ$



1. In the figure above, what is the area of the semicircle?

(A) $\frac{25\pi}{4}$

(B) $\frac{25\pi}{2}$

(C) $\frac{25\pi}{8}$

(D) $\frac{5\pi}{2}$

Solution: Answer: (C)

There is a 3-4-5 right triangle.

Diameter = 5, Radius = $\frac{5}{2}$

$$A_{\text{semicircle}} = \frac{A_{\text{circle}}}{2}$$

$$\Downarrow$$

$$A_{\text{semicircle}} = \frac{r^2\pi}{2} = \frac{\left(\frac{5}{2}\right)^2\pi}{2} = \frac{25\pi}{8}$$

2. In the figure above, what is the arc of the semicircle?

(A) $\frac{25\pi}{4}$

(B) $\frac{25\pi}{2}$

(C) 5π

(D) $\frac{5\pi}{2}$

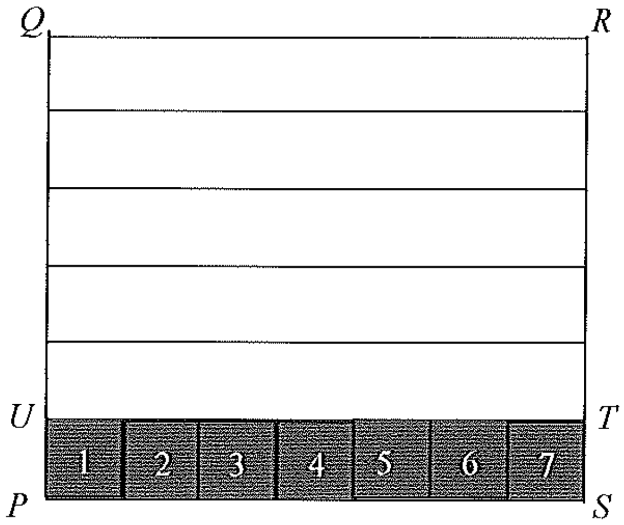
Solution: Answer: (D)

There is a 3-4-5 right triangle.
Diameter = 5

$$C_{\text{semicircle}} = \frac{D_{\text{circle}}\pi}{2}$$

$$\Downarrow$$

$$\frac{5\pi}{2}$$



Note: Figure above not drawn to scale.

3. In the figure above, if $PQRS$ is a rectangle and squares from 1 to 7 each have an area of 4, and $QU = 5PU$, what is the area of rectangle $QRTU$?

Solution: Answer: 140

$$\begin{cases} PU = \sqrt{A_{\text{small square}}} = \sqrt{4} = 2 \\ A_{QRTU} = QU \times TU = 5PU \times 7PU \end{cases}$$

$$\Downarrow$$

$$A_{QRTU} = (5\times 2)\times(7\times 2) = 140$$

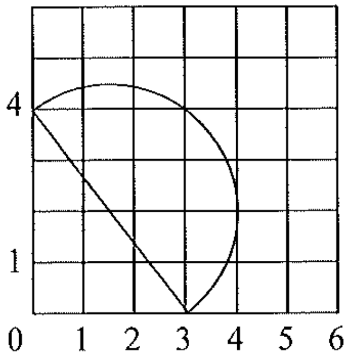
4. In the figure above, if $PQRS$ is a rectangle and squares from 1 to 7 each have an area of 4, and $QU = 5PU$, what is the area of rectangle $PQRS$?

Solution: Answer: 168

$$\begin{cases} PU = \sqrt{A_{\text{small square}}} = \sqrt{4} = 2 \\ A_{PQRS} = PQ \times PS = 6PU \times 7PU \end{cases}$$

$$\Downarrow$$

$$A_{PQRS} = (6\times 2)\times(7\times 2) = 168$$



1. In the figure above, what is the area of the semicircle?

(A) $\frac{25\pi}{4}$

(B) $\frac{25\pi}{2}$

(C) $\frac{25\pi}{8}$

(D) $\frac{5\pi}{2}$

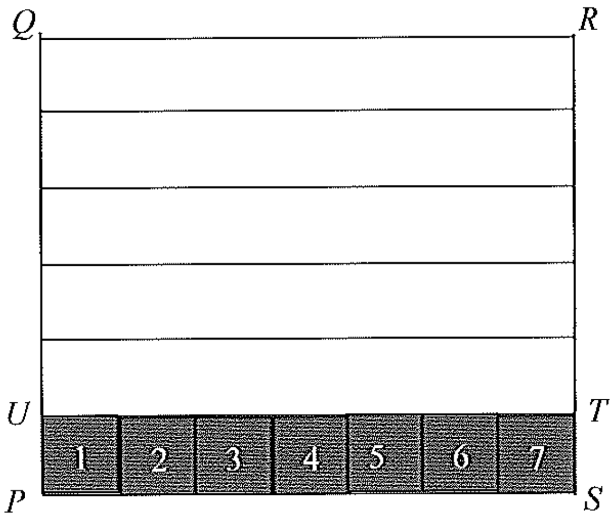
2. In the figure above, what is the arc of the semicircle?

(A) $\frac{25\pi}{4}$

(B) $\frac{25\pi}{2}$

(C) 5π

(D) $\frac{5\pi}{2}$



Note: Figure above not drawn to scale.

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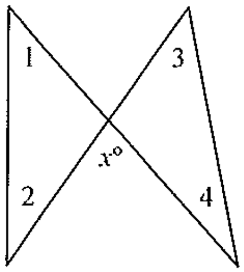
4. In the figure above, if $PQRS$ is a rectangle and squares from 1 to 7 each have an area of 4, and $QU = 5PU$, what is the area of rectangle $PQRS$?

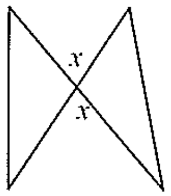


1. In the figure above, what is the sum, in terms of x, of the degree measures of the 4 angles marked 1, 2, 3, and 4?

(A) 180 - x
(B) 180 - $2x$
(C) 360 - x
(D) $2x$

Solution: Answer: (D)

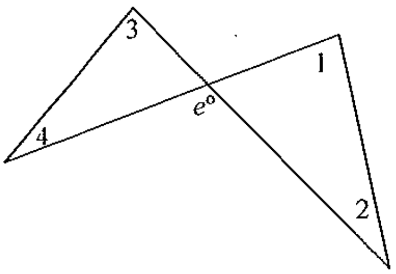
An exterior angle of a triangle is equal in measure to the sum of its 2 nonadjacent interior angles.



$$m\angle 1 + m\angle 2 = x,\ m\angle 3 + m\angle 4 = x$$

$$\Downarrow$$

$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 2x$$

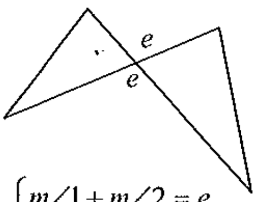


2. In the figure above, what is the sum, in terms of e, of the degree measures of the 4 angles marked 1, 2, 3, and 4?

(A) 180 - e
(B) 180 - $2e$
(C) 360 - e
(D) $2e$

Solution: Answer: (D)

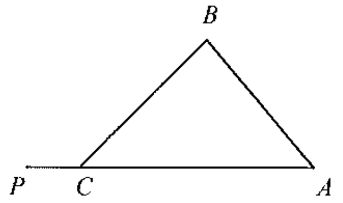
One exterior angle of a triangle is equal in measure to the sum of its two nonadjacent interior angles.



$$\begin{cases} m\angle 1 + m\angle 2 = e \\ m\angle 3 + m\angle 4 = e \end{cases}$$

$$\Downarrow$$

$$\text{Sum}_{\text{4-angles}} = 2e$$



Note: Figure above not drawn to scale.

3. What is $m\angle B$ if in the figure above, $m\angle A = (2x-10)^\circ$, $m\angle B = (5x-20)^\circ$, and $m\angle BCP = (2x+70)^\circ$?

Solution: Answer: 80

One exterior angle of a triangle is equal in measure to the sum of its two nonadjacent interior angles.

$$m\angle BCP = m\angle A + m\angle B$$

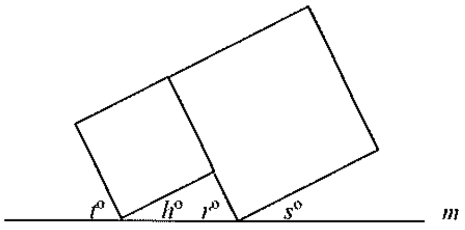
$$2x + 70 = (2x-10) + (5x-20)$$

$$2x + 70 = 7x - 30$$

$$100 = 5x,\ x = 20$$

$$\Downarrow$$

$$m\angle B = 5x - 20 = 5 \cdot 20 - 20 = \boxed{80}$$

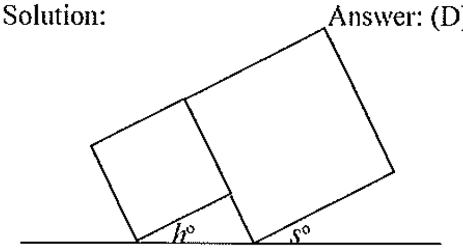


Note: Figure above not drawn to scale.

4. In the figure above, the 2 rectangles are squares. Which of the following must be true?

(A) $h^\circ = t^\circ$
(B) $h^\circ = r^\circ$
(C) $s^\circ = r^\circ$
(D) $h^\circ = s^\circ$

Solution: Answer: (D)



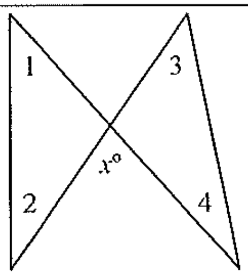
One exterior angle in measure equals the sum of two non-adjacent interior angles.

$$\Downarrow$$

$$\underline{s^\circ + 90^\circ} = \underline{h^\circ + 90^\circ},\ h^\circ = s^\circ$$

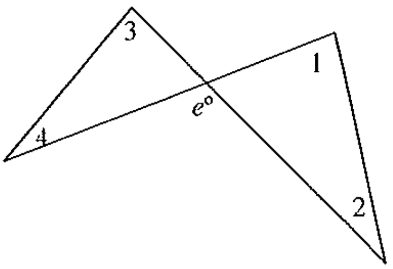
1 exterior angle, 2 interior angles

One Exterior Angle = Sum of Two Nonadjacent Interior Angles



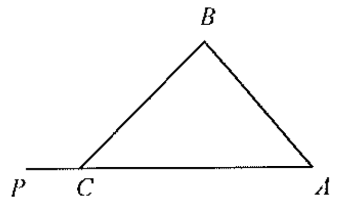
1. In the figure above, what is the sum, in terms of x, of the degree measures of the 4 angles marked 1, 2, 3, and 4?

(A) 180 - x
(B) 180 - $2x$
(C) 360 - x
(D) $2x$



2. In the figure above, what is the sum, in terms of e, of the degree measures of the 4 angles marked 1, 2, 3, and 4?

(A) 180 - e
(B) 180 - $2e$
(C) 360 - e
(D) $2e$



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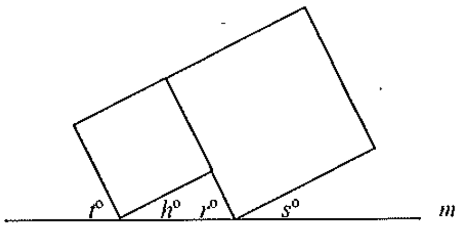

Note: Figure above not drawn to scale.

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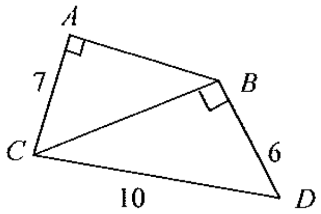
(A) $h^\circ = f^\circ$
(B) $h^\circ = t^\circ$
(C) $s^\circ = t^\circ$
(D) $h^\circ = s^\circ$



Note: Figure above not drawn to scale.

1. In quadrilateral $ABCD$ above, what is the length of $\overline{AB}$?

(A) $\sqrt{15}$

(B) $\sqrt{11}$

(C) $\sqrt{13}$

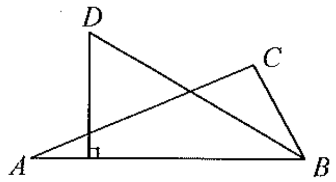
(D) $\sqrt{14}$

Solution: Answer: (A)

BCD is a 6-8-10 right triangle.

$$\Downarrow$$

$$AB = \sqrt{8^2 - 7^2} = \sqrt{64 - 49} = \sqrt{15}$$

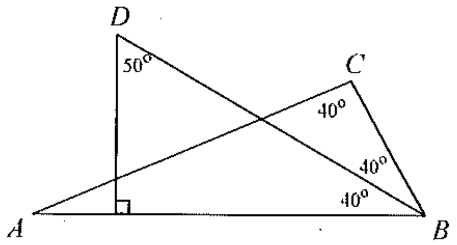


Note: Figure above not drawn to scale.

2. In the figure above, $\overline{BD}$ bisects $\angle B$. The measure of $\angle D$ is 50° and the measure of $\angle C$ is 40°. What is the measure of $\angle A$?

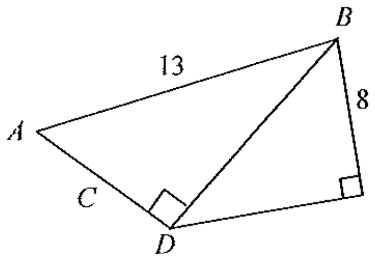
(A) 80
(B) 70
(C) 60
(D) 50

Solution: Answer: (C)



$$\Downarrow$$

$$m\angle A = 180 - 40 - (40 + 40) = 60$$



Note: Figure above not drawn to scale.

3. In quadrilateral $ABCD$ above, what is the area of triangle BCD?

(A) $32\sqrt{5}$

(B) $16\sqrt{5}$

(C) $8\sqrt{5}$

(D) $10\sqrt{14}$

Solution: Answer: (B)

ABD is a $5-12-13$ right triangle.

$$\begin{cases} DC = \sqrt{12^2 - 8^2} = \sqrt{144 - 64} = \sqrt{80} \\ \text{Area} = \dfrac{\text{base} \times \text{height}}{2} \end{cases}$$

$$\Downarrow$$

$$\frac{\sqrt{80} \times 8}{2} = 4\sqrt{16} \cdot \sqrt{5} = 16\sqrt{5}$$

4. In quadrilateral $ABCD$ above, what is the length of $\overline{DC}$?

(A) $\sqrt{208}$

(B) $\sqrt{110}$

(C) $\sqrt{80}$

(D) 70

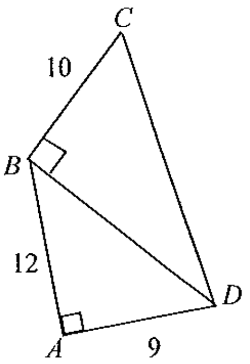
Solution: Answer: (C)

ABD is a $5-12-13$ right triangle.

$$DC = \sqrt{12^2 - 8^2}$$

$$\Downarrow$$

$$\sqrt{144 - 64} = \sqrt{80}$$



Note: Figure above not drawn to scale.

5. In quadrilateral $ABCD$ above, what is the area of triangle DCB?

(A) 150
(B) 75
(C) 85
(D) 65

Solution: Answer: (B)

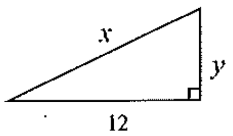
ABD is a $9-12-15$ right triangle.

$\downarrow$

$BD = 12$

$\Downarrow$

$$\text{Area}_{DBC} = \frac{15 \times 10}{2} = 75$$



6. In the triangle above, if $y = 5$, what is the value of x?

(A) 13
(B) 14
(C) 15
(D) 16

Solution: Answer: (A)

Method 1:

It is a 5-12-13 right triangle.

$\Downarrow$

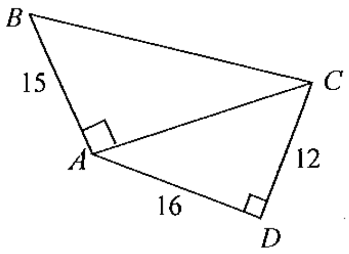
$x = 13$

Method 2:

$$x = \sqrt{5^2 + 12^2}$$

$\Downarrow$

$$\sqrt{169} = 13$$



Note: Figure above not drawn to scale.

7. In quadrilateral $ABCD$ above, what is the perimeter of $ABCD$?

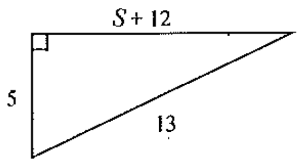
(A) 63
(B) 86
(C) 68
(D) 53

Solution: Answer: (C)

$\begin{cases} ACD \text{ is a } 12-16-20 \text{ right triangle.} \\ ABC \text{ is a } 15-20-25 \text{ right triangle.} \end{cases}$

$\Downarrow$

$\text{Perimeter} = 25 + 16 + 12 + 15 = 68$



8. In the triangle above, what is the value of $2S^2 - 32$?

(A) 32
(B) -18
(C) 20
(D) -32

Solution: Answer: (D)

By $5-12-13$ right triangle, $S = 0$

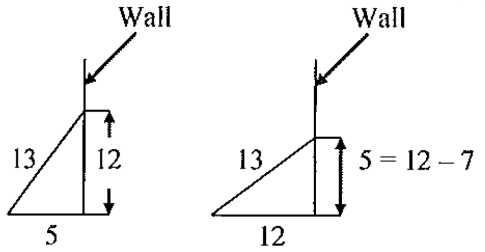
$\Downarrow$

$$2S^2 - 32 = -32$$

9. James places a 13-foot ladder against a vertical wall of a building, with the bottom of the ladder standing on concrete 5 feet from the base of the building. If the top of the ladder slips down 7 feet, the bottom of the ladder will slide out

(A) 4 ft
(B) 3 ft
(C) 6 ft
(D) 7 ft

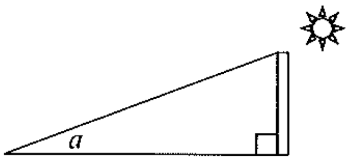
Solution: Answer: (D)



The bottom of the ladder will slide out:

$\Downarrow$

$12 - 5 = 7.$

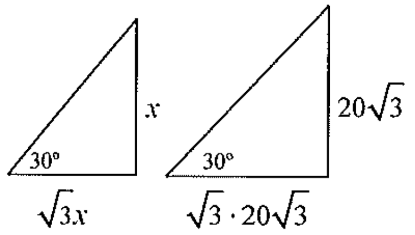


Note: Figure above not drawn to scale.

10. In the figure above, the sun is $a = 30°$ above the flag pole. What is the length of a shadow cast by another flag pole that is $P = 20\sqrt{3}$ feet tall?

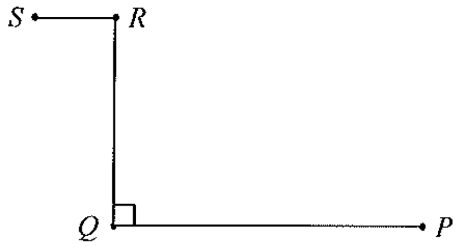
(A) 20
(B) $20\sqrt{6}$
(C) 30
(D) 60

Solution: Answer: (D)



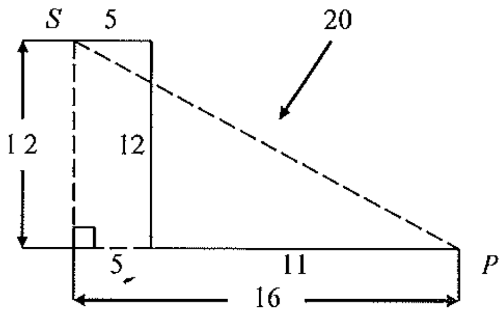
$\Downarrow$

$\sqrt{3} \times 20\sqrt{3} = 20 \times 3 = 60$



11. The figure above shows Pamela's route from her house to her relative house. Pamela travels eleven miles from P to Q, twelve miles from Q to R, and five miles from R to S. If Pamela were able to travel from P to S directly, how much, in mile, would the distance of travel be?

Solution: Answer: 20



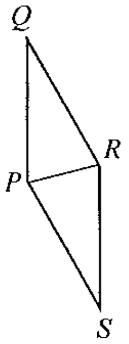
A 12-16-20 right triangle is a transmutation of a 3-4-5 right triangle.

$\Downarrow$

$PS = 20$

or

$$PS = \sqrt{12^2 + 16^2} = \sqrt{144 + 256} = \sqrt{400} = 20$$

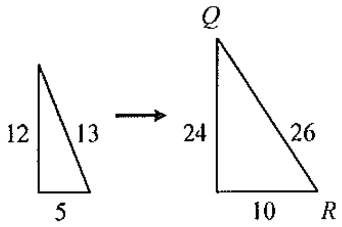


Note: Figure above not drawn to scale.

12. In the quadrilateral $PQRS$ above, two identical isosceles triangles with side length of 26 are joined at bases, which have a length of 20. What is the area of quadrilateral $PQRS$?

(A) 120
(B) 240
(C) 480
(D) 360

Solution: Answer: (C)



$$A=\frac{\text{base}\times\text{height}}{2}$$

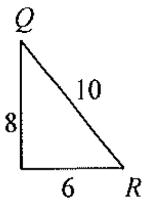
$$\Downarrow$$

$$A_{PQRS}=4\times\frac{10\times 24}{2}=480$$

13. In the quadrilateral $PQRS$ above, two identical isosceles triangles with side length of 10 are joined at bases, which have a length of 12. What is the area of quadrilateral $PQRS$?

(A) 24
(B) 192
(C) 120
(D) 96

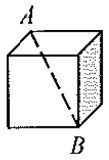
Solution: Answer: (D)



$$A=\frac{\text{base}\times\text{height}}{2}$$

$$\Downarrow$$

$$A_{PQRS}=4\times\frac{6\times 8}{2}=96$$



Note: Figure above not drawn to scale.

14. In the square cube above, it has a side length of 5. What is the length of the diagonal $\overline{AB}$?

(A) $5\sqrt{2}$
(B) 15
(C) $5\sqrt{3}$
(D) 5

Solution: Answer: (C)

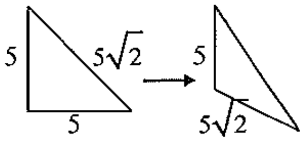
Method 1 You may use the conclusion:

Diagonal of a cube is $s\sqrt{3}$.

$$\Downarrow$$

$$AB=5\sqrt{3}$$

Method 2

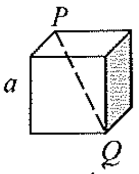


$$AB=\sqrt{5^2+\left(5\sqrt{2}\right)^2}$$

$$\sqrt{25+25\times 2}$$

$$\Downarrow$$

$$\sqrt{25(1+2)}=5\sqrt{3}$$



Note: Figure above not drawn to scale.

15. In the square cube above, what is the length of the diagonal $\overline{PQ}$?

(A) $a\sqrt{2}$
(B) a
(C) $a\sqrt{3}$
(D) a

Solution: Answer: (C)

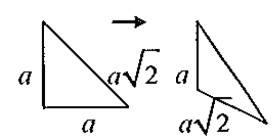
Method 1 You may use the conclusion:

Diagonal of a cube is $s\sqrt{3}$.

$$\Downarrow$$

$$AB=a\sqrt{3}$$

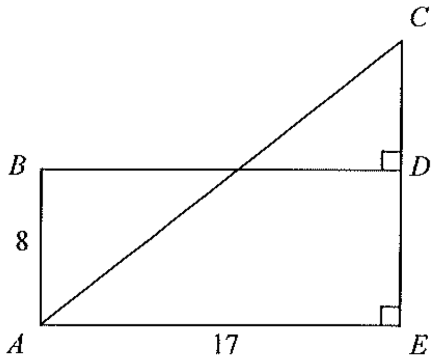
Method 2



$$PQ=\sqrt{a^2+\left(a\sqrt{2}\right)^2}$$

$$\Downarrow$$

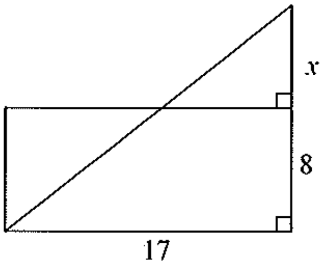
$$PQ=\sqrt{a^2+a^2\times 2}=a\sqrt{3}$$



16. In the figure above, the area of triangle ACE equals the area of rectangle $ABDE$. What is the length of segment $\overline{CD}$?

(A) 6
(B) 7
(C) 8
(D) 9

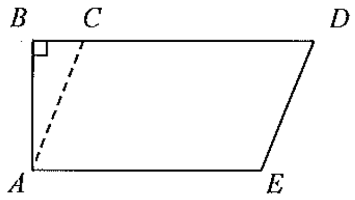
Solution: Answer: (C)



$$\frac{(x+8)\cdot \cancel{17}}{2} = 8\cdot \cancel{17}$$

$$\Downarrow$$

$$x+8=16,\ x=\boxed{8}$$

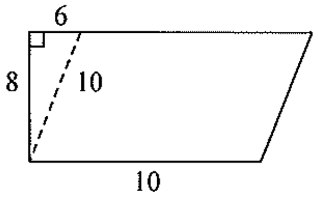


Note: Figure above not drawn to scale.

17. In the figure above, the length of segment $\overline{AB}$ is 8, and the length of each side of quadrilateral $ACDE$ is 10. What is the area of quadrilateral $ABDE$?

(A) 128
(B) 104
(C) 108
(D) 106

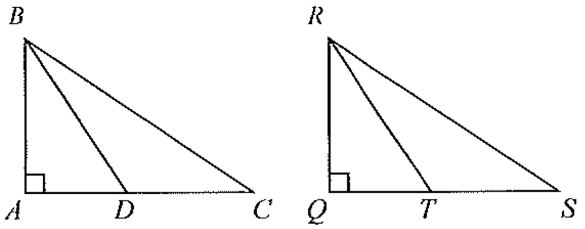
Solution: Answer: (B)



$$\Downarrow$$

$$A_{ACDE} + A_{\text{Tri-}ABC} = 10\cdot 8 + \frac{8\times 6}{2} = 104$$

Note: Figures below not drawn to scale.



18. In the triangles ABC and QRS above, if $AD = 5$, $CD = 7$, $BC = 13$, $QT = 3$, $ST = 8$, and $QR = 5$, what is the sum of areas of the triangles ABC and TRS?

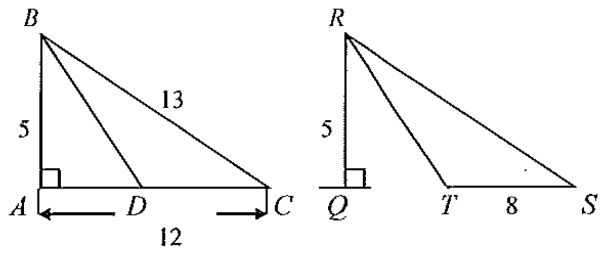
(A) 90

(B) 30

(C) 40

(D) 50

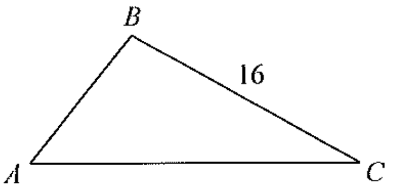
Solution: Answer: (D)



$$A_{ABC} = \frac{12 \cdot 5}{2} = 30,\ A_{TRS} = \frac{8 \cdot 5}{2} = 20$$

$$\Downarrow$$

$$A_{\text{sum}} = A_{ABC} + A_{TRS} = 50$$



Note: Figure above not drawn to scale.

19. If in triangle ABC, $m\angle A = 45^\circ$ and $m\angle C = 30^\circ$, then $AB =$

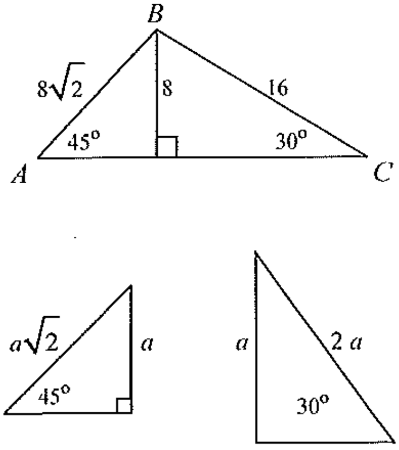
(A) $8\sqrt{3}$

(B) $8\sqrt{2}$

(C) 16

(D) 8

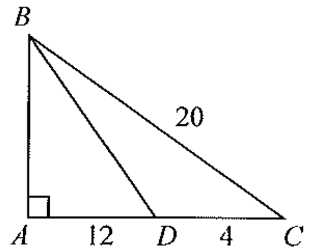
Solution: Answer: (B)



$$\Downarrow$$

Answer is (B).

Note: Figure below not drawn to scale.



20. In the figure above, what is the area of triangle ABD?

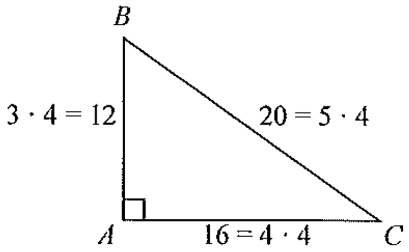
(A) 144

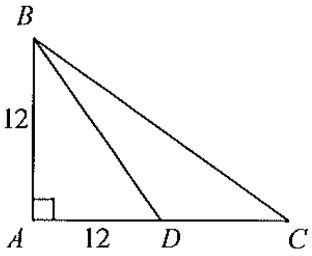
(B) 89

(C) 18

(D) 72

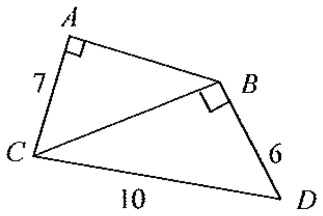
Solution: Answer: (D)





$$\Downarrow$$

$$A_{\text{Tri}} = \frac{b \cdot h}{2} = \frac{12 \cdot 12}{2} = 72$$



Note: Figure above not drawn to scale.

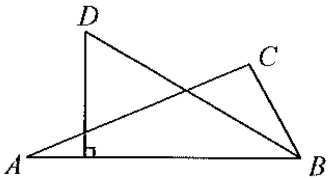
1. In quadrilateral $ABCD$ above, what is the length of $\overline{AB}$?

(A) $\sqrt{15}$

(B) $\sqrt{11}$

(C) $\sqrt{13}$

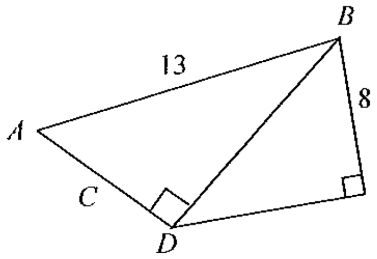
(D) $\sqrt{14}$



Note: Figure above not drawn to scale.

2. In the figure above, $\overline{BD}$ bisects $\angle B$. The measure of $\angle D$ is 50° and the measure of $\angle C$ is 40°. What is the measure of $\angle A$?

(A) 80
(B) 70
(C) 60
(D) 50



Note: Figure above not drawn to scale.

3. In quadrilateral $ABCD$ above, what is the area of triangle BCD?

(A) $32\sqrt{5}$

(B) $16\sqrt{5}$

(C) $8\sqrt{5}$

(D) $10\sqrt{14}$

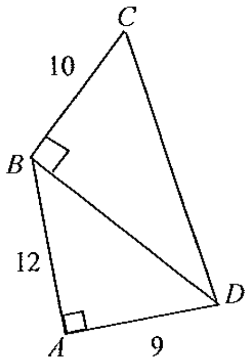
4. In quadrilateral $ABCD$ above, what is the length of $\overline{DC}$?

(A) $\sqrt{208}$

(B) $\sqrt{110}$

(C) $\sqrt{80}$

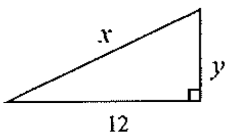
(D) 70



Note: Figure above not drawn to scale.

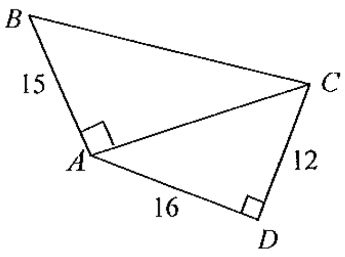
5. In quadrilateral *ABCD* above, what is the area of triangle *DCB*?

(A) 150
(B) 75
(C) 85
(D) 65



6. In the triangle above, if $y = 5$, what is the value of x?

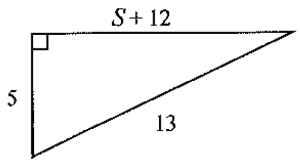
(A) 13
(B) 14
(C) 15
(D) 16



Note: Figure above not drawn to scale.

7. In quadrilateral *ABCD* above, what is the perimeter of *ABCD*?

(A) 63
(B) 86
(C) 68
(D) 53

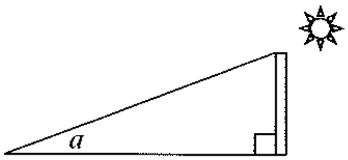


8. In the triangle above, what is the value of $2S^2 - 32$?

(A) 32
(B) -18
(C) 20
(D) -32

9. James places a 13-foot ladder against a vertical wall of a building, with the bottom of the ladder standing on concrete 5 feet from the base of the building. If the top of the ladder slips down 7 feet, the bottom of the ladder will slide out

(A) 4 ft
(B) 3 ft
(C) 6 ft
(D) 7 ft



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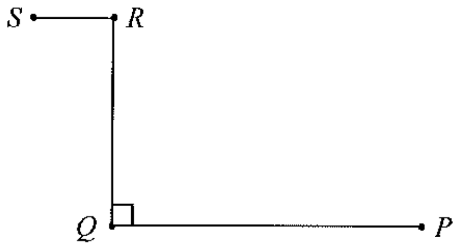
10. In the figure above, the sun is $a = 30°$ above the flag pole. What is the length of a shadow cast by another flag pole that is $P = 20\sqrt{3}$ feet tall?

(A) 20
(B) $20\sqrt{6}$
(C) 30
(D) 60



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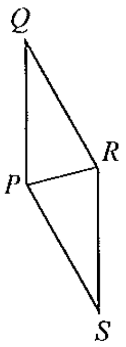

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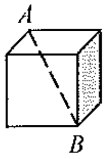
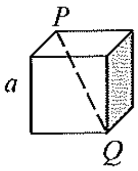


Note: Figure above not drawn to scale.

14. In the square cube above, it has a side length of 5. What is the length of the diagonal $\overline{AB}$?

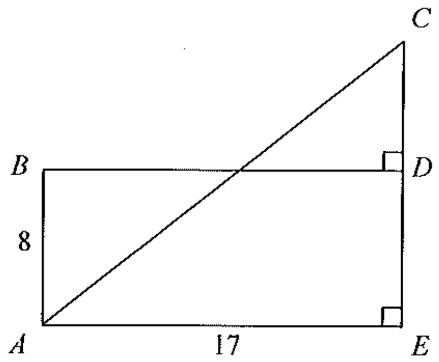
(A) $5\sqrt{2}$
(B) 15
(C) $5\sqrt{3}$
(D) 5



Note: Figure above not drawn to scale.

15. In the square cube above, what is the length of the diagonal $\overline{PQ}$?

(A) $a\sqrt{2}$
(B) a
(C) $a\sqrt{3}$
(D) a



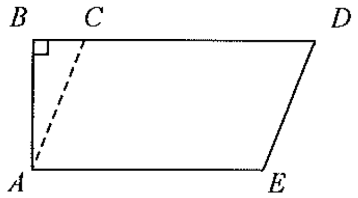
16. In the figure above, the area of triangle ACE equals the area of rectangle $ABDE$. What is the length of segment $\overline{CD}$?

(A) 6

(B) 7

(C) 8

(D) 9



Note: Figure above not drawn to scale.

17. In the figure above, the length of segment $\overline{AB}$ is 8, and the length of each side of quadrilareral $ACDE$ is 10. What is the area of quadrilareral $ABDE$?

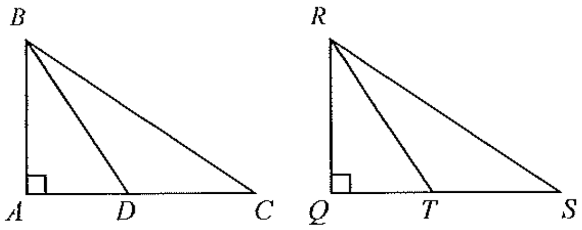
(A) 128

(B) 104

(C) 108

(D) 106

Note: Figures below not drawn to scale.



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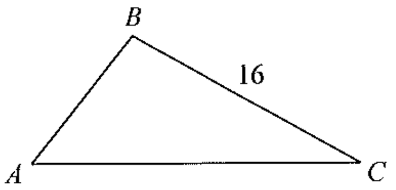
(A) 90

(B) 30

(C) 40

(D) 50



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19. If in triangle ABC, $m\angle A = 45^\circ$ and $m\angle C = 30^\circ$, then $AB =$

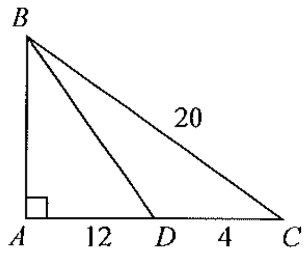
(A) $8\sqrt{3}$

(B) $8\sqrt{2}$

(C) 16

(D) 8

Note: Figure below not drawn to scale.



20. In the figure above, what is the area of triangle ABD?

(A) 144

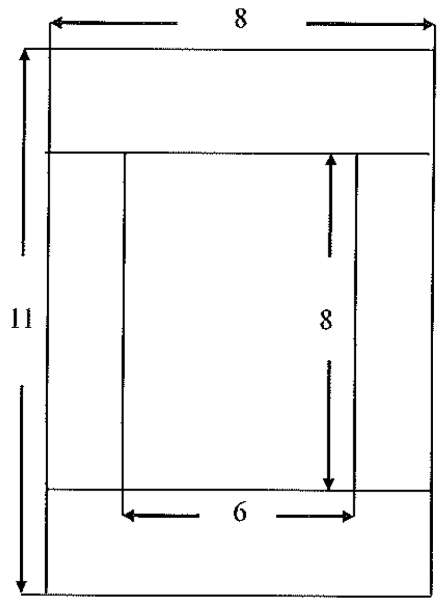
(B) 89

(C) 18

(D) 72

1. Robert uses an 8×11-inch sheet of paper to type an essay. Robert leaves 1-inch margin on both left side and right side, and 3/2-inch margin on both bottom and top. What fractional part of the page is used for typing?

Solution: Answer: 6/11 or .545



$$\Downarrow$$

$$\frac{A_{\text{typing}}}{A_{\text{page}}}=\frac{6\times 8}{8\times 11}=\frac{6}{11}\text{ or }0.545$$

2. A rectangular garden is twice as its length as its width. If its length is increased by 3 meters and its width is decreased by 1 meter. The new garden formed has an area of 33 square meters. Which of the following is the perimeter, in meters, of the original garden?

(A) 44
(B) 24
(C) 34
(D) 33

Solution: Answer: (B)

$$(2w+3)(w-1)=33$$

$$2w^2-2w+3w-3=33$$

$$2w^2+w-36=0$$

$$(2w+9)(w-4)=0$$

$$2w+9=0,\rightarrow \underbrace{w=-\frac{9}{2}}_{\text{It doesn't exist in the real life.}}$$

$$w-4=0,\ \rightarrow\ w=\boxed{4}$$

$$2w=2\cdot 4=\boxed{8}$$

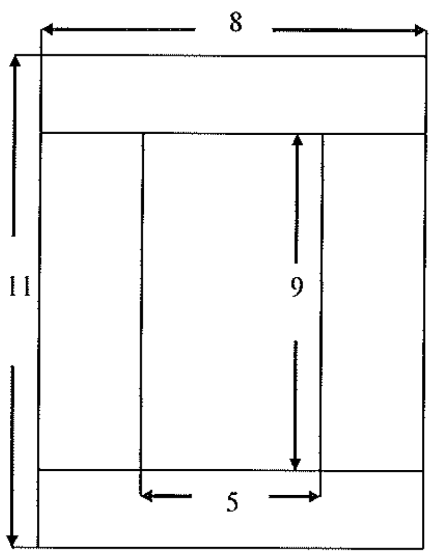
$$\Downarrow$$

$$A=2(4+8)=24$$

3. Amy uses an 8×11-inch sheet of paper to type an essay. She leaves 3/2-inch margin on both left side and right side, and 1-inch margin on both bottom and top. What fractional part of the page is used for typing?

(A) $\frac{88}{45}$
(B) $\frac{86}{45}$
(C) $\frac{45}{88}$
(D) $\frac{35}{88}$

Solution: Answer: (C)



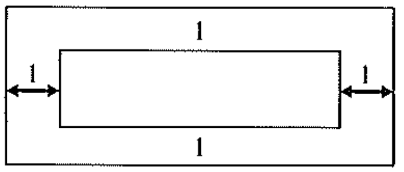
$$\Downarrow$$

$$\frac{A_{\text{typing}}}{A_{\text{page}}}=\frac{5\times 9}{8\times 11}=\frac{45}{88}$$

4. A rectangle garden is composes of a garden part and a uniform walkway part. The garden part is twice as its length as its width, and the walkway is 1 yard wide. If the area of the entire rectangle garden is 264 square yards, what is the perimeter of garden part?

(A) 60
(B) 50
(C) 90
(D) 120

Solution: Answer: (A)



$$(2w+2)(w+2)=264$$

$$2w^2+4w+2w+4=264$$

$$2w^2+6w+4=264,\ w^2+3w+2=132$$

$$w^2+3w-130=0,(w+13)(w-10)=0$$

$$w+13=0,\rightarrow \underbrace{w=-13}_{\text{It doesn't exist in the real life.}}$$

$$w-10=0,\rightarrow w=\boxed{10},2w=2\cdot 10=\boxed{20}$$

$$\Downarrow$$

$$A=2(L+W)=2(10+20)=60$$

Margins

1. Robert uses an 8×11-inch sheet of paper to type an essay. Robert leaves 1-inch margin on both left side and right side, and 3/2-inch margin on both bottom and top. What fractional part of the page is used for typing?

2. A rectangular garden is twice as its length as its width. If its length is increased by 3 meters and its width is decreased by 1 meter. The new garden formed has an area of 33 square meters. Which of the following is the perimeter, in meters, of the original garden?

(A) 44

(B) 24

(C) 34

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(C) $\frac{45}{88}$

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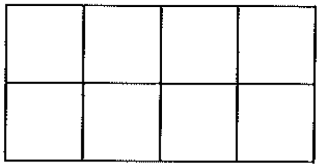
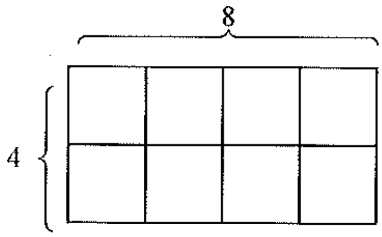


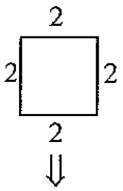
1. In the figure above, the large rectangle is divided into 8 identical small squares. If the area of the large rectangle is 32, what is the perimeter of the small square?

(A) 4
(B) 10
(C) 6
(D) 8

Solution: Answer: (D)

$$32 = 8 \times 4$$





$$\Downarrow$$

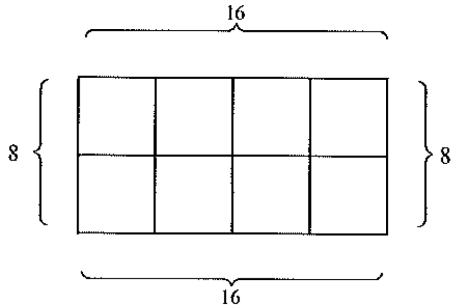
$$P = 2 + 2 + 2 + 2 = 8$$

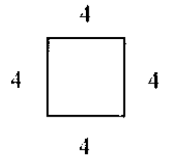
2. In the figure above, the large rectangle is divided into 8 identical small squares. If the perimeter of the large rectangle is 48, what is the perimeter of the small square?

(A) 4
(B) 16
(C) 6
(D) 8

Solution: Answer: (B)

$$48 = 2(16 + 8)$$





$$P = 4 + 4 + 4 + 4 = 16$$

3. If the length and width of rectangle P are 20 percent less and 10 percent more, respectively, than the length and width of rectangle Q, the area of P equals what percent of the area Q?

(A) 72
(B) 81
(C) 88
(D) 68

Solution: Answer: (C)

$$\frac{0.8x \cdot 1.1y}{x \cdot y}$$

$$\downarrow$$

$$\frac{0.88\cancel{xy}}{\cancel{xy}}$$

$$\Downarrow$$

$$0.88 = 88\%$$

4. If the length and width of rectangle M are 30 percent more and 20 percent less, respectively, than the length and width of rectangle N, the area of M equals what percent of the area N?

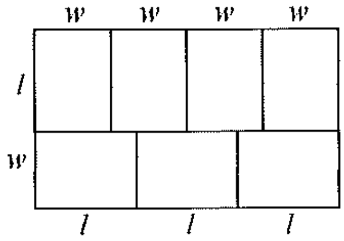
(A) 104
(B) 102
(C) 88
(D) 68

Solution: Answer: (A)

$$\frac{1.3x \cdot 0.8y}{x \cdot y} = \frac{1.04\cancel{xy}}{\cancel{xy}}$$

$$\Downarrow$$

$$1.04 \times 100\% = 104\%$$



Note: Figure above not drawn to scale.

5. The pattern shown above is composed of seven of the same rectangles l units long and w units wide. This pattern is used repeatedly to completely cover a rectangle region $10l$ units long and $9l$ units wide. How many rectangles of dimension l by w are needed?

(A) 90
(B) 100
(C) 110
(D) 120

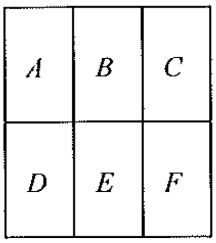
Solution: Answer: (D)

$$\begin{cases} \dfrac{10l \times 9l}{l \times w} \\ 4w = 3l \end{cases}$$

$$4w = 3l \rightarrow w = \frac{3}{4}l$$

$$\Downarrow$$

$$\frac{10l \times 9l}{l \times w} = \frac{90\cancel{l^2}}{\cancel{l} \times \frac{3}{4}\cancel{l}} = \frac{\cancel{90}^{30} \cdot 4}{\cancel{3}} = 120$$



Note: Figure above not drawn to scale.

6. In the figure above, a large rectangle is divided to six smaller rectangles that have integer lengths and widths. The areas of rectangles A, D, and E are 18, 24, and 16, respectively. If the area of the entire figure is 140, what is the area of rectangle C?

(A) 10
(B) 20
(C) 30
(D) 40

Solution: Answer: (C)

(1)

	3	2	x
6	18 A	B	C
8	24 D	16 E	

(2)

$$\begin{cases} \text{Area} = l \times w = 140 \\ \text{Area} = (6+8) \times (3+2+x) \end{cases}$$

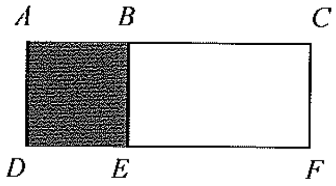
$$\downarrow$$

$$14 \times (5+x) = 140$$

$$\Downarrow$$

$$x = 5$$

(3) $\text{Area}_C = 6x = 6 \times 5 = 30$

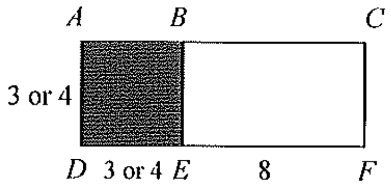


Note: Figure above not drawn to scale.

7. In the figure above, $ACFD$ is a rectangle and $ABED$ is a square. $EF = 8$ and DE is an integer great than 0. If the area of $ACFD$ must be more than 20 but less than 50, what is one possible value of DE?

(A) 3
(B) 2
(C) 5
(D) 6

Solution: Answer: (A)



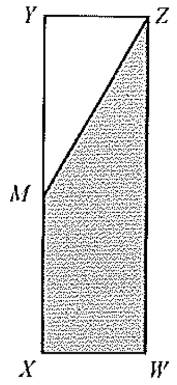
When $DE = 1$ or $DE = 2$, area of $ACFD$ is less or equal to 20.

When $DE = 5$, area of $ACFD$ is more than 50.

When $DE = 3$ or $DE = 4$, area of $ACFD$ is between 20 and 50.

$\Downarrow$

Answer is (A).

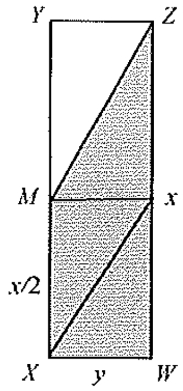


Note: Figure above not drawn to scale.

8. In rectangle $WXYZ$ above, point M is the midpoint of XY. If the area of quadrilateral $WXMZ$ is $\frac{3}{5}$, what is the area of rectangle $WXYZ$?

(A) $\frac{8}{9}$
(B) $\frac{4}{5}$
(C) $\frac{7}{9}$
(D) $\frac{6}{5}$

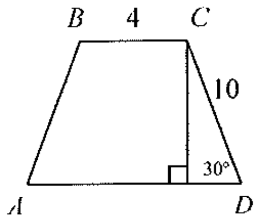
Solution: Answer: (B)



$$\frac{\cancel{2}}{4}xy = \frac{\cancel{2}}{5}$$

$\Downarrow$

$$A_{WXYZ} = xy = \frac{4}{5}$$

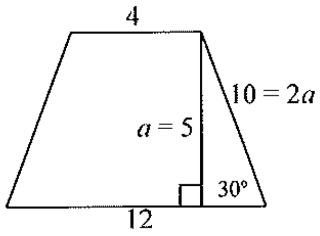


Note: Figure above not drawn to scale.

9. In the figure above, If $AD = 12$, what is the area of trapezoid $ABCD$ in the figure above?

(A) $40\sqrt{2}$
(B) 40
(C) 50
(D) $40\sqrt{3}$

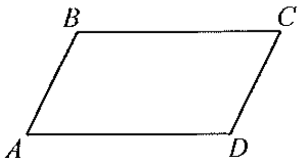
Solution: Answer: (B)



$$A_{\text{trapezoid}} = \frac{\text{Top} + \text{Bottom}}{2} \cdot \text{Height}$$

$$\Downarrow$$

$$\frac{4+12}{2} \cdot 5 = 40$$



Note: Figure above not drawn to scale.

10. In parallelogram $ABCD$, $AB = x + 8$, $BC = 3x$, and $CD = 4x - 4$. $ABCD$ could be a

(A) Trapezoid
(B) Triangle
(C) Square
(D) Line

Solution: Answer: (C)

$ABCD$ is a parallelogram.

$$\downarrow$$

$AB = CD$ and $BC = AD$

$x + 8 = 4x - 4 \rightarrow x = 4$

$$\downarrow$$

$AB = x + 8 = 12$, $CD = 4x - 4 = 12$

$BC = 3x = 12$, $BC = AD \rightarrow AD = 12$

$$\downarrow$$

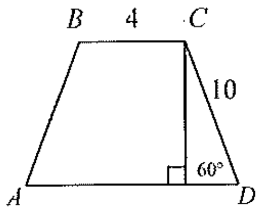
$AB = CD = BC = AD = 12$

A square is a special rhombus.

It is a square if each its interior angle is 90°.

$$\Downarrow$$

It could be a square.

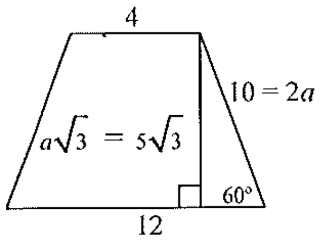


Note: Figure above not drawn to scale.

11. In the figure above, If $AD = 12$, what is the area of trapezoid $ABCD$ in the figure above?

(A) $40\sqrt{2}$
(B) 35
(C) 50
(D) $40\sqrt{3}$

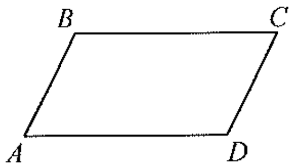
Solution: Answer: (D)



$$A_{\text{trapezoid}} = \frac{\text{Top} + \text{Bottom}}{2} \cdot \text{Height}$$

$$\Downarrow$$

$$\frac{4+12}{2} \cdot 5\sqrt{3} = 40\sqrt{3}$$



Note: Figure above not drawn to scale.

12. In parallelogram $ABCD$, $AB = x + 8$, $BC = 3x$, and $CD = 4x - 4$. $ABCD$ must be a

(A) One surface of a cube
(B) Triangle
(C) Square
(D) Rhombus

Solution: Answer: (D)

$ABCD$ is a parallelogram.

$$\downarrow$$

$AB = CD$ and $BC = AD$

$x + 8 = 4x - 4 \rightarrow x = 4$

$$\downarrow$$

$AB = x + 8 = 12$, $CD = 4x - 4 = 12$,

$BC = 3x = 12$, $BC = AD \rightarrow AD = 12$

$$\downarrow$$

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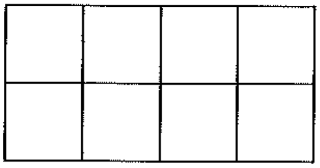
It is a square if each its interior angles is 90°.

A square is a special rhombus.

$$\Downarrow$$

It must be a rhombus.



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(B) 10
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2. In the figure above, the large rectangle is divided into 8 identical small squares. If the perimeter of the large rectangle is 48, what is the perimeter of the small square?

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3. If the length and width of rectangle P are 20 percent less and 10 percent more, respectively, than the length and width of rectangle Q, the area of P equals what percent of the area Q?

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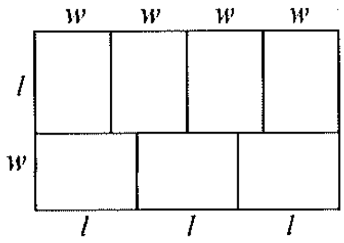

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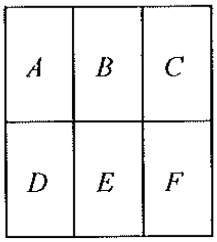

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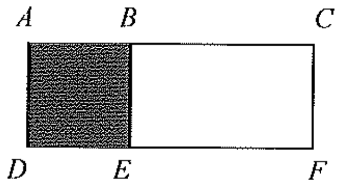
(C) 30
(D) 40



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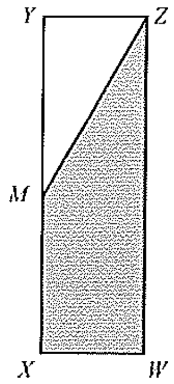

Note: Figure above not drawn to scale.

8. In rectangle *WXYZ* above, point *M* is the midpoint of *XY*. If the area of quadrilateral *WXMZ* is $\frac{3}{5}$, what is the area of rectangle *WXYZ*?

(A) $\frac{8}{9}$
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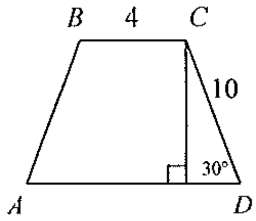
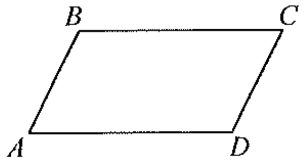


Note: Figure above not drawn to scale.

9. In the figure above, If $AD = 12$, what is the area of trapezoid $ABCD$ in the figure above?

(A) $40\sqrt{2}$
(B) 40
(C) 50
(D) $40\sqrt{3}$



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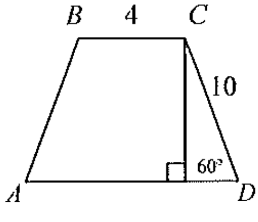

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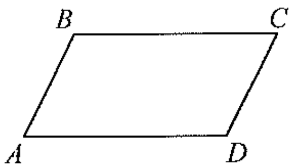

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(A) One surface of a cube
(B) Triangle
(C) Square
(D) Rhombus

1. If the perimeter of a regular polygon is 18, which of the following could be the length of one side of the polygon?

(A) 18
(B) 3
(C) 5
(D) 7

Solution: Answer: (B)

(1) Any regular polygon has at least 3 sides.

↓

Choices (A) and (E) are eliminated.

(2) $S_{\text{side length}} = \dfrac{\text{Perimeter}}{\#\text{ of sides}} \rightarrow \dfrac{\text{Perimeter}}{\text{Integer}}$

$\dfrac{18}{\text{Integer}} \neq 5 \text{ or } 7$ $\qquad$ $\dfrac{18}{\text{Integer}} = \dfrac{18}{6} = 3$

↓ $\qquad$ ⇓

(C) and (D) are liminated. Answer is (B).

2. If the perimeter of a regular polygon is 28, which of the following could be the length of one side of the polygon?

(A) 28
(B) 3
(C) 5
(D) 7

Solution: Answer: (D)

(1) Any regular polygon has at least 3 sides.

↓

Choices (A) and (E) are eliminated.

(2) $S_{\text{side length}} = \dfrac{\text{Perimeter}}{\#\text{ of sides}} \rightarrow \dfrac{\text{Perimeter}}{\text{Integer}}$

$\dfrac{28}{\text{Integer}} \neq 3 \text{ or } 5$ $\qquad$ $\dfrac{28}{\text{Integer}} = \dfrac{28}{4} = 7$

↓ $\qquad$ ⇓

(B) and (C) are liminated. Answer is (D).

3. If the perimeter of a regular polygon is 28, which of the following could be the length of one side of the polygon?

(A) 28
(B) 3
(C) 2
(D) 5

Solution: Answer: (C)

(1) Any regular polygon has at least 3 sides.

↓

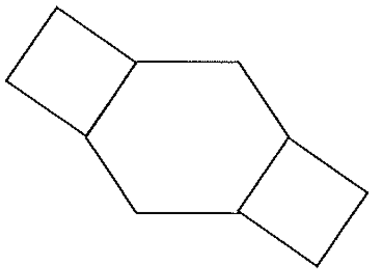
Choices (A) and (E) are eliminated.

(2) $S_{\text{side length}} = \dfrac{\text{Perimeter}}{\#\text{ of sides}} \rightarrow \dfrac{\text{Perimeter}}{\text{Integer}}$

$\dfrac{28}{\text{Integer}} \neq 3 \text{ or } 5$ $\qquad$ $\dfrac{28}{\text{Integer}} = \dfrac{28}{14} = 2$

↓ $\qquad$ ⇓

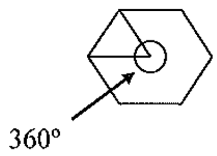
(B) and (D) are liminated. Answer is (C).



4. The figure above shows a regular hexagon with side length s and two squares with side length s. If the area of the hexagon is $96\sqrt{3}$ square feet, what is the sum of the areas, in square feet, of the two squares?

(A) 256
(B) 64
(C) 128
(D) 512

Solution: Answer: (C)



$\dfrac{1}{6} \times 360 = 60 \rightarrow$ 6 equilatera triangles.

Use area formula of an equilatera triangle

$$A = \frac{s^2\sqrt{3}}{4}$$

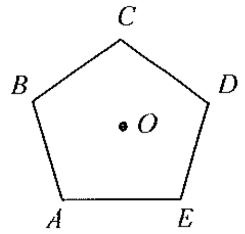
$$6A = 6 \cdot \frac{s^2\sqrt{3}}{4} = 96\sqrt{3},\ s^2 = 64$$

⇓

For 2 squares,

$$2 \cdot s^2 = 128$$

Regular Polygons

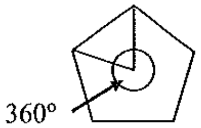


Note: Figure above not drawn to scale.

5. In the regular pentagon $ABCDE$ above, if O is the center of the pentagon, what is the degree measure of $\angle BOC$?

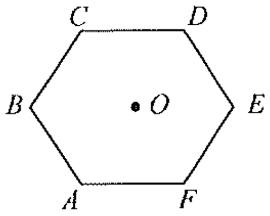
(A) 45°
(B) 60°
(C) 72°
(D) 58°

Solution: Answer: (C)



$$\Downarrow$$

$$\frac{1}{5}\times 360 = 72$$

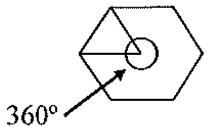


Note: Figure above not drawn to scale.

6. In the regular hexagon $ABCDEF$ above, if O is the center of the hexagon, what is the $m\angle BOC$?

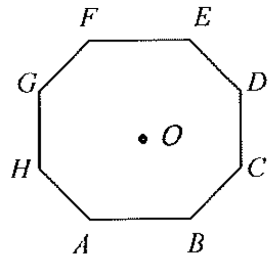
(A) 45°
(B) 60°
(C) 76°
(D) 58°

Solution: Answer: (B)



$$\Downarrow$$

$$\frac{1}{6}\times 360 = 60$$



Note: Figure above not drawn to scale.

7. In the regular octagon $ABCDEFGH$ above, if O is the center of the octagon, what is the degree measure of $\frac{\angle B}{2}$?

(A) 135°
(B) 67.5°
(C) 45°
(D) 90°

Solution: Answer: (B)

$$(n-2)\times 180$$

$$\downarrow$$

$$(8-2)\times 180 = 1080$$

$$\Downarrow$$

$$\frac{m\angle B}{2} = \frac{\frac{1}{8}\cdot 1040}{2} = \frac{1080}{8\cdot 2} = 67.5$$

8. The sum of measures of the angles in a nonagon is

(A) 1260°
(B) 720°
(C) 1440°
(D) 540°

Solution: Answer: (A)

Use the formula: $(n-2)\times 180$

$$\Downarrow$$

$$(9-2)\times 180 = 1260$$

9. The sum of measures of the angles in a hexagon is

(A) 1260°
(B) 720°
(C) 1440°
(D) 540°

Solution: Answer: (B)

Use the formula: $(n-2)\times 180$

$$\Downarrow$$

$$(6-2)\times 180 = 720$$

10. The sum of measures of the angles in a pentagon is

(A) 1440°
(B) 720°
(C) 270°
(D) 540°

Solution: Answer: (D)

Use the formula: $(n-2)\times 180$

$$\Downarrow$$

$$(5-2)\times 180 = 540$$

1. If the perimeter of a regular polygon is 18, which of the following could be the length of one side of the polygon?

(A) 18
(B) 3
(C) 5
(D) 7

2. If the perimeter of a regular polygon is 28, which of the following could be the length of one side of the polygon?

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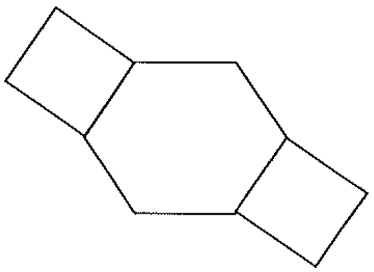

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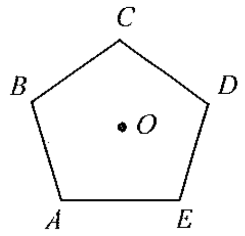
(A) 256
(B) 64
(C) 128
(D) 512

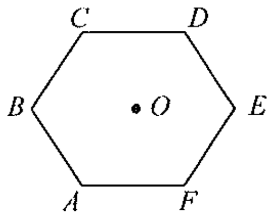
Regular Polygons



Note: Figure above not drawn to scale.

5. In the regular pentagon $ABCDE$ above, if O is the center of the pentagon, what is the degree measure of $\angle BOC$?

(A) 45°
(B) 60°
(C) 72°
(D) 58°



Note: Figure above not drawn to scale.

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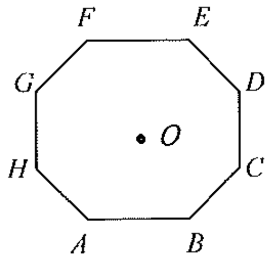

Note: Figure above not drawn to scale.

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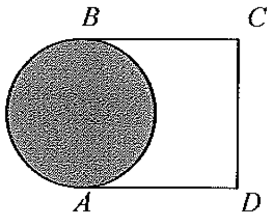
(A) 1260°
(B) 720°
(C) 1440°
(D) 540°

9. The sum of measures of the angles in a hexagon is

(A) 1260°
(B) 720°
(C) 1440°
(D) 540°

10. The sum of measures of the angles in a pentagon is

(A) 1440°
(B) 720°
(C) 270°
(D) 540°



Note Figure above not drawn to scale.

1. In the figure above, side AB of square $ABCD$ passes through the center of circle. If the length of diagonal of square $ABCD$ is $4a$, what is the area of the shaded portion?

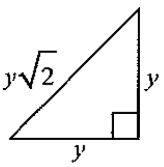
(A) $2a^2\pi$

(B) $a^2\pi$

(C) $a\pi^2$

(D) $\dfrac{a^2}{\pi}$

Solution: Answer: (A)



$$y = \frac{4a}{\sqrt{2}} = 4a \times \frac{\sqrt{2}}{2} = 2a\sqrt{2}$$

$$\downarrow$$

$$r = \frac{y}{2} = \frac{2a\sqrt{2}}{2} = a\sqrt{2}$$

$$\Downarrow$$

$$\text{Area} = r^2\pi = \left(a\sqrt{2}\right)^2\pi = 2a^2\pi$$

2. In the figure above, side AB of square $ABCD$ passes through the center of circle. If the length of a side of square $ABCD$ is 2π, what is the area of the shaded region?

(A) π^4

(B) $2\pi^2$

(C) π^3

(D) $2\pi^3$

Solution: Answer: (C)

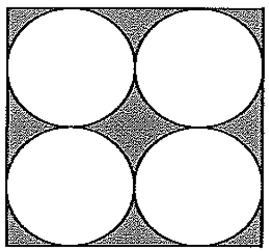
$$s = 2\pi$$

$$\downarrow$$

$$r = \frac{s}{2} = \frac{2\pi}{2} = \pi$$

$$\Downarrow$$

$$\text{Area} = r^2\pi = \pi^2\pi = \pi^3$$



3. There are one bigger square and four circles touching together as the figure shown above. The radius of each circle is π. Of the following, which is the area of the shaded region?

(A) $\pi^2(16-\pi)$

(B) $4\pi^2(4-\pi)$

(C) $16\pi^3 - \pi^2$

(D) $16\pi^3 - \pi^3$

Solution: Answer: (B)

$$A_{\text{shaded}} = A_{\text{square}} - 4A_{\text{circles}}$$

$$r = \pi,\ s = 4r = 4\pi$$

$$\begin{cases} A_{\text{square}} = s^2 = (4\pi)^2 = 16\pi^2 \\ 4A_{\text{circles}} = 4 \times r^2\pi = 4\pi^2\pi = 4\pi^3 \end{cases}$$

$$\Downarrow$$

$$A_{\text{shaded}} = 16\pi^2 - 4\pi^3 = 4\pi^2(4-\pi)$$

4. There are one bigger square and four circles touching together as the figure shown above. The length of the square side is π. Of the following, which is the area of the shaded region?

(A) $\pi^2 - \dfrac{\pi^3}{16}$

(B) $16\pi^3 - \pi$

(C) $16\pi^3 - \pi^2$

(D) $\pi^2\left(1 - \dfrac{\pi}{4}\right)$

Solution: Answer: (D)

$$A_{\text{shaded}} = A_{\text{square}} - 4A_{\text{circle}}$$

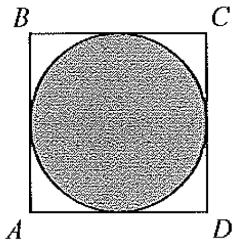
$$s = \pi,\ r = \frac{s}{4} = \frac{\pi}{4}$$

$$\begin{cases} A_{\text{square}} = s^2 = \pi^2 \\ 4A_{\text{circle}} = 4r^2\pi = 4\left(\dfrac{\pi}{4}\right)^2\pi = \dfrac{\pi^3}{4} \end{cases}$$

$$\Downarrow$$

$$A_{\text{shaded}} = \pi^2 - \frac{\pi^3}{4} = \pi^2\left(1 - \frac{\pi}{4}\right)$$

Shaded Regions



5. In the figure above, if the length of diagonal of square *ABCD* is 1, what is the half area of the shaded region?

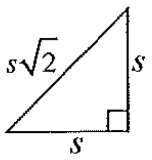
(A) $\frac{\pi}{8}$

(B) $\frac{\pi}{16}$

(C) $\frac{\pi^2}{8}$

(D) $\frac{\pi^2}{4}$

Solution: Answer: (B)



$$1 = s\sqrt{2}\ ,\ s = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\Downarrow$$

$$r = \frac{s}{2} = \frac{1}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}$$

$$\Downarrow$$

$$\frac{1}{2}\text{Area} = \frac{1}{2}r^2\pi = \frac{1}{2}\left(\frac{\sqrt{2}}{4}\right)^2\pi = \frac{\pi}{16}$$

6. In the figure above, if the length of a side of square *ABCD* is 1, what is the area of the non-shaded region?

(A) $\frac{1-\pi}{2}$

(B) $1-\pi$

(C) $\frac{4-\pi}{4}$

(D) $2-\pi$

Solution: Answer: (C)

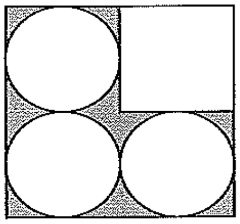
$$A_{\text{non-shaded}} = A_{\text{square}} - A_{\text{circle}}$$

$$r = \frac{1}{2}$$

$$\begin{cases} A_{\text{square}} = s^2 = 1^2 = 1 \\ A_{\text{circle}} = r^2\pi = \left(\frac{1}{2}\right)^2\pi = \frac{\pi}{4} \end{cases}$$

$$\Downarrow$$

$$A_{\text{non-shaded}} = 1 - \frac{\pi}{4} = \frac{4-\pi}{4}$$



7. There are one bigger square, three circles and one smaller square touching together as the figure shown above. The radius of each circle is 1. Of the following, which is the area of the shaded region?

(A) $16-3\pi$

(B) $12-3\pi$

(C) $4-3\pi$

(D) $8-3\pi$

Solution: Answer: (B)

$$A_{\text{shaded}} = A_{\text{square}} - 3A_{\text{circle}} - A_{\text{smaller-sq}}$$

$$r = 1,\ \ s = 4$$

$$\begin{cases} A_{\text{square}} = s^2 = 4^2 = 16 \\ A_{\text{smaller-sq}} = 2^2 = 4 \\ 3A_{\text{circle}} = 3 \times r^2\pi = 3 \times 1^2\pi = 3\pi \end{cases}$$

$$\Downarrow$$

$$A_{\text{shaded}} = 16 - 3\pi - 4 = 12 - 3\pi$$

8. There are one bigger square, three circles and one smaller square touching together as the figure shown above. The circumference of each circle is 2π. Of the following, which is the area of the shaded region?

(A) $12-3\pi$

(B) $8-3\pi$

(C) $16-3\pi$

(D) $10-3\pi$

Solution: Answer: (A)

$$A_{\text{shaded}} = A_{\text{square}} - 3A_{\text{circle}} - A_{\text{smaller-sq}}$$

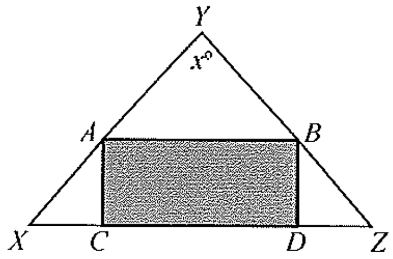
$$C = 2r\pi,\ r = \frac{C}{2\pi} = \frac{2\pi}{2\pi} = 1$$

$$r = 1,\ \ s = 4$$

$$\begin{cases} A_{\text{square}} = s^2 = 4^2 = 16 \\ A_{\text{smaller-sq}} = 2^2 = 4 \\ 3A_{\text{circle}} = 3 \times r^2\pi = 3 \times 1^2\pi = 3\pi \end{cases}$$

$$\Downarrow$$

$$A_{\text{shaded}} = 16 - 3\pi - 4 = 12 - 3\pi$$



Note: Figure above not drawn to scale.

9. In the figure above, XYZ is a triangle with $x = 90^\circ$ and $ABCD$ is a rectangle. A *is* midpoint of $\overline{XY}$ and B is midpoint of $\overline{YZ}$. If $XY = YZ = 2a\sqrt{2}$, what is the area of the shaded portion?

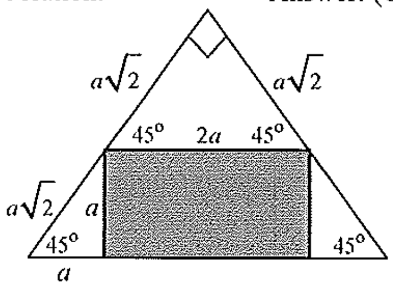
(A) $\sqrt{2}a$

(B) $\left(2\sqrt{a}\right)^2$

(C) $2a^2$

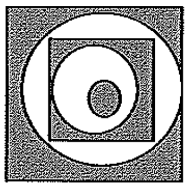
(D) $4a^2$

Solution: Answer: (C)



$$\Downarrow$$

$$A_{\text{shaded}} = 2a \cdot a = 2a^2$$



Note: Figure above not drawn to scale.

10. In the figure above, the two squares have sides of 4 and 7 and the three circles have radii of 1, 2 and 3, respectively. Which of the following is the sum of the area of the shaded portion?

(A) $65\pi - 12$ (B) $65 - 12\pi$ (C) $12\pi - 6.5$ (D) $62\pi - 12$

Solution: Answer: (B)

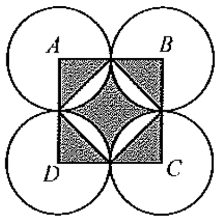
$$\underset{\text{shaded}}{A} = \underset{\text{greater square}}{A} - \underbrace{\underset{\text{greatest circle}}{A} + \underset{\text{smaller square}}{A} - \underset{\text{smaller circle}}{A} + \underset{\text{smallest circle}}{A}}$$

$$7^2 - 3^2\pi + 4^2 - 2^2\pi + 1^2\pi$$

$$49 - 9\pi + 16 - 4\pi + \pi$$

$$\Downarrow$$

$$65 - 12\pi$$



Note: Figure above not drawn to scale.

11. In the figure above, the vertices of square $ABCD$ are the centers of circles A, B, C, and D. If the perimeter of the square is 8, what is the area of the shaded region?

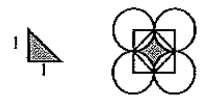
(A) $6 - \pi$

(B) $8 - \pi$

(C) $2 - \pi$

(D) $5 - \pi$

Solution: Answer: (A)



$$A_{\text{Total}} = A_{\text{Sha-1}} + A_{\text{Sha-2}}$$

$$s = \frac{8}{4} = 2,\ A_{\text{Squa}} = s^2 = 2^2 = 4$$

$$r = \frac{s}{2} = \frac{2}{2} = 1,\ A_{\text{Tri}} = \frac{1 \times 1}{2} = \frac{1}{2}$$

$$A_{\text{Cir}} = r^2\pi = 1^2\pi = \pi$$

$$A_{\text{Sha-1}} = 4A_{\text{Tri}} = 4 \times \frac{1}{2} = 2$$

$$A_{\text{Sha-2}} = A_{\text{Squa}} - \underset{4\times 1/4}{A_{\text{Cir}}} = 4 - \pi$$

$$\Downarrow$$

$$A_{\text{Total}} = 2 + \left(4 - \pi\right) = 6 - \pi$$

12. In the figure above, the vertices of square $ABCD$ are the centers of circles A, B, C, and D. If the radius of each circle is 2, what is the area of the shaded region?

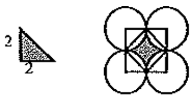
(A) 8

(B) $12 - 2\pi$

(C) $16 - 4\pi$

(D) $24 - 4\pi$

Solution: Answer: (D)



$$A_{\text{Total}} = A_{\text{Sha-1}} + A_{\text{Sha-2}}$$

$$r = 2,\ s = 4$$

$$A_{\text{Suqa}} = s^2 = 4^2 = 16$$

$$A_{\text{TRi}} = \frac{2 \times 2}{2} = 2$$

$$A_{\text{Cir}} = r^2\pi = 2^2\pi = 4\pi$$

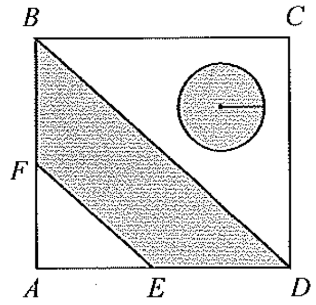
$$A_{\text{Sh-1}} = 4A_{\text{Tri}} = 4 \times 2 = 8$$

$$A_{\text{Sh-2}} = A_{\text{Suqa}} - \underset{4\times 1/4}{A_{\text{Cir}}} = 16 - 4\pi$$

$$\Downarrow$$

$$A_{\text{Total}} = 8 + \left(16 - 4\pi\right) = 24 - 4\pi$$

Shaded Regions



Note: Figure above not drawn to scale.

13. In the square $ABCD$ above, $CD = 2$, $AF = BF$, $AE = DE$, and the radius of the circle equals 1. What is the area of the shaded portion?

(A) $\frac{3}{2} + 2\pi$

(B) $\frac{2}{3} + \pi$

(C) $3 + \pi$

(D) $\frac{3}{2} + \pi$

Solution: Answer: (D)

$$A_{\text{Sha}} = A_{\text{Squ}} - A_{\text{Tri-}L} - A_{\text{Tri-}S} + A_{\text{Cir}}$$

$$\begin{cases} A_{\text{Squ}} = 2 \times 2 = 4 \\ A_{\text{Tri-}L} = \dfrac{2 \times 2}{2} = 2 \\ A_{\text{Tri-}S} = \dfrac{1 \times 1}{2} = \dfrac{1}{2} \\ A_{\text{Cir}} = 1^2 \pi = \pi \end{cases}$$

$$\Downarrow$$

$$A_{\text{sha}} = \frac{3}{2} + \pi$$

14. In the square $ABCD$ above, if $BC = 4$, $AF = BF$, $AE = DE$, and the radius of the circle equals 1, what is the area of the white region?

(A) $10 - \pi$

(B) $10 + \pi$

(C) $\pi - 10$

(D) $12 - \pi$

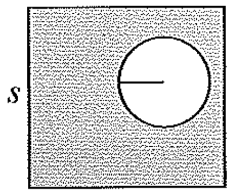
Solution: Answer: (A)

$$A_{\text{white}} = A_{\text{Tri-Smaller}} + A_{\text{Tri-Langger}} - A_{\text{Cir}}$$

$$\begin{cases} A_{\text{Tri-Langger}} = \dfrac{4 \times 4}{2} = 8 \\ A_{\text{Tri-Smaller}} = \dfrac{2 \times 2}{2} = 2 \\ A_{\text{Cir}} = 1^2 \pi \end{cases}$$

$$\Downarrow$$

$$A_{\text{White}} = 8 + 2 - \pi = 10 - \pi$$



Note: Figure above not drawn to scale.

15. In the figure above, a circle inside a square. If the radius of the circle is 4, what is the area, in terms of s, of the shaded region?

(A) $s^2 - 4\pi$

(B) $s^2 - 16\pi$

(C) $s - 4\pi$

(D) $3s - 4\pi$

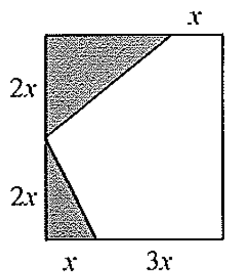
Solution: Answer: (B)

$$A_{\text{Sha}} = A_{\text{Squa}} - A_{\text{Cir}}$$

$$\begin{cases} A_{\text{Cir}} = r^2 \pi = 4^2 \pi = 16\pi \\ A_{\text{Squa}} = s^2 \end{cases}$$

$$\Downarrow$$

$$A_{\text{Sha}} = s^2 - 16\pi$$



Note: Figure above not drawn to scale.

16. In the rectangle above, the area of the white portion is 12. What is the sum of the area of the 2 shaded portions?

Solution: Answer: 4

$$A_{\text{Entire}} = 4x \cdot 4x = 16x^2$$

$$A_{\text{Sha}} = \frac{\cancel{2}x \cdot 3x}{\cancel{2}} + \frac{\cancel{2}x \cdot x}{\cancel{2}} = 3x^2 + x^2 = 4x^2$$

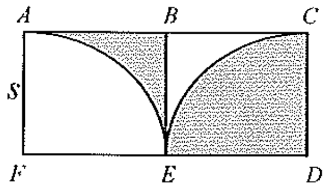
$$A_{\text{Un-sha}} = A_{\text{Entire}} - A_{\text{Sha}} = 12$$

$$\downarrow$$

$$16x^2 - 4x^2 = 12,\ 12x^2 = 12,\ x = \pm 1 \rightarrow x = 1$$

$$\Downarrow$$

$$A_{\text{Sha}} = 4x^2 = 4 \cdot 1^2 = 4$$

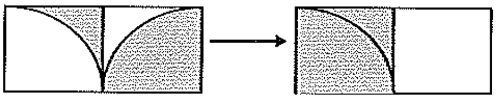


Note: Figure above not drawn to scale.

17. In the figure above, $ABEF$ and $BCDE$ are squares with sides of length 4. Arc AE and arc CE are quarter circles. What is the area of the shaded region?

(A) 10
(B) 12
(C) 16
(D) 14

Solution: Answer: (C)

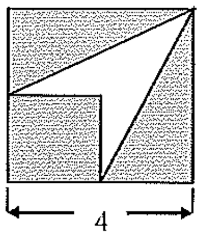


$$A_{sh} = A_{sh\text{-}1} + A_{sh\text{-}2}$$

$$\begin{cases} \text{2 shaded regions} = \text{1 square} \\ s = r = 4 \end{cases}$$

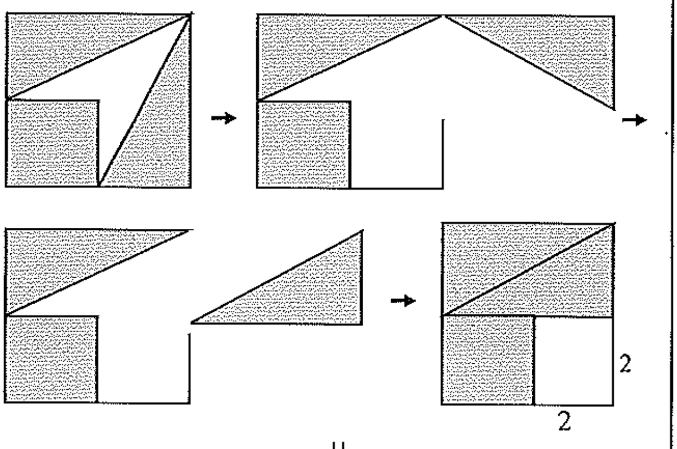
$$\Downarrow$$

$$A_{sh} = 4 \times 4 = 16$$



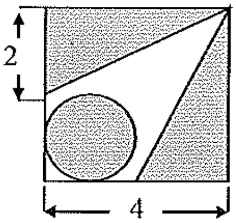
18. In the figure above, one smaller square with sides of length 2 and two equal triangles touch the sides of a larger square. What is the area of the non-shaded region?

Solution: Answer: 4



$$\Downarrow$$

$$A_{white} = 2 \times 2 = 4$$



19. In the figure above, one circle and two equal triangles touch the sides of a larger square. The diameter of circle is 2. What is the area of the non-shaded region?

(A) $8-\pi$
(B) $12-\pi$
(C) $10-\pi$
(D) $16-\pi$

Solution: Answer: (A)

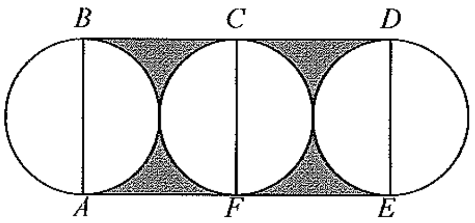
$$A = A_{Squa} - A_{Cir} - 2A_{Tri}$$

$$\Downarrow$$

$$\begin{cases} A_{Squa} = 4^2 = 16 \\ A_{Cir} = 1^2 \pi = \pi \\ 2A_{Tri} = 2 \times \dfrac{2 \times 4}{2} = 8 \end{cases}$$

$$\Downarrow$$

$$A = 16 - \pi - 8 = 8 - \pi$$



20. In the squares $ABCF$ and $CDEF$ shown above, AB, CF, and DE pass through the centers of 3 circles. If $AB = 2$, what is best approximation of the area of the shaded region?

(A) 1
(B) 2
(C) 3
(D) 4

Solution: Answer: (B)

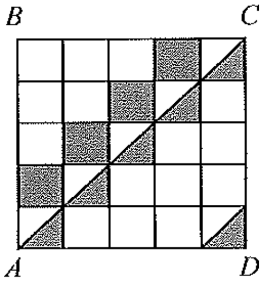
$$A_{Sha} = A_{Rec} - 2A_{Cir}$$

$$\begin{cases} A_{Rec} = AE \times AB = 4 \times 2 = 8 \\ \text{Word "approximation"} \rightarrow \pi = 3 \\ \downarrow \\ 2A_{Cir} = 2 \times r^2 \pi = 2 \times 1^2 \cdot 3 = 6 \end{cases}$$

$$\Downarrow$$

$$A_{Sha} = 8 - 6 = 2$$

Shaded Regions



Note: Figure above not drawn to scale.

21. If length of each side of square *ABCD* shown above is 10, what is the area of the shaded region?

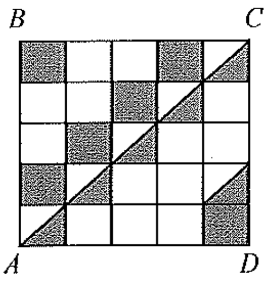
(A) 16
(B) 28
(C) 22
(D) 18

Solution: Answer: (B)

There are 7 shaded units totally.

$$\Downarrow$$

$$A_{sh} = 7 \times 2^2 = 28$$



Note: Figure above not drawn to scale.

22. If the diagonal of square *ABCD* shown above is $10\sqrt{2}$, what is the area of the shaded region?

(A) 14
(B) 18
(C) 22
(D) 36

Solution: Answer: (D)

$$\text{Diagonal} = 10\sqrt{2}$$

$$\downarrow$$

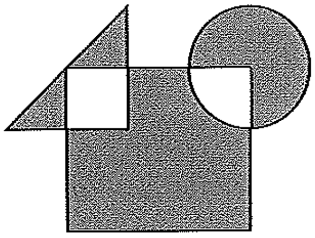
$$AD = CD = 10$$

$$\frac{10}{5} = 2, \quad \text{Area}_{\text{1-shaded}} = 2^2 = 4$$

There are 9 shaded units in total.

$$\Downarrow$$

$$\text{Area}_{\text{total-shaded}} = 9 \times 4 = 36$$



Note: Figure above not drawn to scale.

23. In the figure above, the square, right triangle and one-fourth of circle are overlapped as shown. The length of side of the square is 8, the one of legs of the triangle is 6 and hypotenuse of triangle is 10, and the radius of circle is 2. The length of each side of the smaller square is 2. What is the sum of the areas of the shaded portions?

(A) $84 - 3\pi$
(B) $80 - 3\pi$
(C) $80 + 2\pi$
(D) $84 - 4\pi$

Solution: Answer: (C)

$$A_{\text{sha}} = A_{\text{l-squ}} + A_{\text{circle}} + A_{\text{tri}} - 2 \times (A_{\text{s-squ}} + \frac{1}{4} A_{\text{circle}})$$

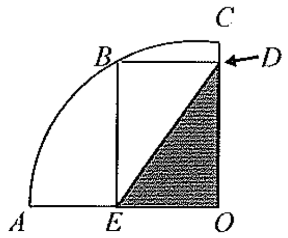
$$A_{\text{tri}} = \frac{\text{leg}_1 \times \text{leg}_2}{2}$$

$\text{leg}_2 = 8 \leftarrow (6 - 8 - 10 \text{ right triangle})$

$$\begin{bmatrix} A_{\text{tri}} = \frac{6 \times 8}{2} = 24 \\ A_{\text{s-squ}} = 2 \times 2 = 4 \\ A_{\text{circle}} = 2^2 \pi = 4\pi \\ A_{\text{l-squ}} = 8 \times 8 = 64 \end{bmatrix}$$

$$\Downarrow$$

$$64 + 4\pi + 24 - 2 \times (4 + \frac{1}{4} \times 4\pi) = 80 + 2\pi$$



Note: Figure above not drawn to scale.

24. In the figure above, arc ABC is one-fourth of a circle with center O, and its radius is 10. The sum of the length and width of rectangle $EBDO$ is 14. Which of the following is the perimeter of the non-shaded portion?

(A) $10+5\pi$
(B) $16+5\pi$
(C) $13+5\pi$
(D) $14+5\pi$

Solution: Answer: (B)

Perimeter $= ABC + DE + (EA + CD)$

(1) $\text{Arc}_{ABC} = \dfrac{\text{Circumference}}{4} = \dfrac{2r\pi}{4} = \dfrac{10\pi}{2} = 5\pi$

(2) $BDOE$ is a rectangle. $\rightarrow DE = BO =$ radius $= 10$

(3) Put $\overline{OC}$ down to know $AC = 2$ radius.

A E O D C

14

$2r = 2\cdot 10 = 20$

$\downarrow$

$EA + CD = 2r - (EO + OD) = 2\cdot 10 - 14 = 6$

$\Downarrow$

(4) Perimeter $= 5\pi + 10 + 6 = 16 + 5\pi$

25. In the figure above, arc ABC is one-fourth of a circle with center O, and its radius is 13. If the length of rectangle $EBDO$ is 5, what is the area of the non-shaded portion?

(A) $30 - \dfrac{13^2\pi}{4}$
(B) $\dfrac{13^2\pi}{4}$
(C) $\dfrac{13^2\pi}{4} - 30$
(D) 30π

Solution: Answer: (C)

(1) $DE =$ radius $= 13$, $OD = 5$

$\downarrow$

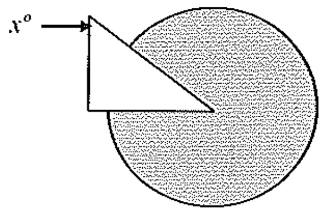
EDO is a 5-12-13 right triangle. $OE = 12$

(2) $A_{\text{Tri}} = \dfrac{5\times 12}{2} = 30$

$A_{\frac{1}{4}\text{circle}} = \dfrac{r^2\pi}{4} = \dfrac{13^2\pi}{4}$

$\Downarrow$

(3) $A_{\text{shaded}} = A_{\frac{1}{4}\text{circle}} - A_{\text{Tri}} = \dfrac{13^2\pi}{4} - 30$



Note: Figure above not drawn to scale.

26. The circle and right triangle are shown above. The circle has a diameter of length 12. If the area of the shaded portion is 30π, what is the value of x?

(A) 20
(B) 30
(C) 36
(D) 60

Solution: Answer: (B)

$D = 12$, $r = 6$

$A_{\text{entire}} = r^2\pi = 36\pi$

Arc
Degree } Obtain the circle ratio from area to be used for degree.
Area

$\text{Ratio} = \dfrac{30\pi}{36\pi} = \dfrac{5}{6}$

$\dfrac{1}{6}\times 360^\circ = 60$

$\uparrow$

Ratio of non-shaded portion

$\Downarrow$

$x = 90 - 60 = 30$

27. The circle and right triangle are shown above. The circle has a radius of length 8. If the arc of the shaded portion is 14π, what is the value of x?

(A) 45
(B) 30
(C) 36
(D) 60

Solution: Answer: (A)

$C = 2r\pi = 2\cdot 8\pi = 16\pi$

Arc
Degree } Obtain a ratio from arc to be used for degree.
Area

$\text{Ratio} = \dfrac{14\pi}{16\pi} = \dfrac{7}{8}$, $1 - \dfrac{7}{8} = \dfrac{1}{8}$

$\dfrac{1}{8}\times 360^\circ = 45$

$\uparrow$

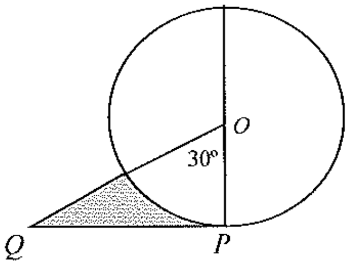
Non-shadedportion

$\Downarrow$

$x = 90 - 45 = 45$

Shaded Regions

Note: Figure below not drawn to scale.



28. In the figure above, the center of the circle with a radius of 1 is at O and segment $\overline{PQ}$ is tangent to the circle at point P. What is the area of the shaded portion?

(A) $\frac{\sqrt{3}}{6}+\frac{\pi}{12}$

(B) $\frac{\pi}{12}-\frac{\sqrt{3}}{6}$

(C) $\frac{\pi}{12}-\frac{\sqrt{3}}{12}$

(D) $\frac{\sqrt{3}}{6}-\frac{\pi}{12}$

Solution: Answer: (D)

$A_{shaded} = A_{triangle} - A_{sector}$

Arc

Degree / Area — Obtain a ratio from degree to be used for area.

$\text{Ratio} = \frac{30}{360} = \frac{1}{12}$

$A_{Circle} = 1^2\pi = \pi$

$A_{Sector} = \frac{1}{12}\cdot\pi = \frac{\pi}{12}$

30°-60°-Right Triangle

$\downarrow$

$1 = QP\sqrt{3}$

$\downarrow$

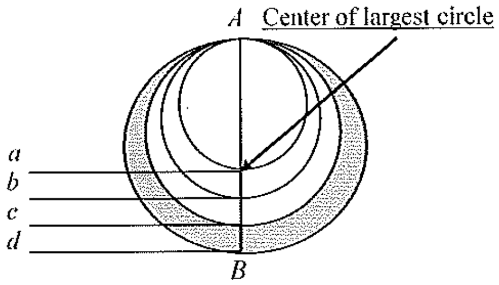
$QP = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\downarrow$

$A_{triangle} = \frac{\frac{\sqrt{3}}{3}\cdot 1}{2} = \frac{\sqrt{3}}{6}$

$\Downarrow$

$A_{shaded} = \frac{\sqrt{3}}{6} - \frac{\pi}{12}$



29. The four circles form the figure above. The each center of them is on segment $\overline{AB}$. ab, bc, and cd represent the distances between a and b, between b and c, and between c and d, respectively. $ab = bc = cd = 2$. What is the area of the shaded region?

(A) 10π

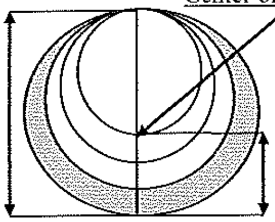
(B) 11π

(C) 9π

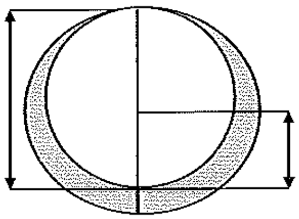
(D) 8π

Solution: Answer: (B)

Center of largest circle



$D_{largest} = 12$, $r_{largest} = \frac{12}{2} = 6$



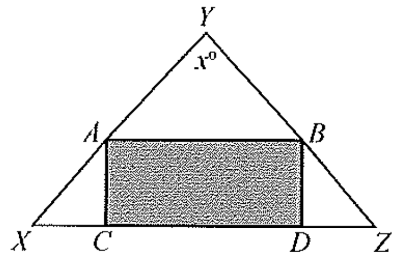
$D_{\text{second largest}} = 10$, $r_{\text{second largest}} = \frac{10}{2} = 5$

$\downarrow$

$A_{Shaded} = A_{Largest} - A_{\text{second Largest}}$

$\Downarrow$

$6^2\pi - 5^2\pi = \boxed{11\pi}$



Note: Figure above not drawn to scale.

30. In the figure above, XYZ is a triangle with $x = 60^\circ$ and $ABCD$ is a rectangle. A *is* midpoint of $\overline{XY}$ and B is midpoint of $\overline{YZ}$. If $XY = YZ = 2a\sqrt{2}$, what is the area of the shaded portion?

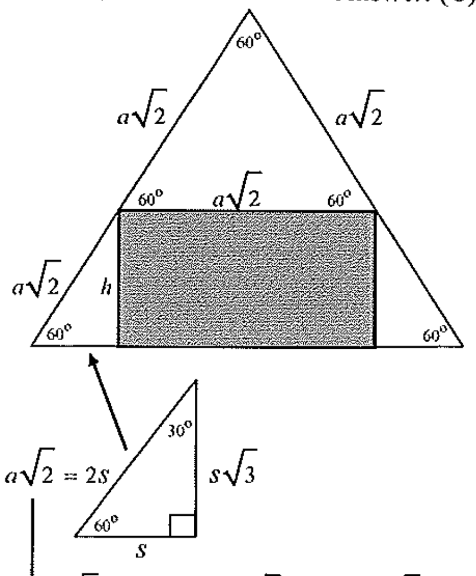
(A) $\sqrt{2}a$

(B) $\left(2\sqrt{a}\right)^2$

(C) $\sqrt{3}\cdot a^2$

(D) $4a^2$

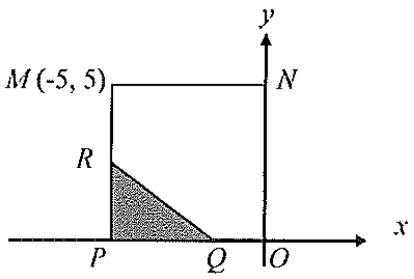
Solution: Answer: (C)



$$s = \frac{a\sqrt{2}}{2}, \quad s\sqrt{3} = \frac{a\sqrt{2}}{2}\cdot\sqrt{3} = \frac{a\sqrt{6}}{2} = h$$

$$\Downarrow$$

$$A_{\text{Shaded}} = \frac{a\sqrt{6}}{2}\cdot a\sqrt{2} = \frac{a^2\sqrt{4\times 3}}{2} = a^2\sqrt{3}$$



Note: Figure above not drawn to scale.

31. In the figure above, the coordinates of R are $(-5, k)$ and the coordinates of Q are $(-2, 0)$. One point in square $MNOP$ is to be chosen randomly. If the probability that the point will be in the shaded triangle is 12%, what is the value of k?

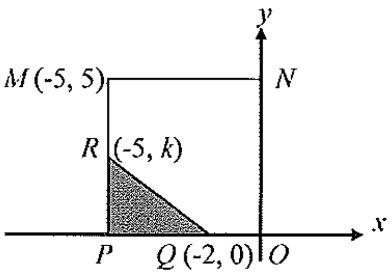
(A) 2

(B) 3

(C) 4

(D) 5

Solution: Answer: (A)



$$|-5-(-2)| = 3$$

$$\downarrow$$

$$\frac{\frac{3\times k}{2}}{5\times 5} = \frac{12}{100}$$

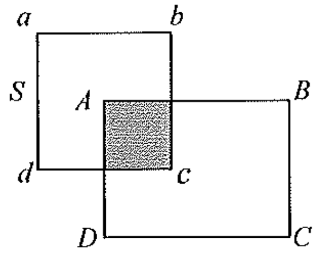
$$\downarrow$$

$$\frac{3k}{50} = \frac{12}{100}$$

$$\Downarrow$$

$$k = \frac{12\cdot 50}{3\cdot 100} = 2$$

Shaded Regions



Note: Figure above not drawn to scale.

32. In the figure above, the bigger square *abcd* and rectangle *ABCD* are overlapped each other. The point *A* is located at the center of the bigger square. If the area of the bigger square is 16, what is the area of the shaded region?

Solution: Answer: 4

Point A is at the center of the bigger square.

↓

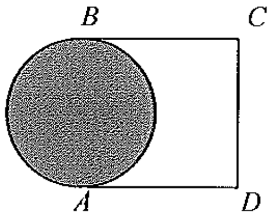
The shaded region is a square.

↓

The length of side of shaded square is $\sqrt{16} \div 2 = 2$.

⇓

$$A_{\text{shaded}} = 2 \times 2 = 4$$



Note Figure above not drawn to scale.

1. In the figure above, side AB of square $ABCD$ passes through the center of circle. If the length of diagonal of square $ABCD$ is $4a$, what is the area of the shaded portion?

(A) $2a^2\pi$

(B) $a^2\pi$

(C) $a\pi^2$

(D) $\dfrac{a^2}{\pi}$

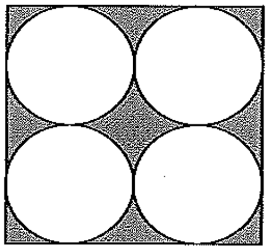
2. In the figure above, side AB of square $ABCD$ passes through the center of circle. If the length of a side of square $ABCD$ is 2π, what is the area of the shaded region?

(A) π^4

(B) $2\pi^2$

(C) π^3

(D) $2\pi^3$



3. There are one bigger square and four circles touching together as the figure shown above. The radius of each circle is π. Of the following, which is the area of the shaded region?

(A) $\pi^2(16-\pi)$

(B) $4\pi^2(4-\pi)$

(C) $16\pi^3-\pi^2$

(D) $16\pi^3-\pi^3$

4. There are one bigger square and four circles touching together as the figure shown above. The length of the square side is π. Of the following, which is the area of the shaded region?

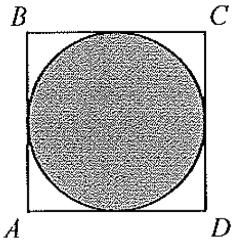
(A) $\pi^2-\dfrac{\pi^3}{16}$

(B) $16\pi^3-\pi$

(C) $16\pi^3-\pi^2$

(D) $\pi^2\left(1-\dfrac{\pi}{4}\right)$

Shaded Regions



5. In the figure above, if the length of diagonal of square *ABCD* is 1, what is the half area of the shaded region?

(A) $\frac{\pi}{8}$

(B) $\frac{\pi}{16}$

(C) $\frac{\pi^2}{8}$

(D) $\frac{\pi^2}{4}$

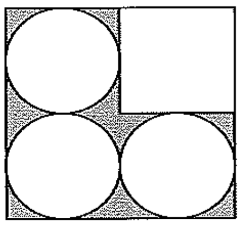
6. In the figure above, if the length of a side of square *ABCD* is 1, what is the area of the non-shaded region?

(A) $\frac{1-\pi}{2}$

(B) $1-\pi$

(C) $\frac{4-\pi}{4}$

(D) $2-\pi$

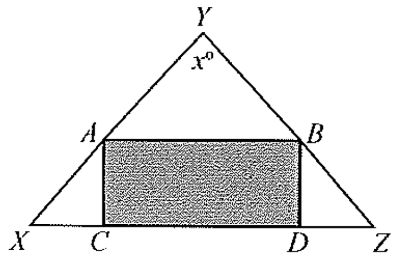


7. There are one bigger square, three circles and one smaller square touching together as the figure shown above. The radius of each circle is 1. Of the following, which is the area of the shaded region?

(A) $16-3\pi$
(B) $12-3\pi$
(C) $4-3\pi$
(D) $8-3\pi$

8. There are one bigger square, three circles and one smaller square touching together as the figure shown above. The circumference of each circle is 2π. Of the following, which is the area of the shaded region?

(A) $12-3\pi$
(B) $8-3\pi$
(C) $16-3\pi$
(D) $10-3\pi$



Note: Figure above not drawn to scale.

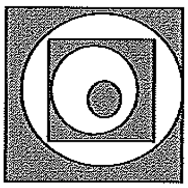
9. In the figure above, XYZ is a triangle with $x = 90^\circ$ and $ABCD$ is a rectangle. A is midpoint of $\overline{XY}$ and B is midpoint of $\overline{YZ}$. If $XY = YZ = 2a\sqrt{2}$, what is the area of the shaded portion?

(A) $\sqrt{2}a$

(B) $\left(2\sqrt{a}\right)^2$

(C) $2a^2$

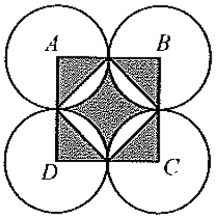
(D) $4a^2$



Note: Figure above not drawn to scale.

10. In the figure above, the two squares have sides of 4 and 7 and the three circles have radii of 1, 2 and 3, respectively. Which of the following is the sum of the area of the shaded portion?

(A) $65\pi - 12$ (B) $65 - 12\pi$ (C) $12\pi - 6.5$ (D) $62\pi - 12$



Note: Figure above not drawn to scale.

11. In the figure above, the vertices of square $ABCD$ are the centers of circles A, B, C, and D. If the perimeter of the square is 8, what is the area of the shaded region?

(A) $6 - \pi$

(B) $8 - \pi$

(C) $2 - \pi$

(D) $5 - \pi$

12. In the figure above, the vertices of square $ABCD$ are the centers of circles A, B, C, and D. If the radius of each circle is 2, what is the area of the shaded region?

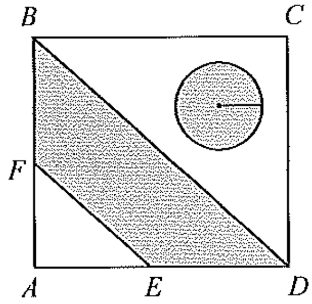
(A) 8

(B) $12 - 2\pi$

(C) $16 - 4\pi$

(D) $24 - 4\pi$

Shaded Regions



Note: Figure above not drawn to scale.

13. In the square $ABCD$ above, $CD = 2$, $AF = BF$, $AE = DE$, and the radius of the circle equals 1. What is the area of the shaded portion?

(A) $\frac{3}{2} + 2\pi$

(B) $\frac{2}{3} + \pi$

(C) $3 + \pi$

(D) $\frac{3}{2} + \pi$

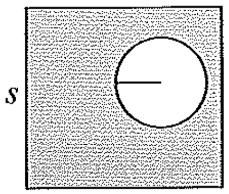
14. In the square $ABCD$ above, if $BC = 4$, $AF = BF$, $AE = DE$, and the radius of the circle equals 1, what is the area of the white region?

(A) $10 - \pi$

(B) $10 + \pi$

(C) $\pi - 10$

(D) $12 - \pi$



Note: Figure above not drawn to scale.

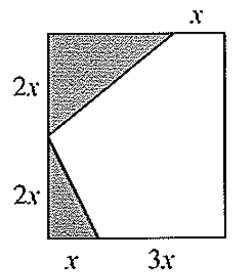
15. In the figure above, a circle inside a square. If the radius of the circle is 4, what is the area, in terms of s, of the shaded region?

(A) $s^2 - 4\pi$

(B) $s^2 - 16\pi$

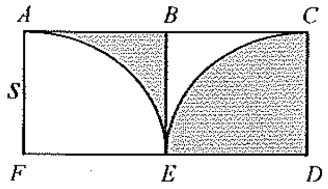
(C) $s - 4\pi$

(D) $3s - 4\pi$



Note: Figure above not drawn to scale.

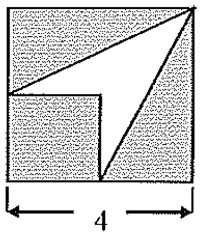
16. In the rectangle above, the area of the white portion is 12. What is the sum of the area of the 2 shaded portions?



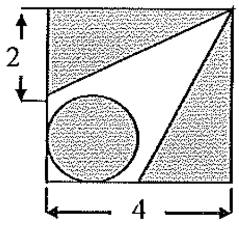
Note: Figure above not drawn to scale.

17. In the figure above, *ABEF* and *BCDE* are squares with sides of length 4. Arc *AE* and arc *CE* are quarter circles. What is the area of the shaded region?

(A) 10
(B) 12
(C) 16
(D) 14

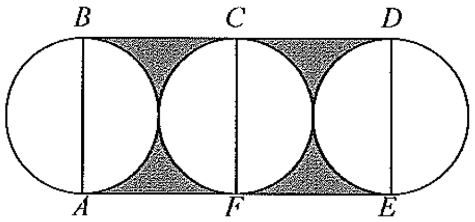


18. In the figure above, one smaller square with sides of length 2 and two equal triangles touch the sides of a larger square. What is the area of the non-shaded region?



19. In the figure above, one circle and two equal triangles touch the sides of a larger square. The diameter of circle is 2. What is the area of the non-shaded region?

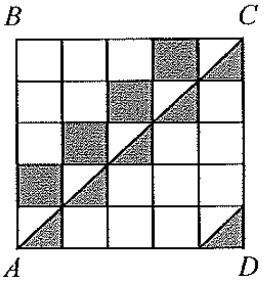
(A) $8-\pi$
(B) $12-\pi$
(C) $10-\pi$
(D) $16-\pi$



20. In the squares *ABCF* and *CDEF* shown above, *AB, CF,* and *DE* pass through the centers of 3 circles. If $AB = 2$, what is best approximation of the area of the shaded region?

(A) 1
(B) 2
(C) 3
(D) 4

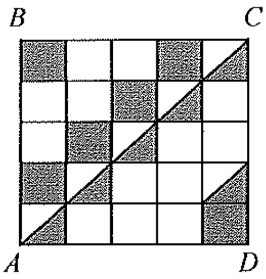
Shaded Regions



Note: Figure above not drawn to scale.

21. If length of each side of square *ABCD* shown above is 10, what is the area of the shaded region?

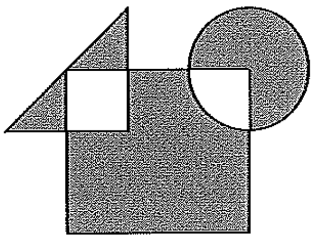
(A) 16
(B) 28
(C) 22
(D) 18



Note: Figure above not drawn to scale.

22. If the diagonal of square *ABCD* shown above is $10\sqrt{2}$, what is the area of the shaded region?

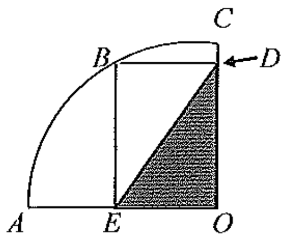
(A) 14
(B) 18
(C) 22
(D) 36



Note: Figure above not drawn to scale.

23. In the figure above, the square, right triangle and one-fourth of circle are overlapped as shown. The length of side of the square is 8, the one of legs of the triangle is 6 and hypotenuse of triangle is 10, and the radius of circle is 2. The length of each side of the smaller square is 2. What is the sum of the areas of the shaded portions?

(A) $84-3\pi$
(B) $80-3\pi$
(C) $80+2\pi$
(D) $84-4\pi$



Note: Figure above not drawn to scale.

24. In the figure above, arc *ABC* is one-fourth of a circle with center *O*, and its radius is 10. The sum of the length and width of rectangle *EBDO* is 14. Which of the following is the perimeter of the non-shaded portion?

(A) $10+5\pi$

(B) $16+5\pi$

(C) $13+5\pi$

(D) $14+5\pi$

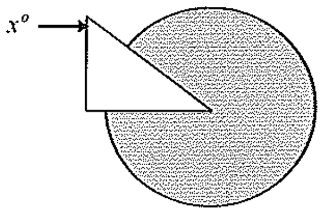
25. In the figure above, arc *ABC* is one-fourth of a circle with center *O*, and its radius is 13. If the length of rectangle *EBDO* is 5, what is the area of the non-shaded portion?

(A) $30-\frac{13^2\pi}{4}$

(B) $\frac{13^2\pi}{4}$

(C) $\frac{13^2\pi}{4}-30$

(D) 30π



Note: Figure above not drawn to scale.

26. The circle and right triangle are shown above. The circle has a diameter of length 12. If the area of the shaded portion is 30π, what is the value of x?

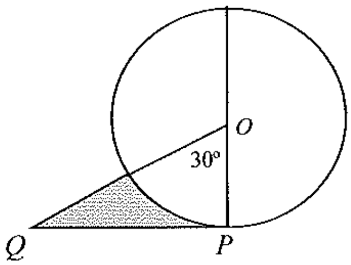
(A) 20
(B) 30
(C) 36
(D) 60

27. The circle and right triangle are shown above. The circle has a radius of length 8. If the arc of the shaded portion is 14π, what is the value of x?

(A) 45
(B) 30
(C) 36
(D) 60

Shaded Regions

Note: Figure below not drawn to scale.



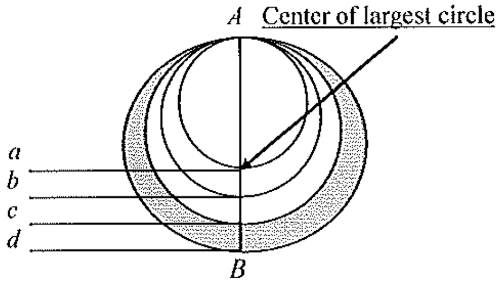
28. In the figure above, the center of the circle with a radius of 1 is at O and segment $\overline{PQ}$ is tangent to the circle at point P. What is the area of the shaded portion?

(A) $\frac{\sqrt{3}}{6}+\frac{\pi}{12}$

(B) $\frac{\pi}{12}-\frac{\sqrt{3}}{6}$

(C) $\frac{\pi}{12}-\frac{\sqrt{3}}{12}$

(D) $\frac{\sqrt{3}}{6}-\frac{\pi}{12}$



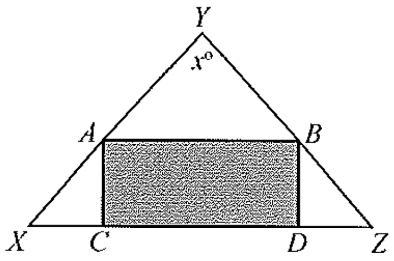
29. The four circles form the figure above. The each center of them is on segment $\overline{AB}$. ab, bc, and cd represent the distances between a and b, between b and c, and between c and d, respectively. $ab = bc = cd = 2$. What is the area of the shaded region?

(A) 10π

(B) 11π

(C) 9π

(D) 8π



Note: Figure above not drawn to scale.

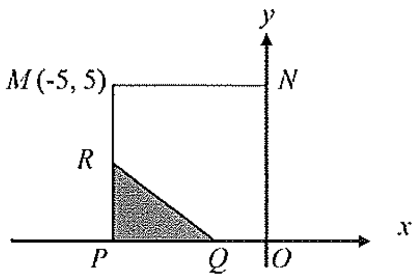
30. In the figure above, XYZ is a triangle with $x = 60^\circ$ and $ABCD$ is a rectangle. A *is* midpoint of $\overline{XY}$ and B is midpoint of $\overline{YZ}$. If $XY = YZ = 2a\sqrt{2}$, what is the area of the shaded portion?

(A) $\sqrt{2}a$

(B) $\left(2\sqrt{a}\right)^2$

(C) $\sqrt{3} \cdot a^2$

(D) $4a^2$



Note: Figure above not drawn to scale.

31. In the figure above, the coordinates of R are $(-5, k)$ and the coordinates of Q are $(-2, 0)$. One point in square $MNOP$ is to be chosen randomly. If the probability that the point will be in the shaded triangle is 12%, what is the value of k?

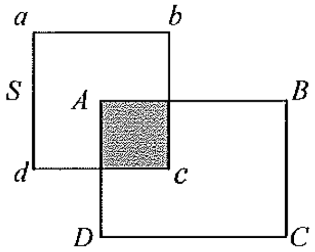
(A) 2

(B) 3

(C) 4

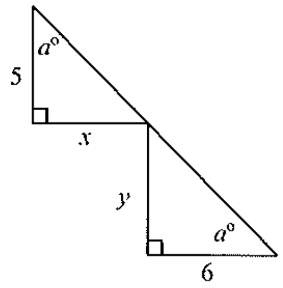
(D) 5

Shaded Regions



Note: Figure above not drawn to scale.

32. In the figure above, the bigger square *abcd* and rectangle *ABCD* are overlapped each other. The point *A* is located at the center of the bigger square. If the area of the bigger square is 16, what is the area of the shaded region?



Note: Figure above not drawn to scale.

1. In the figure above, what is the value of $\frac{y}{x}$?

(A) $\frac{5}{6}$
(B) 30
(C) $\frac{6}{5}$
(D) 1

Solution: Answer: (C)

The 2 triangles have the same 3 pairs of angles. (interior)

↓

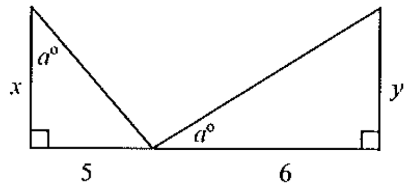
They are similar triangles.

↓

The ratios of the corresponding sides are equal.

⇓

$$\frac{y}{x} = \frac{6}{5}$$



Note: Figure above not drawn to scale.

2. In the figure above, what is the value of xy?

(A) $\frac{5}{6}$
(B) 30
(C) 11
(D) 1

Solution: Answer: (B)

The 2 triangles have the same 3 pairs of angles. (interior)

↓

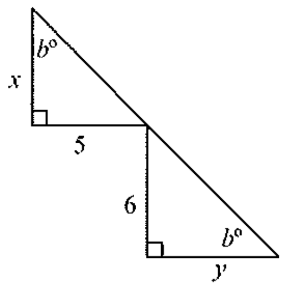
They are similar triangles.

↓

The ratios of the corresponding sides are equal.

⇓

$$\frac{y}{5} = \frac{6}{x}, \quad xy = \boxed{30}$$



Note: Figure above not drawn to scale.

3. In the figure above, what is the value of $\frac{x}{y}$?

(A) $\frac{5}{6}$
(B) 30
(C) 11
(D) 1

Solution: Answer: (A)

The 2 triangles have the same 3 pairs of angles. (interior)

↓

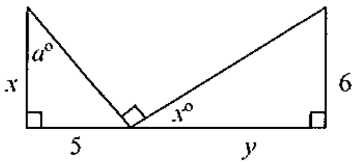
They are similar triangles.

↓

The ratios of the corresponding sides are equal.

⇓

$$\frac{x}{y} = \frac{5}{6}$$



Note: Figure above not drawn to scale.

4. In the figure above, what is the value of $\frac{x}{y}$?

(A) $\frac{6}{5}$
(B) 16
(C) 11
(D) $\frac{5}{6}$

Solution: Answer: (D)

one exterior angle = sum of two interior angles (in measure)

↓

$$\cancel{90} + a = \cancel{90} + x, \quad x = a$$

The 2 triangles have the same 3 pairs of angles. (interior)

↓

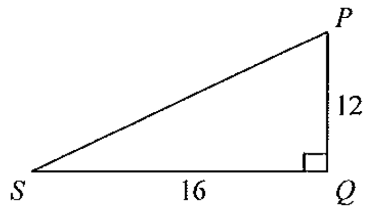
They are similar triangles.

↓

The ratios of the corresponding sides are equal.

⇓

$$\frac{x}{y} = \frac{5}{6}$$



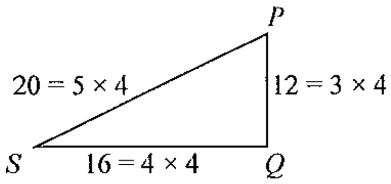
Note: Figure above not drawn to scale.

5. In the figure above, each angle of triangle PQS has the same measure as an angle in triangle ABC (not shown). If the $AB = 240$, what is one possible perimeter of triangle ABC?

Solution: Answer: 960

"Multiple of multiple" of 3-4-5 right triangle

↓



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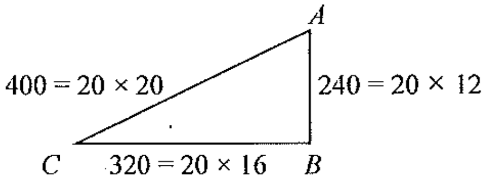


⇓

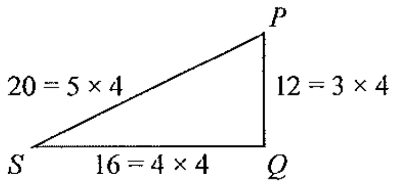
$400 + 320 + 240 = 960$

(One possible perimeter)

6. In the figure above, each angle of triangle PQS has the same measure as an angle in triangle ABC (not shown). If $AB = 6$, what is one possible area of triangle ABC?

(A) 48
(B) 84
(C) 24
(D) 12

Solution: Answer: (C)



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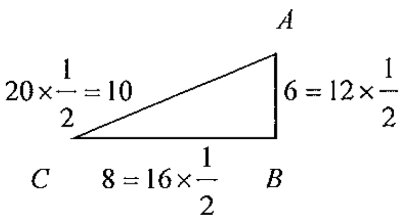
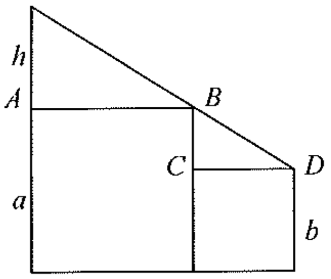


⇓

$$\text{Area} = \frac{b \times h}{2} = \frac{6 \times 8}{2} = 24$$

(One possible perimeter)



Note: Figure above not drawn to scale.

7. The figure above shows two squares with sides a and b, respectively. What is the value of h?

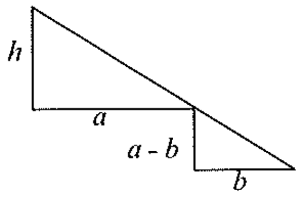
(A) $\frac{b^2}{a} - b$

(B) $\frac{b}{a} - b$

(C) $\frac{a}{b} - b$

(D) $\frac{a^2}{b} - a$

Solution: Answer: (D)



$CB = a - b$

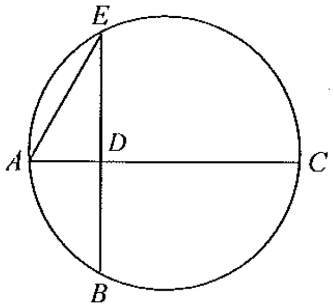
The 2 triangles have the 3 pairs of interior angles. Then they are the similar triangles.

↓

The ratios of the corresponding sides are equal.

⇓

$$\frac{h}{a-b} = \frac{a}{b},\ h = \frac{a(a-b)}{b} = \boxed{\frac{a^2}{b} - a}$$

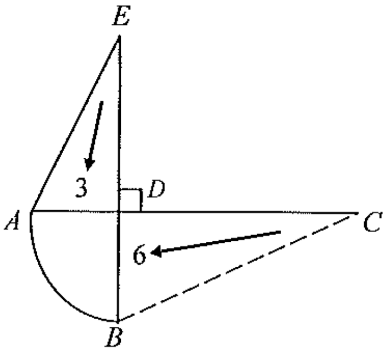


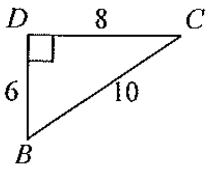
Note: Figure above not drawn to scale.

8. If $\overline{BE}$ is perpendicular to $\overline{AC}$, $CD = 8$, $BD = 6$, and $AD = 3$, what is AE ?

(A) 8
(B) 10
(C) 5
(D) 39

Solution: Answer: (C)





$\angle AEB$ and $\angle ACB$ face the same arc AB.

→ They have the same angle measure.

The 2 triangles have the same 3 pairs of interior angles.

→ They are similar triangles.

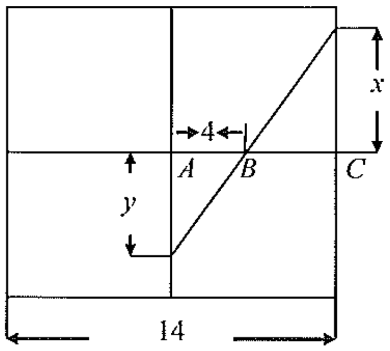
→ The ratios of the corresponding sides are equal.

$$\downarrow$$

$$\frac{AE}{BC} = \frac{3}{6}$$

$$\Downarrow$$

$$\frac{AE}{10} = \frac{3}{6},\ AE = \boxed{5}$$



Note: Figure above not drawn to scale.

9. In the figure above, the side of the large square is 14. Joining the midpoints of opposite sides forms the 4 smaller squares. $y = 5$. What is the value of x?

(A) $\frac{32}{3}$
(B) 9
(C) $\frac{15}{4}$
(D) 6

Solution: Answer: (C)

The 2 triangles have the same 3 pairs of interior angles.

$$\downarrow$$

They are similar triangles.

$$\downarrow$$

The ratios of the corresponding sides are equal.

$AC = 14 / 2 = 7,\ BC = 7 - 4 = 3$

$$\Downarrow$$

$$\frac{x}{5} = \frac{3}{4},\ x = \boxed{\frac{15}{4}}$$

10. In the figure above, the side of the large square is 14. Joining the midpoints of opposite sides forms the 4 smaller squares. $x = 5$. What is the value of y?

(A) $\frac{32}{3}$
(B) 9
(C) 10
(D) $\frac{20}{3}$

Solution: Answer: (D)

The 2 triangles have the same 3 pairs of interior angles.

$$\downarrow$$

They are similar triangles.

$$\downarrow$$

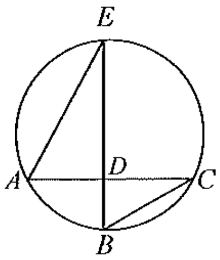
The ratios of the corresponding sides are equal.

$AC = 14 / 2 = 7,\ BC = 7 - 4 = 3$

$$\Downarrow$$

$$\frac{y}{5} = \frac{4}{3},\ y = \boxed{\frac{20}{3}}$$

Similar Triangles

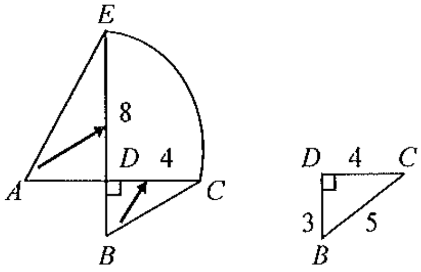


Note: Figure above not drawn to scale.

11. If $\overline{BE}$ is perpendicular to $\overline{AC}$, $DE = 8$, $BD = 3$, and $DC = 4$, what is AE?

(A) 8
(B) 10
(C) 6
(D) 39

Solution: Answer: (B)



$\angle AEB$ and $\angle ACB$ face the same arc AB.

↓

They have the same angle measure.

The 2 triangles have the same 3 pairs of angles.

interior

↓

They are similar triangles.

↓

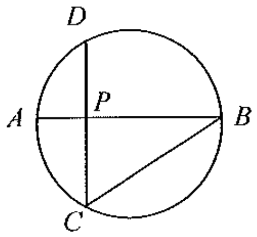
The ratios of the corresponding sides are equal.

↓

$$\frac{AE}{BC} = \frac{8}{4}$$

⇓

$$\frac{AE}{5} = \frac{8}{4},\ AE = \boxed{10}$$

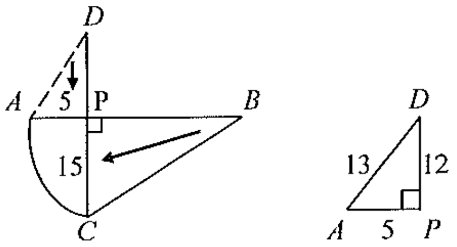


Note: Figure above not drawn to scale.

12. If $\overline{DC}$ is perpendicular to $\overline{AB}$, $AP = 5$, $DP = 12$, and $CP = 15$, what is CB?

(A) 18
(B) 20
(C) 16
(D) 39

Solution: Answer: (D)



$\angle ADC$ and $\angle ABC$ face the same arc AC.

↓

They have the same angle measure.

The 2 triangles have the same 3 pairs of angles.

interior

↓

They are similar triangles.

↓

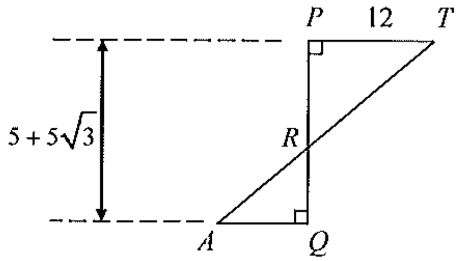
The ratios of the corresponding sides are equal.

↓

$$\frac{CB}{AD} = \frac{15}{5}$$

⇓

$$\frac{CB}{13} = \frac{15}{5},\ CB = \boxed{39}$$



Note: Figure above not drawn to scale.

13. In the figure above, the segment $\overline{PQ} = 5+5\sqrt{3}$ and $RT = 13$. What is the area of the bottom triangle?

(A) 50

(B) $60+\sqrt{3}$

(C) 70

(D) 90

Solution: Answer: (D)

$$AR = \sqrt{13^2 - 12^2} = \sqrt{169-144} = \sqrt{25} = 5$$

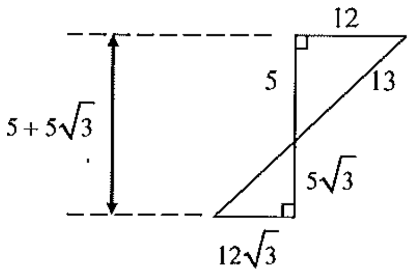
or

RPT is a 5-12-13 right triangle. $\rightarrow AR = 5$.

$$BR = 5+5\sqrt{3}-5 = 5\sqrt{3}$$

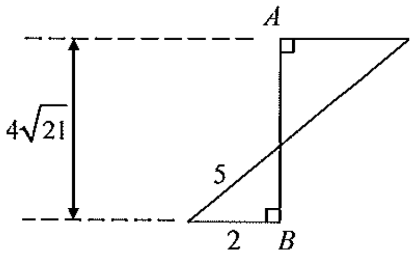
Because the two triangles have 3 pairs of the same interior angles, they are the similar triangles

$\downarrow$



$\Downarrow$

$$A = \frac{5\sqrt{3}\cdot 12\sqrt{3}}{2} = 90$$



Note: Figure above not drawn to scale.

14. In the figure above, the segment $\overline{AB} = 4\sqrt{21}$. What is the perimeter P of the top triangle?

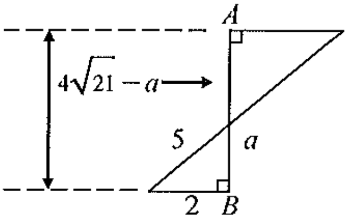
(A) $12+2\sqrt{21}$

(B) $18+3\sqrt{21}$

(C) $5+2\sqrt{21}$

(D) $21+3\sqrt{21}$

Solution: Answer: (D)

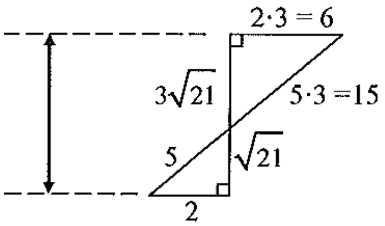


$$a = \sqrt{5^2 - 2^2} = \sqrt{21}$$

$$4\sqrt{21} - a = 4\sqrt{21} - \sqrt{21} = 3\sqrt{21}$$

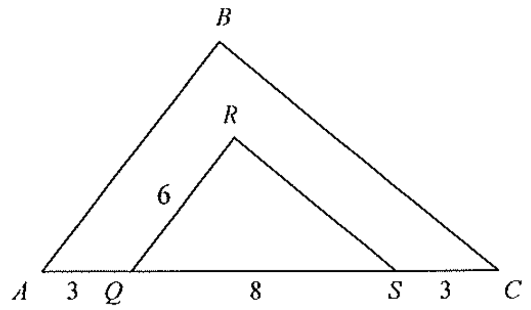
Because the two triangles have 3 pairs of the same interior angles, they are the similar triangles.

$\downarrow$



$\Downarrow$

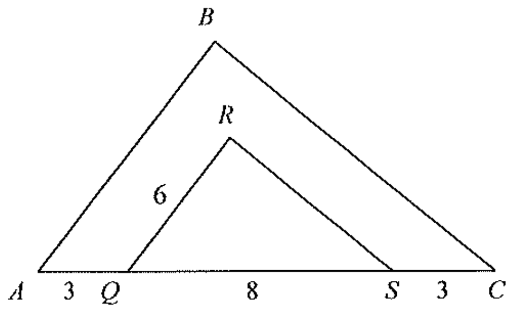
$$P = 6+15+3\sqrt{21} = 21+3\sqrt{21}$$



15. In triangle ABC above, points Q and S lie on $\overline{AC}$. $\overline{AB}$ is parallel to $\overline{QR}$, and $\overline{BC}$ is parallel to $\overline{RS}$. Which of the following is AB ?

(A) 11.5 (B) 12.0 (C) 12.5 (D) 10.5

Solution: Answer: (D)



$\overline{AB}$ is parallel to $\overline{QR}$. $\overline{BC}$ is parallel to $\overline{RS}$.

$\downarrow$ $\downarrow$

$m\angle A = m\angle Q$, $m\angle C = m\angle S$

$\downarrow$

The rest angles $m\angle B = m\angle R$

$\downarrow$

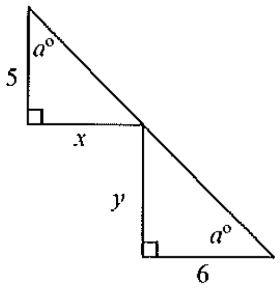
ABC and QRS are similar triangles.

$\downarrow$

$$\frac{AB}{QR} = \frac{AC}{QS}, \quad \frac{AB}{6} = \frac{3+8+3}{8}$$

$\Downarrow$

$$AB = 10.5$$



Note: Figure above not drawn to scale.

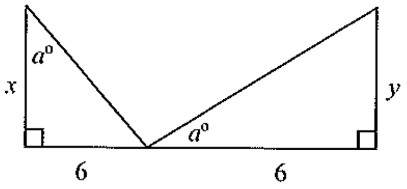
1. In the figure above, what is the value of $\frac{y}{x}$?

(A) $\frac{5}{6}$

(B) 30

(C) $\frac{6}{5}$

(D) 1



Note: Figure above not drawn to scale.

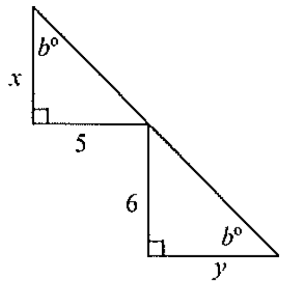
2. In the figure above, what is the value of xy?

(A) $\frac{5}{6}$

(B) 30

(C) 11

(D) 1



Note: Figure above not drawn to scale.

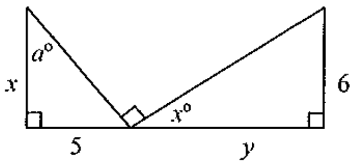
3. In the figure above, what is the value of $\frac{x}{y}$?

(A) $\frac{5}{6}$

(B) 30

(C) 11

(D) 1



Note: Figure above not drawn to scale.

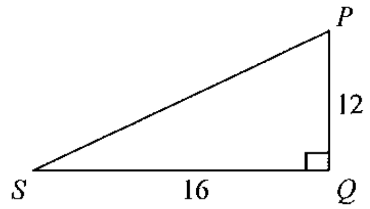
4. In the figure above, what is the value of $\frac{x}{y}$?

(A) $\frac{6}{5}$

(B) 16

(C) 11

(D) $\frac{5}{6}$

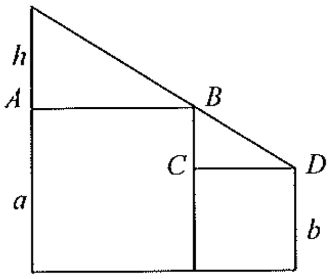


Note: Figure above not drawn to scale.

5. In the figure above, each angle of triangle PQS has the same measure as an angle in triangle ABC (not shown). If the $AB = 240$, what is one possible perimeter of triangle ABC?

6. In the figure above, each angle of triangle PQS has the same measure as an angle in triangle ABC (not shown). If $AB = 6$, what is one possible area of triangle ABC?

(A) 48
(B) 84
(C) 24
(D) 12



Note: Figure above not drawn to scale.

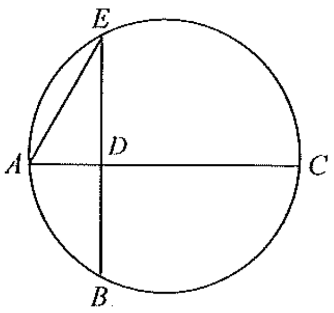
7. The figure above shows two squares with sides a and b, respectively. What is the value of h?

(A) $\frac{b^2}{a} - b$

(B) $\frac{b}{a} - b$

(C) $\frac{a}{b} - b$

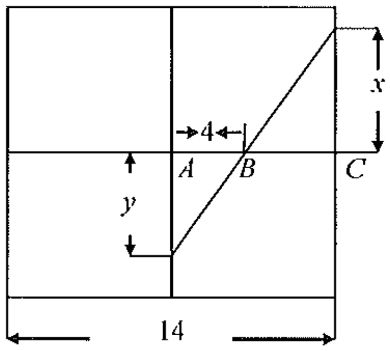
(D) $\frac{a^2}{b} - a$



Note: Figure above not drawn to scale.

8. If $\overline{BE}$ is perpendicular to $\overline{AC}$, $CD = 8$, $BD = 6$, and $AD = 3$, what is AE ?

(A) 8
(B) 10
(C) 5
(D) 39



Note: Figure above not drawn to scale.

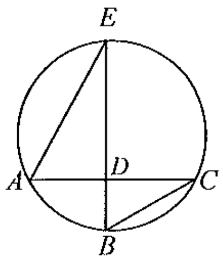
9. In the figure above, the side of the large square is 14. Joining the midpoints of opposite sides forms the 4 smaller squares. $y = 5$. What is the value of x?

(A) $\frac{32}{3}$
(B) 9
(C) $\frac{15}{4}$
(D) 6

10. In the figure above, the side of the large square is 14. Joining the midpoints of opposite sides forms the 4 smaller squares. $x = 5$. What is the value of y?

(A) $\frac{32}{3}$
(B) 9
(C) 10
(D) $\frac{20}{3}$

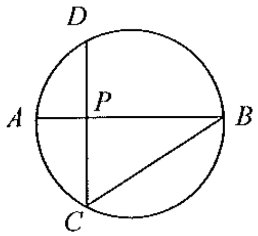
Similar Triangles



Note: Figure above not drawn to scale.

11. If $\overline{BE}$ is perpendicular to $\overline{AC}$, $DE = 8$, $BD = 3$, and $DC = 4$, what is AE?

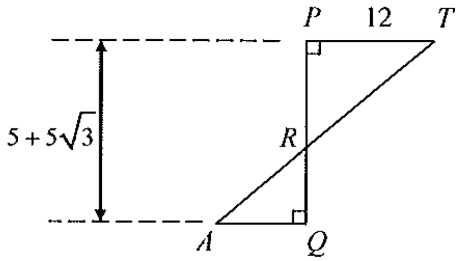
(A) 8
(B) 10
(C) 6
(D) 39



Note: Figure above not drawn to scale.

12. If $\overline{DC}$ is perpendicular to $\overline{AB}$, $AP = 5$, $DP = 12$, and $CP = 15$, what is CB?

(A) 18
(B) 20
(C) 16
(D) 39



Note: Figure above not drawn to scale.

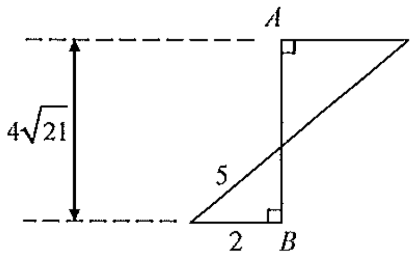
13. In the figure above, the segment $\overline{PQ} = 5+5\sqrt{3}$ and $RT = 13$. What is the area of the bottom triangle?

(A) 50

(B) $60+\sqrt{3}$

(C) 70

(D) 90



Note: Figure above not drawn to scale.

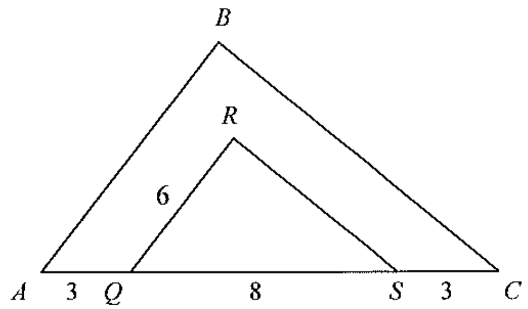
14. In the figure above, the segment $\overline{AB} = 4\sqrt{21}$. What is the perimeter P of the top triangle?

(A) $12+2\sqrt{21}$

(B) $18+3\sqrt{21}$

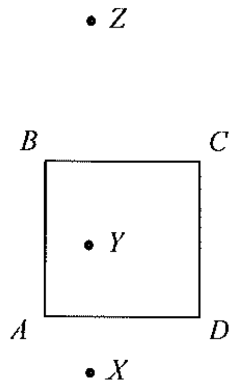
(C) $5+2\sqrt{21}$

(D) $21+3\sqrt{21}$



15. In triangle ABC above, points Q and S lie on $\overline{AC}$. $\overline{AB}$ is parallel to $\overline{QR}$, and $\overline{BC}$ is parallel to $\overline{RS}$. Which of the following is AB ?

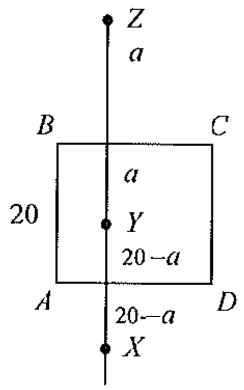
(A) 11.5 (B) 12.0 (C) 12.5 (D) 10.5



1. In the figure above, $ABCD$ is a square with side = 20. Points X, Y, and Z are distinct points on a line (not shown) that is parallel to segment $\overline{AB}$. Points Y and Z are symmetric about line BC and points X and Y are symmetric about line AD. What is the length of segment $\overline{XZ}$?

(A) 32
(B) 38
(C) 36
(D) 40

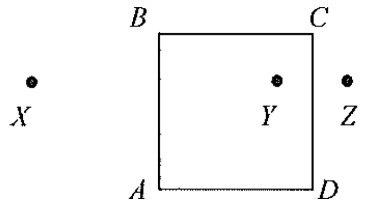
Solution: Answer: (D)



Let $YZ = 2a$.

$\Downarrow$

$XZ = a + a + (20 - a) + (20 - a) = 40$

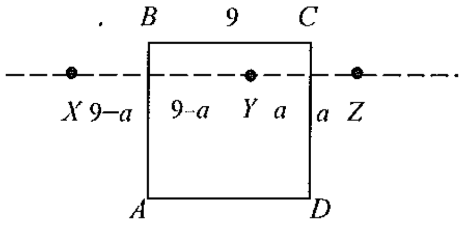


Note: Figure above not drawn to scale.

2. In the figure above, $ABCD$ is a square with side = 9. Points X, Y, and Z are distinct points on a line (not shown) that is parallel to segment $\overline{BC}$. Points Y and Z are symmetric about line CD and points X and Y are symmetric about line AB. What is the length of segment $\overline{XZ}$?

(A) 18
(B) 20
(C) 23
(D) 17

Solution: Answer: (A)



Let $YZ = 2a$.

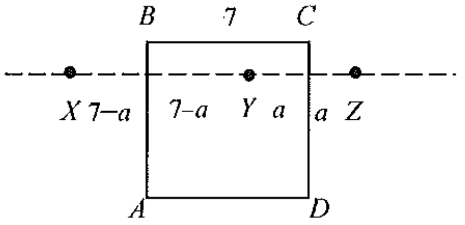
$\Downarrow$

$XZ = a + a + (9 - a) + (9 - a) = 18$

3. In the figure above, $ABCD$ is a rectangle with $AD = 7$. Points X, Y, and Z are distinct points on a line (not shown) that is parallel to segment $\overline{AD}$. Points Y and Z are symmetric about line CD and points X and Y are symmetric about line AB. What is the length of segment $\overline{XZ}$?

(A) 15
(B) 18
(C) 23
(D) 14

Solution: Answer: (D)



Let $YZ = 2a$.

$\Downarrow$

$XZ = a + a + (7 - a) + (7 - a) = 14$

4. Which of the following has exactly four symmetric lines?

(A) Circle

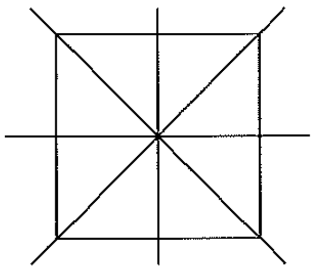
(B) Triangle

(C) Square

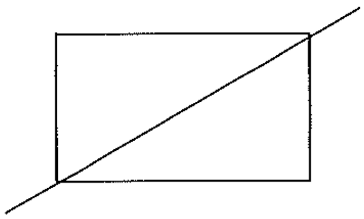
(D) Rectangle

Solution: Answer: (C)

Among the 5 choices, only square has exactly 4 symmetric lines: 2 diagonals and 2 lines connecting the midpoints of opposite sides.

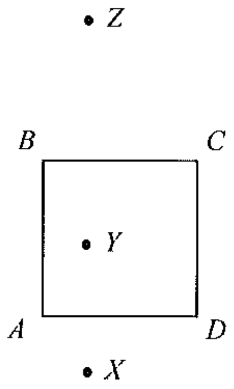


A square is a special rectangle. But a rectangle may be not a square.



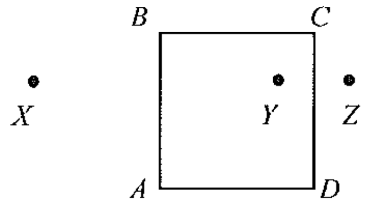
$\Downarrow$

Answer is (C).



1. In the figure above, $ABCD$ is a square with side = 20. Points X, Y, and Z are distinct points on a line (not shown) that is parallel to segment $\overline{AB}$. Points Y and Z are symmetric about line BC and points X and Y are symmetric about line AD. What is the length of segment $\overline{XZ}$?

(A) 32
(B) 38
(C) 36
(D) 40



Note: Figure above not drawn to scale.

2. In the figure above, $ABCD$ is a square with side = 9. Points X, Y, and Z are distinct points on a line (not shown) that is parallel to segment $\overline{BC}$. Points Y and Z are symmetric about line CD and points X and Y are symmetric about line AB. What is the length of segment $\overline{XZ}$?

(A) 18
(B) 20
(C) 23
(D) 17

3. In the figure above, $ABCD$ is a rectangle with $AD = 7$. Points X, Y, and Z are distinct points on a line (not shown) that is parallel to segment $\overline{AD}$. Points Y and Z are symmetric about line CD and points X and Y are symmetric about line AB. What is the length of segment $\overline{XZ}$?

(A) 15
(B) 18
(C) 23
(D) 14

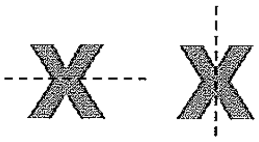
4. Which of the following has exactly four symmetric lines?

(A) Circle

(B) Triangle

(C) Square

(D) Rectangle



1. In the figure above, the letter X is symmetric with respect to two distinct lines, as shown by the dotted lines. Which of the following letters is symmetric with respect to at least 2 distinct lines?

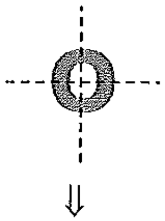
(A) M

(B) O

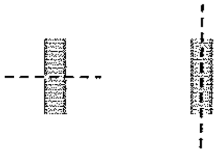
(C) T

(D) V

Solution: Answer: (B)



Answer is (B).



2. In the figure above, the letter I is symmetric with respect to two distinct lines, as shown by the dotted lines. Which of the following letters is symmetric with respect to at least 2 distinct lines?

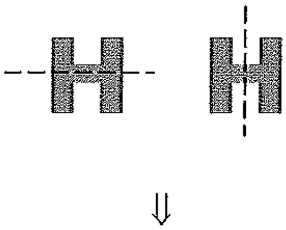
(A) U

(B) B

(C) T

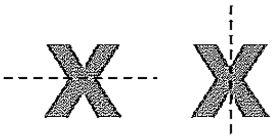
(D) H

Solution: Answer: (D)



Answer is (D).

Symmetric Letters in the Plane



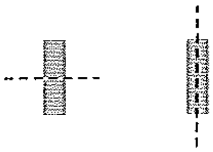
1. In the figure above, the letter X is symmetric with respect to two distinct lines, as shown by the dotted lines. Which of the following letters is symmetric with respect to at least 2 distinct lines?

(A) M

(B) O

(C) T

(D) V



2. In the figure above, the letter I is symmetric with respect to two distinct lines, as shown by the dotted lines. Which of the following letters is symmetric with respect to at least 2 distinct lines?

(A) U

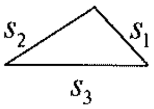
(B) B

(C) T

(D) H

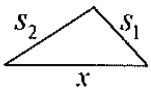
Legend:

$s_1 + s_2 > s_3$



and

$s_2 - s_1 < x < s_1 + s_2$

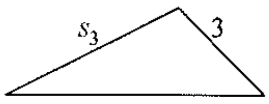


where $s_2 > s_1$.

1. In a triangle, if the length of its one side of is 7 and the length of another side is 3, which of the following CANNOT be the length of the third side of the triangle?

(A) 9
(B) 7
(C) 8
(D) 10

Solution: Answer: (D)



$s_1 + s_2 > s_3$

↓

$s_1 + s_2 \neq s_3$

↓

$3 + 7 = 10$

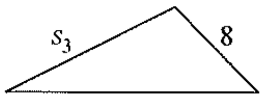
⇓

Answer is (D).

2. In a triangle, if the length of its one side of is 8 and the length of another side is 13, which of the following CANNOT be the length of the third side of the triangle?

(A) 8
(B) 13
(C) 21
(D) 14

Solution: Answer: (C)



$s_1 + s_2 > s_3$

↓

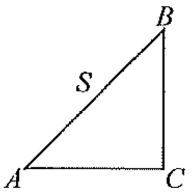
$s_1 + s_2 \neq s_3$

↓

$13 + 8 = 21$

⇓

Answer is (C).



Note: Figure above not drawn to scale.

3. In the figure above, if the triangle ABC is isosceles, $S = 9$ and BC is not more than 4, which of the following statements must be true?

(A) $AC = BC$
(B) $AB > AC$
(C) $AC = AB$
(D) $BC > AC$

Solution: Answer: (C)

AC may be equal to BC or AB.

$s_1 + s_2 > s_3$

If $BC = 4$ (maximum value of BC) and $AC = 4$,

↓

$4 + 4 = 8 < 9$
$AC = AB = S = 9$

↓

It satisfies the rule:

$AC + AB > BC$
$AC + BC > AB$
$AB + BC > AC$

⇓

Answer is (C).

Note that the question doesn't indicate the triangle is a right triangle.

4. In the figure above, if the triangle ABC is isosceles, $S = 10$ and BC is not more than 5, which of the following statements must be true?

(A) $AC = BC$
(B) $AC = AB$
(C) $AB < AC$
(D) $BC > AC$

Solution: Answer: (B)

AC may be equal to BC or AB.

$s_1 + s_2 > s_3$

If $BC = 5$ (maximum value of BC) and $AC = 5$,

↓

$5 + 5 = 10 = AB$, not $> AB$
$AC = AB = S = 10$

↓

Then it satisfies the rule:

$AC + AB > BC$
$AC + BC > AB$
$AB + BC > AC$

⇓

Answer is (B).

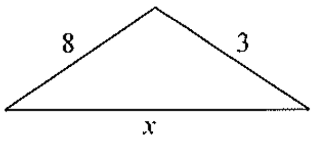
Note that the question doesn't indicate the triangle is a right triangle.

Triangle Inequalities

5. If x is an integer and $3 < x < 8$, how many distinct triangles are there with sides of lengths 3, 8, and x?

(A) 1
(B) 2
(C) 3
(D) 4

Solution: Answer: (B)



$$S_2 - S_1 < S_3 < S_2 + S_1$$

where $S_2 > S_1$.

$$\downarrow$$

$$8 - 3 < x < 8 + 3$$

$$\downarrow$$

$$\begin{cases} \underbrace{5 < x < 11}_{6,\ 7,\ 8,\ 9,\ 10} \\ \text{but,} \\ \underbrace{3 < x < 8}_{4,\ 5,\ 6,\ 7} \end{cases}$$

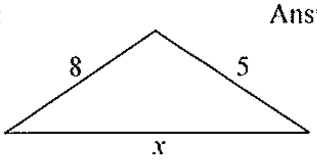
$$\Downarrow$$

$$\underbrace{x = 6 \text{ and } 7}_{\text{Common numbers}}$$

6. If x is an integer and $2 < x < 10$, how many different triangles are there with sides of lengths 5, 8, and x?

(A) 1
(B) 2
(C) 6
(D) 4

Solution: Answer: (C)



$$S_2 - S_1 < S_3 < S_2 + S_1$$

where $S_2 > S_1$.

$$\downarrow$$

$$8 - 5 < x < 8 + 5$$

$$\downarrow$$

$$\begin{cases} \underbrace{3 < x < 13}_{4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12} \\ \text{but} \\ \underbrace{2 < x < 10}_{3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9} \end{cases}$$

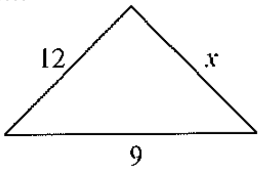
$$\Downarrow$$

$$\underbrace{4, 5, 6, 7, 8, \text{ and } 9}_{\text{Common numbers}}$$

7. If three sides of a triangle have lengths 12, 9 and x that is an integer, respectively, what are the least and greatest possible perimeters of this triangle?

(A) 26 and 42
(B) 9 and 12
(C) 4 and 20
(D) 25 and 41

Solution: Answer: (D)



$$S_2 - S_1 < S_3 < S_2 + S_1$$

where $S_2 > S_1$.

$$\downarrow$$

$$12 - 9 < x < 12 + 9$$

$$\downarrow$$

$$3 < x < 21$$

$$\downarrow$$

$$x_{\text{least}} = 4 \text{ and } x_{\text{greatest}} = 20$$

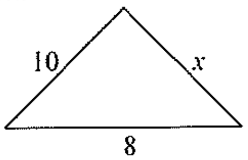
$$\Downarrow$$

$$\begin{cases} P_{\text{least}} = 4 + 9 + 12 = 25 \\ P_{\text{greatest}} = 20 + 9 + 12 = 41 \end{cases}$$

8. If three sides of a triangle have lengths 8 and 10 and x that is an integer, respectively, what is the greatest possible perimeter of this triangle?

(A) 35
(B) 21
(C) 17
(D) 34

Solution: Answer: (A)



$$S_2 - S_1 < S_3 < S_2 + S_1$$

where $S_2 > S_1$.

$$\downarrow$$

$$x < 10 + 8$$

$$\downarrow$$

$$x < 18$$

$$\downarrow$$

$$x = 17$$

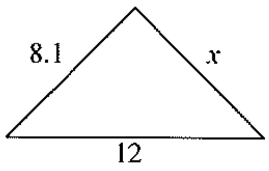
$$\Downarrow$$

$$P_{\text{greatest}} = 10 + 8 + 17 = 35$$

9. 2 sides of a triangle have lengths 8.1 and 12, respectively. If the remaining side of the triangle has a value that is an integer, what is the greatest possible perimeter of this triangle?

(A) 41.1
(B) 39.1
(C) 40
(D) 40.1

Solution: Answer: (D)



$\boxed{s_2 - s_1 < x < s_1 + s_2}$

$\downarrow$

$x < 12 + 8.1$

$\downarrow$

$3.9 < x < 20.1$

$\downarrow$

$x = 20$

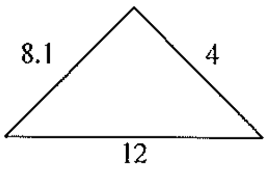
$\Downarrow$

$P = 12 + 8.1 + 20 = 40.1$

10. 2 sides of a triangle have lengths 8.1 and 12, respectively. If the remaining side of the triangle has a value that is an integer, what is the least possible perimeter of this triangle?

(A) 24.1
(B) 25.1
(C) 40
(D) 40.1

Solution: Answer: (A)



$\boxed{s_2 - s_1 < x < s_1 + s_2}$

$\downarrow$

$12 - 8.1 < x$

$\downarrow$

$3.9 < x < 20.1$

$\downarrow$

$x = 4$

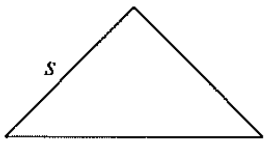
$\Downarrow$

$P = 12 + 8.1 + 4 = 24.1$

11. The measures of the lengths of two sides of an isosceles triangle are s and $2s + 3$. s is a positive integer. Then, the perimeter of the triangle is

(A) $4s + 3$
(B) $5s + 6$
(C) $4s + 5$
(D) $4s + 7$

Solution: Answer: (B)



$s_1 + s_2 > s_3$

(1) Because $s + s = 2s < 2s + 3$, the unknown side $\neq s$.

$\downarrow$

The unknown side $= 2s + 3$.

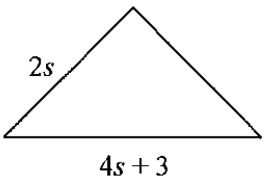
$\Downarrow$

(2) Perimeter $= s + 2(2s + 3) = 5s + 6$

12. The measures of the lengths of two sides of an isosceles triangle are $2s$ and $4s + 3$. s is a positive integer. Then, the perimeter of the triangle is

(A) $10s + 3$
(B) $8s + 6$
(C) $10s + 6$
(D) $4s + 7$

Solution: Answer: (C)



(1) Because $2s + 2s = 4s$ and $4s < 4s + 3$, the unknown side $\neq 2s$.

$\downarrow$

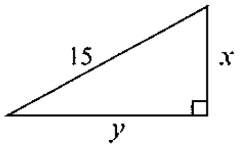
The unknown side $= 4s + 3$.

$\Downarrow$

(2) Perimeter $= 2s + 2(4s + 3) = 10s + 6$

Triangle Inequalities

Note: Figure below not drawn to scale.



13. Which of the following is true about the lengths x and y of the sides of the triangle above?

(A) $0 < (x + y)^2 < 225$

(B) $15 \leq (x + y)^2 < 30$

(C) $0 < (x + y)^2 \leq 225$

(D) $225 < (x + y)^2$

Solution: Answer: (D)

Sum of 2 side lengths > third side length

$\downarrow$

$x + y > 15$

$\downarrow$

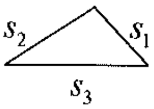
$(x + y)^2 > 15^2$

$\Downarrow$

$225 < (x + y)^2$

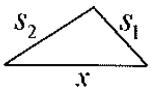
Legend:

$s_1 + s_2 > s_3$



and

$s_2 - s_1 < x < s_1 + s_2$



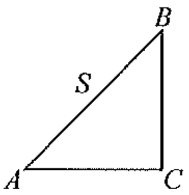
where $s_2 > s_1$.

1. In a triangle, if the length of its one side of is 7 and the length of another side is 3, which of the following CANNOT be the length of the third side of the triangle?

(A) 9
(B) 7
(C) 8
(D) 10

2. In a triangle, if the length of its one side of is 8 and the length of another side is 13, which of the following CANNOT be the length of the third side of the triangle?

(A) 8
(B) 13
(C) 21
(D) 14



Note: Figure above not drawn to scale.

3. In the figure above, if the triangle ABC is isosceles, $S = 9$ and BC is not more than 4, which of the following statements must be true?

(A) $AC = BC$
(B) $AB > AC$
(C) $AC = AB$
(D) $BC > AC$

4. In the figure above, if the triangle ABC is isosceles, $S = 10$ and BC is not more than 5, which of the following statements must be true?

(A) $AC = BC$
(B) $AC = AB$
(C) $AB < AC$
(D) $BC > AC$

Triangle Inequalities

5. If x is an integer and $3 < x < 8$, how many distinct triangles are there with sides of lengths 3, 8, and x?

(A) 1
(B) 2
(C) 3
(D) 4

6. If x is an integer and $2 < x < 10$, how many different triangles are there with sides of lengths 5, 8, and x?

(A) 1
(B) 2
(C) 6
(D) 4

7. If three sides of a triangle have lengths 12, 9 and x that is an integer, respectively, what are the least and greatest possible perimeters of this triangle?

(A) 26 and 42
(B) 9 and 12
(C) 4 and 20
(D) 25 and 41

8. If three sides of a triangle have lengths 8 and 10 and x that is an integer, respectively, what is the greatest possible perimeter of this triangle?

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(C) 17
(D) 34

9. 2 sides of a triangle have lengths 8.1 and 12, respectively. If the remaining side of the triangle has a value that is an integer, what is the greatest possible perimeter of this triangle?

(A) 41.1
(B) 39.1
(C) 40
(D) 40.1

10. 2 sides of a triangle have lengths 8.1 and 12, respectively. If the remaining side of the triangle has a value that is an integer, what is the least possible perimeter of this triangle?

(A) 24.1
(B) 25.1
(C) 40
(D) 40.1

11. The measures of the lengths of two sides of an isosceles triangle are s and $2s + 3$. s is a positive integer. Then, the perimeter of the triangle is

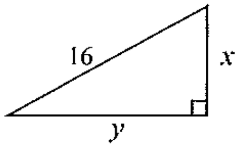
(A) $4s + 3$
(B) $5s + 6$
(C) $4s + 5$
(D) $4s + 7$

12. The measures of the lengths of two sides of an isosceles triangle are $2s$ and $4s + 3$. s is a positive integer. Then, the perimeter of the triangle is

(A) 10s + 3
(B) $8s + 6$
(C) 10s + 6
(D) $4s + 7$

Triangle Inequalities

Note: Figure below not drawn to scale.



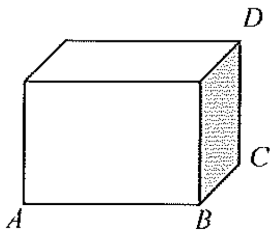
13. Which of the following is true about the lengths x and y of the sides of the triangle above?

(A) $0 < (x + y)^2 < 225$

(B) $15 \leq (x + y)^2 < 30$

(C) $0 < (x + y)^2 \leq 225$

(D) $225 < (x + y)^2$



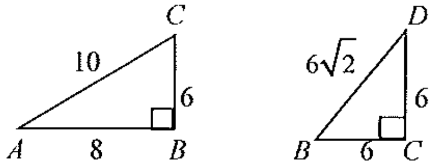
Note: Figure above not drawn to scale.

1. In the rectangular solid above, the segments $AC = 10$, $AB = 8$, and $BD = 6\sqrt{2}$. What is the volume of the solid?

(A) 192
(B) 166
(C) 288
(D) 188

Solution: Answer: (C)

$$V = AB \times BC \times CD$$



$\uparrow$

$\begin{cases} 6-8-10 \text{ triangle} \\ 45^\circ \text{ right triangle} \end{cases}$

$\downarrow$

$BC = CD = 3,\ AB = 4$

$\Downarrow$

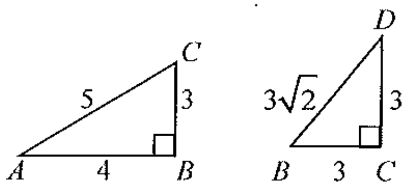
$V = 8 \times 6 \times 6 = 288$

2. In the rectangular solid above, $\angle CBD = \angle CDB$, $BD = 3\sqrt{2}$, and $AC = 5$. What is the volume of the solid?

(A) 36
(B) 48
(C) 96
(D) 24

Solution: Answer: (A)

$$V = AB \times BC \times CD$$



$\uparrow$

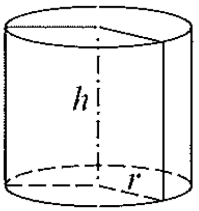
$\begin{cases} 3\text{-}4\text{-}5 \text{ triangle} \\ 45^\circ \text{ triangle} \end{cases}$

$\downarrow$

$BC = 6$ and $CD = 6$

$\Downarrow$

$V = 4 \times 3 \times 3 = 36$



Note: Figure above not drawn to scale.

3. Which of the following has the same volume as the cylinder shown above with radius $r = 3a$ and height $h = \pi a$?

(A) A rectangular solid with dimensions $3\pi a$, πa, and $3a$
(B) A cylinder with volume $3a\pi$
(C) A cylinder with radius $3\pi a$ and height $3a$
(D) A rectangular solid with πa, πa, and $3a^2$

Solution: Answer: (A)

$r = 3a,\ h = \pi a$

$V = r^2\pi \times h = (3a)^2\,\pi \times \pi a = 3^2 a^3 \pi^2$

$\downarrow$

"Two 3s, three as, and two πs"

Use plug-in method.

$\Downarrow$

Answer is (A).

4. Which of the following has the same volume as the cylinder shown above with radius $r = 3a$ and height $h = \pi a^2$?

(A) A rectangular solid with dimensions $3\pi a$, πa, and $3a$
(B) A cylinder with base area $3a\pi$ and height a
(C) A cylinder with radius $3\pi a$ and height $3a$
(D) A rectangular solid with $3\pi a$, πa^2, and $3a$

Solution: Answer: (D)

$r = 3a,\ h = \pi a^2$

$V = r^2\pi \times h = (3a)^2\,\pi \times \pi a^2 = 3^2 a^4 \pi^2$

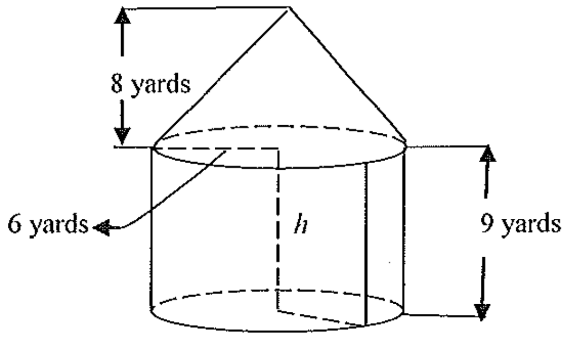
$\downarrow$

Two 3s, four as, two πs

Use plug-in method.

$\Downarrow$

Answer is (D).



5. The figure above shows a container with measurements. The container is built from a right circular cone and a right circular cylinder. What is the volume, in cubic yards, of the container?

(A) 1019.5

(B) 1119.5

(C) 1319.5

(D) 1219.5

Solution: Answer: (C)

$$V = V_{Cy} + V_{Co} = r^2\pi h_1 + \frac{1}{3}r^2\pi h_2$$

$$\downarrow$$

$$r^2\pi\left(h_1 + \frac{1}{3}h_2\right) = \underbrace{6^2\pi\left(9 + \frac{8}{3}\right)}_{\text{Use a calculator.}} \approx 1319.468$$

$$\Downarrow$$

$$V = 1319.5$$

6. If the surface area of a cube is s square inches, what is the volume, in cubic inches, of the cube?

(A) $\frac{s^2}{6}\sqrt{\frac{s}{6}}$

(B) $\frac{s^2}{6}\sqrt{\frac{s}{6^2}}$

(C) $\frac{s}{6}\sqrt{\frac{s}{6}}$

(D) $s\sqrt{\frac{s}{6^2}}$

Solution: Answer: (C)

$$A = \frac{s}{6},\ \text{one side} = \sqrt{\frac{s}{6}}$$

$$\downarrow$$

$$V = \left(\sqrt{\frac{s}{6}}\right)^3 = \left(\sqrt{\frac{s}{6}}\right)^2 \times \left(\sqrt{\frac{s}{6}}\right)$$

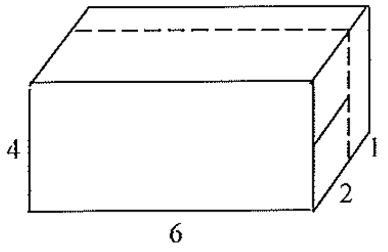
$$\Downarrow$$

$$V = \frac{s}{6}\sqrt{\frac{s}{6}}$$

7. The rectangular container on a ship is 6 meters deep, 3 meters wide, and 4 meters high. If each cube box is 2 meters on each edge, how many cube boxes will the container hold?

(A) 9
(B) 8
(C) 7
(D) 6

Solution: Answer: (D)



$$\Downarrow$$

$$\underbrace{N = \frac{V_{\text{larger}}}{V_{\text{smaller}}} = \frac{3\times4\times6}{2^3} = \frac{72}{8} = 9}_{\text{Wrong}}$$

$$\underbrace{N = \frac{V_{\text{larger}}}{V_{\text{smaller}}} = \frac{\cancel{2}\times4\times6}{\cancel{2^3}} = \boxed{6}}_{\text{Correct}}$$

8. If the total surface area of a cube is 96 square inches, what is the volume, in cubic inches, of the cube?

(A) 64
(B) 16
(C) 8
(D) 56

Solution: Answer: (A)

A cube has 6 faces.

$$A_{\frac{1}{6}} = \frac{96}{6} = 16,$$

$$S = \sqrt{A_{\frac{1}{6}}} = \sqrt{16} = 4$$

$$\Downarrow$$

$$V = S^3 = 4^3 = 64$$

9. If the total surface area of a cube is 24 square inches, what is the volume, in cubic inches, of the cube?

(A) 16
(B) 32
(C) 56
(D) 8

Solution: Answer: (D)

A cube has 6 faces.

$$A_{\frac{1}{6}} = \frac{24}{6} = 4,$$

$$S = \sqrt{A_{\frac{1}{6}}} = \sqrt{4} = 2$$

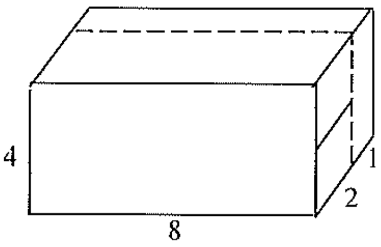
$$\Downarrow$$

$$V = S^3 = 2^3 = 8$$

10. The rectangular container is 8 meters long, 3 meters wide and 4 meters high. If each cube box is 2 meters on each edge, how many cube boxes will the container hold?

(A) 12
(B) 10
(C) 8
(D) 6

Solution: Answer: (C)



$$\Downarrow$$

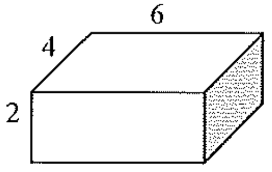
$$N = \underbrace{\frac{V_{\text{larger}}}{V_{\text{smaller}}} = \frac{3\times4\times8}{2^3} = 12}_{\text{Wrong}}$$

$$N = \underbrace{\frac{V_{\text{larger}}}{V_{\text{smaller}}} = \frac{2\times4\times8}{2^3} = \boxed{8}}_{\text{Correct}}$$

11. The rectangular container is 6 meters long, 4 meters wide, and 2 meters high. If each cube box is 2 meters on each edge, how many cube boxes will the container hold?

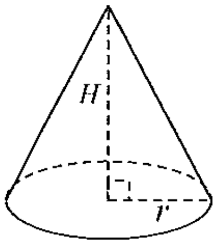
(A) 12
(B) 10
(C) 8
(D) 6

Solution: Answer: (D)



$$\Downarrow$$

$$N = \frac{V_{\text{larger}}}{V_{\text{smaller}}} = \frac{2\times4\times6}{2^3} = 6$$



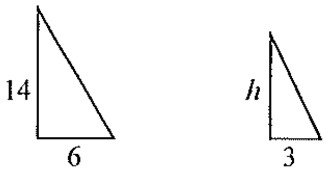
Note: Figure above not drawn to scale.

12. In the figure above, the right circular cone with radius $r = 6$ and height $H = 14$ is cut by a plane parallel to the base to form a new smaller cone. If the diameter of the base of the smaller cone is 6, what is the height of the smaller cone?

(A) 7
(B) 3
(C) 5
(D) 4

Solution: Answer: (A)

Two similar triangles are formed.



$$\Downarrow$$

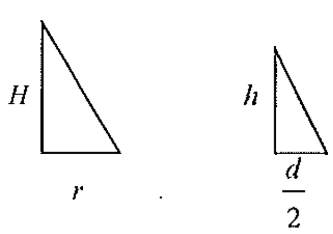
$$\frac{h}{14} = \frac{3}{6}, \quad h = \frac{3\times14}{6} = 7$$

13. In the figure above, the right circular cone with radius r and height H is cut by a plane parallel to the base to form a new smaller cone. If the diameter of the base of the smaller cone is d, what is the height of the smaller cone?

(A) $\frac{dH}{r}$
(B) $\frac{dH}{2r}$
(C) $\frac{dr}{2H}$
(D) $\frac{2H}{dr}$

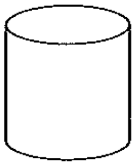
Solution: Answer: (B)

Two similar triangles are formed.



$$\Downarrow$$

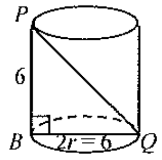
$$\frac{h}{H} = \frac{\frac{d}{2}}{r}, \quad h = \frac{dH}{2r}$$



14. In the cylinder with dimensions radius 3 and height 6 shown above, if point P and Q lie on the circumference of the top and bottom of the cylinder, respectively, what is the greatest possible distance between P and Q?

(A) $\sqrt{45}$
(B) $2\sqrt{6}$
(C) $3\sqrt{5}$
(D) $6\sqrt{2}$

Solution: Answer: (D)



PB = height of cylinder
When BQ = diameter of the circle,
PQ can get the greatest value.

$r = 3,\ d = 6$

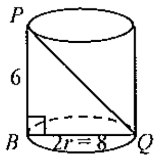
$PB = BQ = 6$
(45° – right triangle)

$\Downarrow$

$PQ = 6\sqrt{2}$

15. In the cylinder shown above with dimensions radius 4 and height 6, if point P and Q lie on the circumference of the top and bottom of the cylinder, respectively, what is the greatest possible distance between P and Q?

Solution: Answer: 10



PB = height of cylinder
When BQ = diameter of circle,
PQ can get the greatest value.

$r = 4,\ h = 6$

$PB = 6,\ BQ = 8$
(6 – 8 – 10 right triangle)

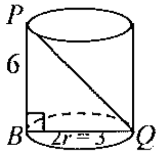
$\Downarrow$

$PQ = 10$

16. In the cylinder shown on the left with dimensions radius 1.5 and height 6. If point P and Q lie on the circumference of the top and bottom of the cylinder, respectively, what is the greatest possible distance between P and Q?

(A) $3\sqrt{5}$
(B) $2\sqrt{6}$
(C) $6\sqrt{2}$
(D) $6\sqrt{3}$

Solution: Answer: (A)



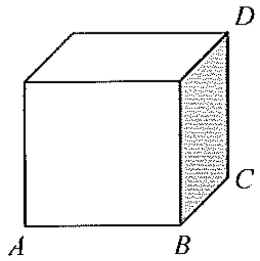
PB = height of cylinder
When BQ = diameter of circle,
PQ can get the greatest value

$r = 1.5,\ d = 3$

$PB = 6\ BQ = 3$

$\Downarrow$

$PQ = \sqrt{3^2 + 6^2} = 3\sqrt{5}$

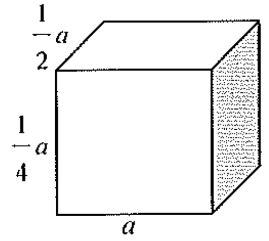


Note: Figure above not drawn to scale.

17. In the rectangular solid above, $AB = a$, $BC = \frac{1}{2}AB$, and $DC = \frac{1}{2}BC$. What is the volume of the solid?

(A) $\frac{1}{2}a^3$
(B) $\frac{1}{8}a^3$
(C) $\frac{1}{4}a^3$
(D) $\frac{1}{6}a^3$

Solution: Answer: (B)

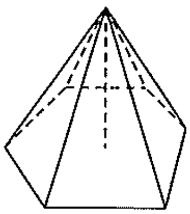


$V = xyz$

$\Downarrow$

$a \times \frac{1}{2}a \times \frac{1}{4}a = \frac{1}{8}a^3$

Note: Figure below not drawn to scale.



18. In the figure above, the solid has a hexagonal base. Each edge of the base has length 6 units and each of the other edges of the solid has the same length as the edge of the base. What is the the total length of the edges of the solid?

(A) 60

(B) 72

(C) 84

(D) 96

Solution: Answer: (B)

$$6\times(6+6)=72$$

$$\Downarrow$$

Answer is (B).

19. Oil is poured into a cylindrical tank at a speed of π cubic inches per minute. If its radius is i inches and its height is h inches, how many minutes will it take to fill up the tank?

(A) $i^2\pi$

(B) $i^2\pi^2$

(C) ih^2

(D) i^2h

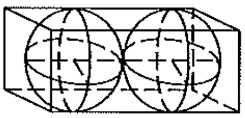
Solution: Answer: (D)

$$\text{Volume} = A\times h = r^2\pi h$$

$$\text{Rate} = \pi$$

$$\Downarrow$$

$$\text{Time} = \frac{\text{Volume}}{\text{Rate}} = \frac{r^2\pi h}{\pi} = \frac{i^2\pi h}{\pi} = i^2h$$



20. In the figure above, two identical spheres fit snugly in a rectangular box. The radii of the spheres are equal to half-length of one side of the smallest surfaces of the rectangular box. What is the ratio of the sum of the volumes of the two spheres to the volume of the rectangular box? (The formula for the volume of a sphere of radius r is $V=\frac{4}{3}\pi r^3$.)

(A) $\frac{6}{\pi}$

(B) $\frac{6}{\pi r}$

(C) $\frac{6r}{\pi}$

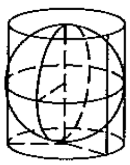
(D) $\frac{\pi}{6}$

Solution: Answer: (D)

$$\frac{V_{\text{2-spheres}}}{V_{\text{rectangular solid}}} = \frac{2\cdot V_{\text{1-spheres}}}{A_{\text{base}}\cdot H}$$

$$\Downarrow$$

$$\frac{2\cdot\frac{4\pi r^3}{3}}{(2r)^2\cdot 4r} = \frac{2\cdot 4\pi r^3}{3\cdot 4r^3} = \frac{\pi}{6}$$



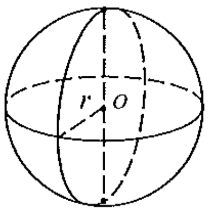
21. In the figure above, one sphere fits snugly in a circular cylinder. The radius of the sphere is equal to the radius of base of the cylinder. The sphere just touches the bottom and top of the cylinder. What fraction of the volume of the cylinder does the volume of the sphere take up? (The formula for the volume of a sphere of radius r is $V=\frac{4}{3}\pi r^3$.)

Solution: Answer: 2/3 or .667 or .666

$$\frac{V_{\text{1-spheres}}}{V_{\text{cylinder}}} = \frac{V_{\text{1-spheres}}}{A_{base}\cdot H}$$

$$\Downarrow$$

$$\frac{\frac{4\pi r^3}{3}}{r^2\pi\cdot 2r} = \frac{2}{3}$$



22. What is the radius of a sphere shown above whose volume equals the volume of a right prism, which is 36π? (The formula for the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.)

(A) 9
(B) 5
(C) 6
(D) 3

Solution: Answer: (D)

$$V_p = 36\pi,\ V_s = \frac{4}{3}\pi r^3$$

$$V_p = V_s$$

$$\downarrow$$

$$\cancel{36}^{9}\ \cancel{\pi} = \frac{4}{\cancel{3}}\cancel{\pi} r^3$$

$$27 = r^3$$

$$\Downarrow$$

$$r = \sqrt[3]{27} = \sqrt[3]{3\cdot 3\cdot 3} = 3$$

23. What is the radius of a sphere shown above whose surface area equals the area of surfaces of a cube whose edges are 2? (The formula for the area of surface of a sphere of radius r is $A = 4\pi r^2$.)

(A) $\sqrt{\frac{6}{2\pi}}$
(B) $\sqrt{6\pi}$
(C) $\sqrt{\frac{6}{\pi}}$
(D) $\sqrt{\frac{\pi}{6}}$

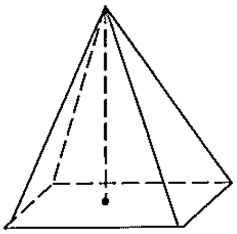
Solution: Answer: (C)

$$A_c = 6A_1 = 6\cdot 2^2 = 24,\ A_S = 4\pi r^2$$

$$A_c = A_s$$

$$\Downarrow$$

$$24 = 4\pi r^2,\quad r = \sqrt{\frac{6}{\pi}}$$



24. If the volume of a pyramid shown above whose base is a square with side = s is w, what is the height of the pyramid?

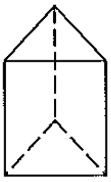
(A) $\frac{s^2}{3w}$
(B) $\frac{3w}{s^2}$
(C) $\frac{s^2}{6w}$
(D) $\frac{3w}{s}$

Solution: Answer: (B)

$$V = \frac{1}{3}A_b h$$

$$\Downarrow$$

$$h = \frac{3V}{A_b} = \frac{3w}{s^2}$$



Note: figure above not drawn to scale.

25. In the figure above, if the height of the triangular prism is 1 and one side of the bases that are equilateral triangles is 1, what is the volume of this prism?

(A) $\frac{\sqrt{4}}{3}$
(B) $\frac{3\sqrt{3}}{4}$
(C) $\frac{2\sqrt{3}}{4}$
(D) $\frac{\sqrt{3}}{4}$

Solution: Answer: (D)

The bases of a triangular prism are equilateral triangles.

$$A = \frac{s^2\sqrt{3}}{4} = \frac{1^2\sqrt{3}}{4} = \frac{\sqrt{3}}{4}$$

$$\Downarrow$$

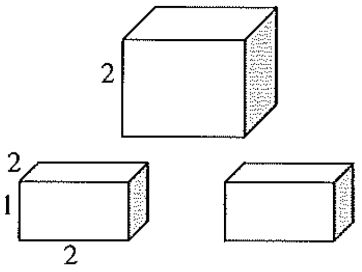
$$V = A\cdot h = \frac{\sqrt{3}}{4}\cdot 1 = \frac{\sqrt{3}}{4}$$

26. Kathie cuts a wooden cube with volume 8 in half horizontally. The two halves are not glued together to form a rectangular solid or a cube. What is the area of the surfaces of these new solids?

(A) 28
(B) 26
(C) 14
(D) 32

Solution: Answer: (D)

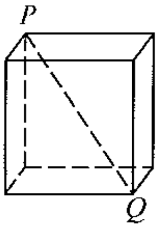
$8 = s^3,\ s = 2$



$$\Downarrow$$

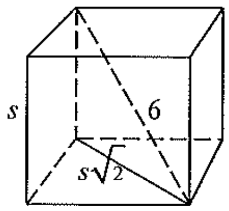
$$A = 2\times 2\left(2\times 1 + 2\times 1 + 2\times 2\right)$$

$$A = 4\times 8 = 32$$



27. In the figure above, the length of diagonal $\overline{PQ}$ of the cube is 6, what is the area of total surfaces of the cube?

Solution: Answer: 72



Method 1

$$6^2 = s^2 + \left(s\sqrt{2}\right)^2,\ 36 = 3s^2$$

$$A_1 = s^2 = 12$$

$$\Downarrow$$

$$A_6 = 6\times s^2 = 72$$

Method 2

Use the conclusion: $\text{Diagonal}_{\text{for any cube}} = s\sqrt{3}$.

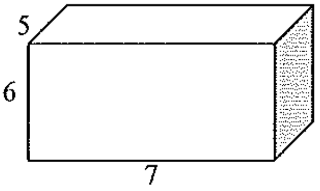
$$6 = s\sqrt{3},\ s = \boxed{\frac{6}{\sqrt{3}}}$$

$$\Downarrow$$

$$A = 6s^2 = 6\left(\frac{6}{\sqrt{3}}\right)^2 = \cancel{6}^{2}\cdot\frac{36}{\cancel{3}} = \boxed{72}$$

28. The three surfaces of a rectangular solid have areas 30, 35, and 42. If the dimensions of the solid are all integers, what is the volume of the solid?

Solution: Answer: 210



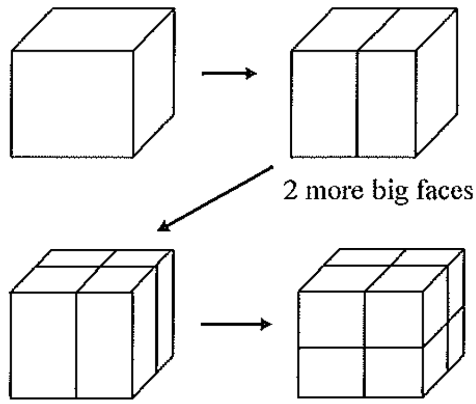
$$V = l\times w\times h$$

$$\Downarrow$$

$$V = 5\times 6\times 7 = 210$$

29. Shirley cuts a solid object with six equal square sides in half in all 3 directions. If the volume of the object is 8 cubic inches, what is the area of all surfaces of the separated smaller cubes in square inches?

Solution: Answer: 48



$$8 = s^3,\ s = 2,\ A_1 = s^2 = 2^2 = 4$$

Each of the 3 cutting creates 2 more surfaces with area 4. The 3 cuttings create 6 more surfaces with area 4.

$$\downarrow$$

$$6 + 6 = 12$$

$$\Downarrow$$

$$A_{\text{total}} = 12\times 4 = 48$$

30. A rectangular container has a length of 9, a width of 7, a height of 9, and a volume of V_1. Which of the following represents the volume of a rectangular container with dimensions length 9, width 7, and height 3 in terms of V_1?

(A) $\frac{2}{3}V_1$

(B) $\frac{1}{2}V_1$

(C) $\frac{1}{3}V_1$

(D) $2V_1$

Solution: Answer: (C)

$$V_1 = 9\times7\times9,\ V_2 = 9\times7\times3$$

$$\downarrow$$

$$\frac{V_2}{V_1} = \frac{9\times7\times3}{9\times7\times9}$$

$$\downarrow$$

$$\frac{V_2}{V_1} = \frac{\cancel{9\times7}\times3}{\cancel{9\times7}\times9} = \frac{1}{3}$$

$$\Downarrow$$

$$V_2 = \frac{1}{3}V_1$$

31. A rectangular container has a length of 9, a width of 6, a height of 7, and a volume of V_1. Which of the following represents the volume of a rectangular container with dimensions length 9, width 2, and height 7 in terms of V_1?

(A) $\frac{2}{3}V_1$

(B) $\frac{1}{2}V_1$

(C) $3V_1$

(D) $\frac{1}{3}V_1$

Solution: Answer: (D)

$$V_1 = 9\times6\times7,\ V_2 = 9\times2\times7$$

$$\downarrow$$

$$\frac{V_2}{V_1} = \frac{9\times2\times7}{9\times6\times7}$$

$$\downarrow$$

$$\frac{V_2}{V_1} = \frac{\cancel{9}\times2\times\cancel{7}}{\cancel{9}\times6\times\cancel{7}} = \frac{1}{3}$$

$$\Downarrow$$

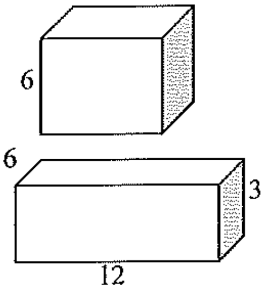
$$V_2 = \frac{1}{3}V_1$$

32. Robert cuts a wooden cube with volume 216 in half horizontally. The two halves are glued together to form a rectangular solid, which is not a cube. What is the surface area of this new solid?

(A) 216
(B) 252
(C) 108
(D) 512

Solution: Answer: (B)

$$216 = s^3,\ s = 6$$



$$A = 2\left((3\times6)+(3\times12)+(6\times12)\right)$$

$$\Downarrow$$

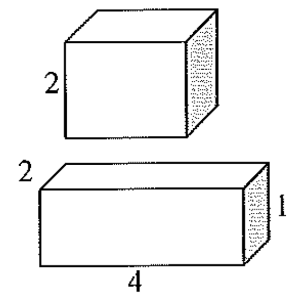
$$A = 2\left(18+36+72\right) = 252$$

33. Stephanie cuts wooden cube with volume 8 in half horizontally. The two halves are glued together to form a rectangular solid, which is not a cube. What is the surface area of this new solid?

(A) 28
(B) 26
(C) 14
(D) 18

Solution: Answer: (A)

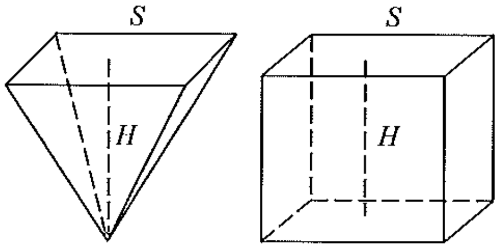
$$8 = s^3,\ s = 2$$



$$A = 2\left((4\times1)+(2\times1)+(4\times2)\right)$$

$$\Downarrow$$

$$A = 2\left(4+2+8\right) = 28$$



34. In the figure above, one container is a pyramid with a square base of side S units and height $H = 9$ units and another container is a rectamgular prism with a square base of side S units. If the pyramid is filled with sand, and then the sand is poured into the empty prism container, what will be the depth of the sand in the prism container?

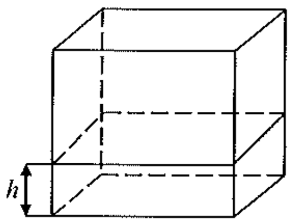
(A) $\frac{1}{3}$

(B) 2

(C) 3

(D) $3\frac{1}{2}$

Solution: Answer: (C)



$$V_{\text{pyramid}} = V_{\text{part of prism}}$$

$$\frac{1}{3}S^2H = S^2h$$

$$\Downarrow$$

$$h = \frac{1}{3}H = \frac{1}{3}\cdot 9 = 3$$

35. Water is poured into a cylindrical tank at a speed of π cubic inches per minute. If its radius is π inches and its height is π inches, how many minutes will it take to fill up the tank?

(A) π^4

(B) π^3

(C) $\pi^{\frac{3}{2}}$

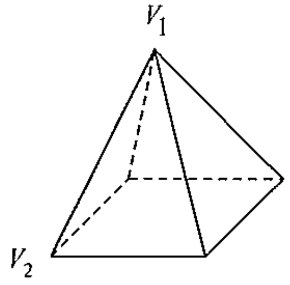
(D) π^2

Solution: Answer: (B)

$$\text{Time} = \frac{\text{Volume}}{\text{Rate}} = \frac{r^2h\times\pi}{\pi}$$

$$\Downarrow$$

$$\frac{\pi^2\times\pi\times\pi}{\pi} = \pi^3$$



36. In the figure above, the pyramid has a square base and four congruent triangular surfaces. If the height of the pyramid is $\sqrt{5.5}$ feet and each side of the base is 5 feet, what is the length of segment $\overline{V_1V_2}$?

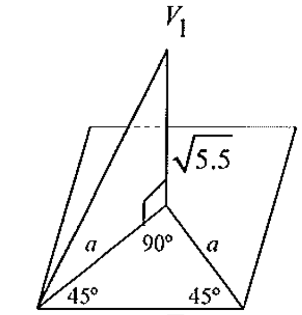
(A) $3\sqrt{2}$

(B) $2\sqrt{3}$

(C) $2\sqrt{2}$

(D) $5.5\sqrt{3}$

Solution: Answer: (A)



$$5 = a\sqrt{2}$$

$$5 = a\sqrt{2}, \rightarrow a = \frac{5}{\sqrt{2}}$$

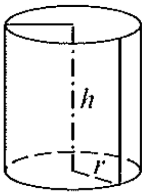
$$V_1V_2 = \sqrt{\left(\sqrt{5.5}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2}$$

$$\downarrow$$

$$\sqrt{5.5 + \frac{25}{2}} = \sqrt{\frac{11}{2} + \frac{25}{2}} = \sqrt{18}$$

$$\Downarrow$$

$$V_1V_2 = \sqrt{9\cdot 2} = 3\sqrt{2}$$



Note: Figure above not drawn to scale.

37. In the figure above, the surface area, without the surface areas of the top and bottom bases with r, of the cylinder with height h is 20π. Which of the following is the volume of this cylinder?

(A) $100\pi h$
(B) $10\pi h$
(C) $\frac{10\pi}{h}$
(D) $\frac{100\pi}{h}$

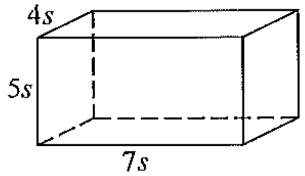
Solution: Answer: (D)

$$A = L \times W \rightarrow 20\pi = 2r\pi \times h$$
$$\downarrow$$
$$r = \frac{10}{h}$$
$$\downarrow$$
$$V = A \cdot h = r^2\pi \cdot h = \left(\frac{10}{h}\right)^2 \pi h$$
$$\Downarrow$$
$$V = \boxed{\frac{100\pi}{h}}$$

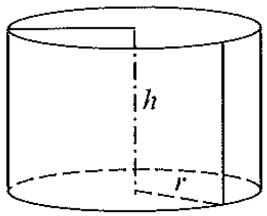
38. The sides of a rectangular solid have lengths $4s$, $5s$, and $7s$. Which of the following is the entire surface area of the rectangular solid, in terms of s ?

(A) $83s^2$
(B) $176s^2$
(C) $166s^2$
(D) $186s^2$

Solution: Answer: (C)



$$2(5s \cdot 7s + 4s \cdot 7s + 4s \cdot 5s)$$
$$\downarrow$$
$$2(35s^2 + 28s^2 + 20s^2)$$
$$\Downarrow$$
$$2 \cdot 83s^2 = 166s^2$$



Note: Figure above not drawn to scale.

39. In the figure above, the surface area, without the surface areas of the top and bottom bases with radius r, of the cylinder with height $h = 5$ is 20π. Which of the following is the volume of this cylinder?

(A) 100π
(B) 10π
(C) 20π
(D) 500π

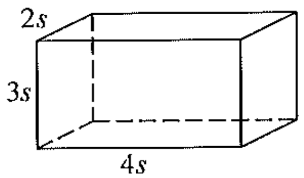
Solution: Answer: (C)

$$A = L \times W \rightarrow 20\pi = 2r\pi \times h$$
$$\downarrow$$
$$r = \frac{10}{h}$$
$$\downarrow$$
$$V = A \cdot h = r^2\pi \cdot h = \left(\frac{10}{h}\right)^2 \pi h$$
$$\Downarrow$$
$$V = \frac{100\pi}{h} = \frac{100\pi}{5} = 20\pi$$

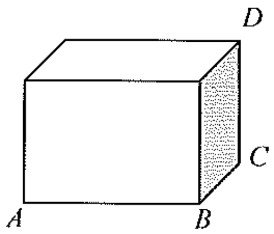
40. The sides of a rectangular solid have lengths $2s$, $3s$, and $4s$. Which of the following is the entire surface area of the rectangular solid, in terms of s ?

(A) $26s^2$
(B) $62s^2$
(C) $52s^2$
(D) $25s^2$

Solution: Answer: (C)



$$2(2s \cdot 3s + 2s \cdot 4s + 3s \cdot 4s)$$
$$\downarrow$$
$$2(6s^2 + 8s^2 + 12s^2)$$
$$\Downarrow$$
$$2 \cdot 26s^2 = 52s^2$$



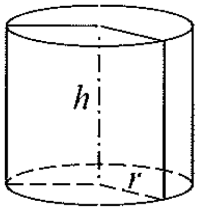
Note: Figure above not drawn to scale.

1. In the rectangular solid above, the segments $AC = 10$, $AB = 8$, and $BD = 6\sqrt{2}$. What is the volume of the solid?

(A) 192
(B) 166
(C) 288
(D) 188

2. In the rectangular solid above, $\angle CBD = \angle CDB$, $BD = 3\sqrt{2}$, and $AC = 5$. What is the volume of the solid?

(A) 36
(B) 48
(C) 96
(D) 24



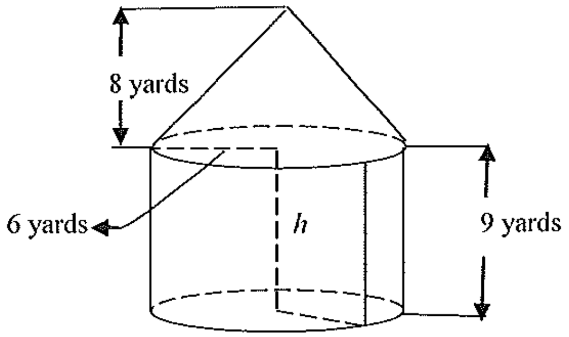
Note: Figure above not drawn to scale.

3. Which of the following has the same volume as the cylinder shown above with radius $r = 3a$ and height $h = \pi a$?

(A) A rectangular solid with dimensions $3\pi a$, πa, and $3a$
(B) A cylinder with volume $3a\pi$
(C) A cylinder with radius $3\pi a$ and height $3a$
(D) A rectangular solid with πa, πa, and $3a^2$

4. Which of the following has the same volume as the cylinder shown above with radius $r = 3a$ and height $h = \pi a^2$?

(A) A rectangular solid with dimensions $3\pi a$, πa, and $3a$
(B) A cylinder with base area $3a\pi$ and height a
(C) A cylinder with radius $3\pi a$ and height $3a$
(D) A rectangular solid with $3\pi a$, πa^2, and $3a$



5. The figure above shows a container with measurements. The container is built from a right circular cone and a right circular cylinder. What is the volume, in cubic yards, of the container?

(A) 1019.5
(B) 1119.5
(C) 1319.5
(D) 1219.5

6. If the surface area of a cube is s square inches, what is the volume, in cubic inches, of the cube?

(A) $\frac{s^2}{6}\sqrt{\frac{s}{6}}$
(B) $\frac{s^2}{6}\sqrt{\frac{s}{6^2}}$
(C) $\frac{s}{6}\sqrt{\frac{s}{6}}$
(D) $s\sqrt{\frac{s}{6^2}}$

7. The rectangular container on a ship is 6 meters deep, 3 meters wide, and 4 meters high. If each cube box is 2 meters on each edge, how many cube boxes will the container hold?

(A) 9
(B) 8
(C) 7
(D) 6

8. If the total surface area of a cube is 96 square inches, what is the volume, in cubic inches, of the cube?

(A) 64
(B) 16
(C) 8
(D) 56

9. If the total surface area of a cube is 24 square inches, what is the volume, in cubic inches, of the cube?

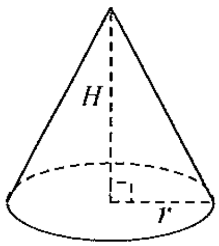
(A) 16
(B) 32
(C) 56
(D) 8

10. The rectangular container is 8 meters long, 3 meters wide and 4 meters high. If each cube box is 2 meters on each edge, how many cube boxes will the container hold?

(A) 12
(B) 10
(C) 8
(D) 6

11. The rectangular container is 6 meters long, 4 meters wide, and 2 meters high. If each cube box is 2 meters on each edge, how many cube boxes will the container hold?

(A) 12
(B) 10
(C) 8
(D) 6



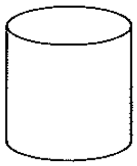
Note: Figure above not drawn to scale.

12. In the figure above, the right circular cone with radius $r = 6$ and height $H = 14$ is cut by a plane parallel to the base to form a new smaller cone. If the diameter of the base of the smaller cone is 6, what is the height of the smaller cone?

(A) 7
(B) 3
(C) 5
(D) 4

13. In the figure above, the right circular cone with radius r and height H is cut by a plane parallel to the base to form a new smaller cone. If the diameter of the base of the smaller cone is d, what is the height of the smaller cone?

(A) $\frac{dH}{r}$
(B) $\frac{dH}{2r}$
(C) $\frac{dr}{2H}$
(D) $\frac{2H}{dr}$



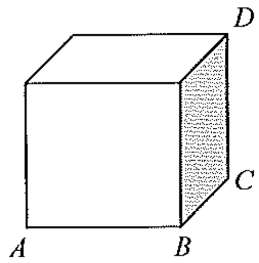
14. In the cylinder with dimensions radius 3 and height 6 shown above, if point P and Q lie on the circumference of the top and bottom of the cylinder, respectively, what is the greatest possible distance between P and Q?

(A) $\sqrt{45}$
(B) $2\sqrt{6}$
(C) $3\sqrt{5}$
(D) $6\sqrt{2}$

15. In the cylinder shown above with dimensions radius 4 and height 6, if point P and Q lie on the circumference of the top and bottom of the cylinder, respectively, what is the greatest possible distance between P and Q?

16. In the cylinder shown on the left with dimensions radius 1.5 and height 6. If point P and Q lie on the circumference of the top and bottom of the cylinder, respectively, what is the greatest possible distance between P and Q?

(A) $3\sqrt{5}$
(B) $2\sqrt{6}$
(C) $6\sqrt{2}$
(D) $6\sqrt{3}$

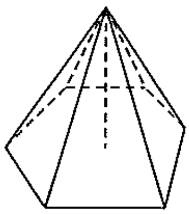


Note: Figure above not drawn to scale.

17. In the rectangular solid above, $AB = a, BC = \frac{1}{2}AB$, and $DC = \frac{1}{2}BC$. What is the volume of the solid?

(A) $\frac{1}{2}a^3$
(B) $\frac{1}{8}a^3$
(C) $\frac{1}{4}a^3$
(D) $\frac{1}{6}a^3$

Note: Figure below not drawn to scale.



18. In the figure above, the solid has a hexagonal base. Each edge of the base has length 6 units and each of the other edges of the solid has the same length as the edge of the base. What is the the total length of the edges of the solid?

(A) 60

(B) 72

(C) 84

(D) 96

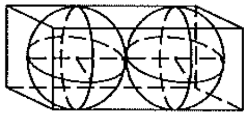
19. Oil is poured into a cylindrical tank at a speed of π cubic inches per minute. If its radius is i inches and its height is h inches, how many minutes will it take to fill up the tank?

(A) $i^2\pi$

(B) $i^2\pi^2$

(C) ih^2

(D) i^2h



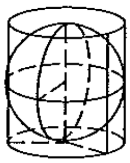
20. In the figure above, two identical spheres fit snugly in a rectangular box. The radii of the spheres are equal to half-length of one side of the smallest surfaces of the rectangular box. What is the ratio of the sum of the volumes of the two spheres to the volume of the rectangular box? (The formula for the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.)

(A) $\frac{6}{\pi}$

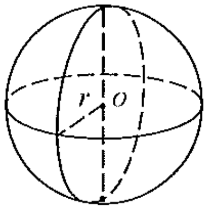
(B) $\frac{6}{\pi r}$

(C) $\frac{6r}{\pi}$

(D) $\frac{\pi}{6}$



21. In the figure above, one sphere fits snugly in a circular cylinder. The radius of the sphere is equal to the radius of base of the cylinder. The sphere just touches the bottom and top of the cylinder. What fraction of the volume of the cylinder does the volume of the sphere take up? (The formula for the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.)

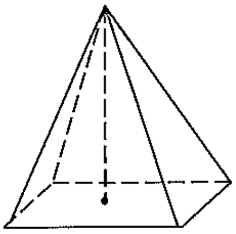


22. What is the radius of a sphere shown above whose volume equals the volume of a right prism, which is 36π? (The formula for the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.)

(A) 9
(B) 5
(C) 6
(D) 3

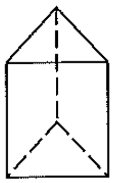
23. What is the radius of a sphere shown above whose surface area equals the area of surfaces of a cube whose edges are 2? (The formula for the area of surface of a sphere of radius r is $A = 4\pi r^2$.)

(A) $\sqrt{\frac{6}{2\pi}}$
(B) $\sqrt{6\pi}$
(C) $\sqrt{\frac{6}{\pi}}$
(D) $\sqrt{\frac{\pi}{6}}$



24. If the volume of a pyramid shown above whose base is a square with side = s is w, what is the height of the pyramid?

(A) $\frac{s^2}{3w}$
(B) $\frac{3w}{s^2}$
(C) $\frac{s^2}{6w}$
(D) $\frac{3w}{s}$



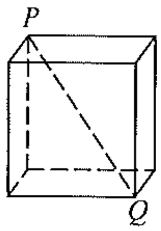
Note: figure above not drawn to scale.

25. In the figure above, if the height of the triangular prism is 1 and one side of the bases that are equilateral triangles is 1, what is the volume of this prism?

(A) $\frac{\sqrt{4}}{3}$
(B) $\frac{3\sqrt{3}}{4}$
(C) $\frac{2\sqrt{3}}{4}$
(D) $\frac{\sqrt{3}}{4}$

26. Kathie cuts a wooden cube with volume 8 in half horizontally. The two halves are not glued together to form a rectangular solid or a cube. What is the area of the surfaces of these new solids?

(A) 28
(B) 26
(C) 14
(D) 32



27. In the figure above, the length of diagonal $\overline{PQ}$ of the cube is 6, what is the area of total surfaces of the cube?

28. The three surfaces of a rectangular solid have areas 30, 35, and 42. If the dimensions of the solid are all integers, what is the volume of the solid?

29. Shirley cuts a solid object with six equal square sides in half in all 3 directions. If the volume of the object is 8 cubic inches, what is the area of all surfaces of the separated smaller cubes in square inches?

30. A rectangular container has a length of 9, a width of 7, a height of 9, and a volume of V_1. Which of the following represents the volume of a rectangular container with dimensions length 9, width 7, and height 3 in terms of V_1?

(A) $\frac{2}{3}V_1$

(B) $\frac{1}{2}V_1$

(C) $\frac{1}{3}V_1$

(D) $2V_1$

31. A rectangular container has a length of 9, a width of 6, a height of 7, and a volume of V_1. Which of the following represents the volume of a rectangular container with dimensions length 9, width 2, and height 7 in terms of V_1?

(A) $\frac{2}{3}V_1$

(B) $\frac{1}{2}V_1$

(C) $3V_1$

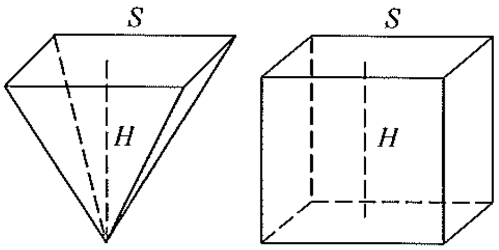
(D) $\frac{1}{3}V_1$

32. Robert cuts a wooden cube with volume 216 in half horizontally. The two halves are glued together to form a rectangular solid, which is not a cube. What is the surface area of this new solid?

(A) 216
(B) 252
(C) 108
(D) 512

33. Stephanie cuts wooden cube with volume 8 in half horizontally. The two halves are glued together to form a rectangular solid, which is not a cube. What is the surface area of this new solid?

(A) 28
(B) 26
(C) 14
(D) 18



34. In the figure above, one container is a pyramid with a square base of side S units and height $H = 9$ units and another container is a rectamgular prism with a square base of side S units. If the pyramid is filled with sand, and then the sand is poured into the empty prism container, what will be the depth of the sand in the prism container?

(A) $\frac{1}{3}$

(B) 2

(C) 3

(D) $3\frac{1}{2}$

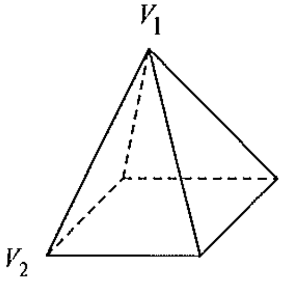
35. Water is poured into a cylindrical tank at a speed of π cubic inches per minute. If its radius is π inches and its height is π inches, how many minutes will it take to fill up the tank?

(A) π^4

(B) π^3

(C) $\pi^{\frac{3}{2}}$

(D) π^2



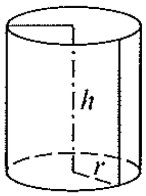
36. In the figure above, the pyramid has a square base and four congruent triangular surfaces. If the height of the pyramid is $\sqrt{5.5}$ feet and each side of the base is 5 feet, what is the length of segment $\overline{V_1V_2}$?

(A) $3\sqrt{2}$

(B) $2\sqrt{3}$

(C) $2\sqrt{2}$

(D) $5.5\sqrt{3}$



Note: Figure above not drawn to scale.

37. In the figure above, the surface area, without the surface areas of the top and bottom bases with r, of the cylinder with height h is 20π. Which of the following is the volume of this cylinder?

(A) $100\pi h$

(B) $10\pi h$

(C) $\frac{10\pi}{h}$

(D) $\frac{100\pi}{h}$

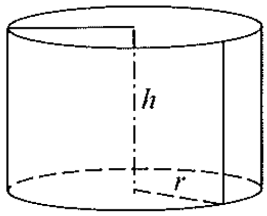
38. The sides of a rectangular solid have lengths $4s$, $5s$, and $7s$. Which of the following is the entire surface area of the rectangular solid, in terms of s ?

(A) $83s^2$

(B) $176s^2$

(C) $166s^2$

(D) $186s^2$



Note: Figure above not drawn to scale.

39. In the figure above, the surface area, without the surface areas of the top and bottom bases with radius r, of the cylinder with height $h = 5$ is 20π. Which of the following is the volume of this cylinder?

(A) 100π

(B) 10π

(C) 20π

(D) 500π

40. The sides of a rectangular solid have lengths $2s$, $3s$, and $4s$. Which of the following is the entire surface area of the rectangular solid, in terms of s ?

(A) $26s^2$

(B) $62s^2$

(C) $52s^2$

(D) $25s^2$

Data Analysis, Statistics, And Probability

Data Analysis, Statistics, and Probability

1. What is the average (arithmetic mean) of $(2x - 7)$, $(2x - 1)$, and $(5x + 2)$?

(A) $2x - 3$
(B) $3x - 2$
(C) $3x - 1$
(D) $3x + 3$

Solution: Answer: (B)

$$\frac{2x-7+2x-1+5x+2}{3}$$

$$\downarrow$$

$$3x-2$$

$$\Downarrow$$

Answer is (B).

2. What is the average (arithmetic mean) of $(2x + 7)$, $(2x + 4)$ and $(5x - 2)$?

(A) $2x - 3$
(B) $3x - 2$
(C) $3x - 1$
(D) $3x + 3$

Solution: Answer: (D)

$$\frac{2x+7+2x+4+5x-2}{3}$$

$$\downarrow$$

$$3x+3$$

$$\Downarrow$$

Answer is (D).

3. The sides of a quadrilateral have lengths of $3x+5$, $6x - 12$, $9x + 7$, and $2x - 8$. The average (arithmetic mean) length of sides is 23. What is the product of lengths of the greatest side and least side?

Solution: Answer: 104

$$\frac{3x+5+6x-12+9x+7+2x-8}{4}=23$$

$$\downarrow$$

$$20x-8=92,\ 20x=100,\ x=5$$

$$\downarrow$$

$9x+7=9\times5+7=52$ (the greatest one)

$2x-8=2\times5-8=2$ (the smallest one)

$$\Downarrow$$

$$\text{Product}=52\times2=104$$

4. What is the average (arithmetic mean) of $(2a+3)$, $(2a-5)$ and $(5a+2)$?

(A) $3a+2$
(B) $3a$
(C) $2a$
(D) $3a-2$

Solution: Answer: (B)

$$\frac{2a+3+2a-5+5a+2}{3}$$

$$\downarrow$$

$$\frac{9a}{3}=3a$$

$$\Downarrow$$

Answer is (B).

Average of Algebraic Expressions

1. What is the average (arithmetic mean) of $(2x - 7)$, $(2x - 1)$, and $(5x + 2)$?

(A) $2x - 3$
(B) $3x - 2$
(C) $3x - 1$
(D) $3x + 3$

2. What is the average (arithmetic mean) of $(2x + 7)$, $(2x + 4)$ and $(5x - 2)$?

(A) $2x - 3$
(B) $3x - 2$
(C) $3x - 1$
(D) $3x + 3$

3. The sides of a quadrilateral have lengths of $3x+5$, $6x - 12$, $9x + 7$, and $2x - 8$. The average (arithmetic mean) length of sides is 23. What is the product of lengths of the greatest side and least side?

4. What is the average (arithmetic mean) of $(2a+3)$, $(2a-5)$ and $(5a+2)$?

(A) $3a+2$
(B) $3a$
(C) $2a$
(D) $3a-2$

1. Ryan drove from Edison Township to New York City at an average speed of 45 miles per hour. The distance between the two cities is 90 miles. Ryan made the return trip at an average speed of 30 miles per hour. What was his average speed for the round trip?

Solution: Answer: 36

Note: Do not use the wrong way :

$$\frac{\text{Rate}_1+\text{Rate}_2}{2} \text{ or } \frac{\text{Speed}_1+\text{Speed}_2}{2}.$$

$$\underbrace{\text{Average Speed}}_{\text{for entire trip}}=\frac{\text{Total Distance}}{\text{Total Time}}=\frac{\text{Distance}_1+\text{Distance}_2}{\text{Time}_1+\text{Time}_2}$$

$$\begin{cases}\text{Time}_1=\dfrac{90}{45}=2,\ \text{Time}_2=\dfrac{90}{30}=3\\ \text{Distance}_1=\text{Distance}_2=90\end{cases}$$

$$\Downarrow$$

$$\underbrace{\text{Average Speed}}_{\text{for entire trip}}=\frac{90+90}{2+3}=\frac{180}{5}=36$$

2. Mark goes for 40-minute jog every morning. Today Mark's average speed for the first 5 minutes was 6 miles per hour. During his remaining time , he jogged 3 miles. Which of the following is Mark's average speed for his entire jog?

(A) $5\frac{3}{4}$ mph

(B) $4\frac{1}{4}$ mph

(C) $3\frac{3}{4}$ mph

(D) $5\frac{1}{4}$ mph

Solution: Answer: (D)

$$\underbrace{\text{Average speed}}_{\text{for entire trip}}=\frac{\text{Total distance}}{\text{Total time}}$$

$$\Downarrow$$

$$\frac{\frac{5}{60}\times 6+3}{\frac{40}{60}}=\frac{\frac{1}{2}+3}{\frac{2}{3}}=\frac{\frac{7}{2}}{\frac{2}{3}}=\frac{21}{4}$$

$$\Downarrow$$

$$\underbrace{\text{Average speed}}_{\text{for entire trip}}=5\frac{1}{4}$$

3. Shirley walked from her school to a nearby store at an average rate of 60 yards per minute. The distance between the two locations is 600 yards. Shirley made the return trip at an average rate of 30 yards per minute. What was her average rate for the round short trip?

Solution: Answer: 40

Note: Do not use the wrong way :

$$\frac{\text{Rate}_1+\text{Rate}_2}{2} \text{ or } \frac{\text{Speed}_1+\text{Speed}_2}{2}.$$

$$\underbrace{\text{Average Speed}}_{\text{for entire trip}}=\frac{\text{Total Distance}}{\text{Total Time}}=\frac{\text{Distance}_1+\text{Distance}_2}{\text{Time}_1+\text{Time}_2}$$

$$\begin{cases}\text{Time}_1=\dfrac{600}{60}=10,\ \text{Time}_2=\dfrac{600}{30}=20\\ \text{Distance}_1=\text{Distance}_2=600\end{cases}$$

$$\Downarrow$$

$$\underbrace{\text{Average Speed}}_{\text{for entire trip}}=\frac{600+600}{10+20}=\frac{120\cancel{0}}{3\cancel{0}}=40$$

4. Frank drives a car from place P to place Q. From P to Q, the average rate of the car is 50 mph; from Q to P, the average rate is 40 mph. Which of the following statements is true?

(A) The average rate for the entire trip is 45 mph.

(B) The average rate for the entire trip is 40 mph shorter.

(C) The average rate for the entire trip is 50 mph longer.

(D) The average rate for the entire trip is less than 45 mph.

Solution: Answer: (D)

$$\frac{D_1+D_2}{t_1+t_2}=\frac{2D}{\frac{D}{50}+\frac{D}{40}}=\frac{2\cancel{D}}{\cancel{D}\left(\frac{1}{50}+\frac{1}{40}\right)}$$

$$\Downarrow$$

$$\underbrace{\text{Average Rate}}_{\text{for entire trip}}=\frac{2}{\frac{40+50}{2000}}=\frac{4000}{90}=44\frac{4}{9}<45$$

Average Speed (Rate) in Motion

1. Ryan drove from Edison Township to New York City at an average speed of 45 miles per hour. The distance between the two cities is 90 miles. Ryan made the return trip at an average speed of 30 miles per hour. What was his average speed for the round trip?

2. Mark goes for 40-minute jog every morning. Today Mark's average speed for the first 5 minutes was 6 miles per hour. During his remaining time , he jogged 3 miles. Which of the following is Mark's average speed for his entire jog?

(A) $5\frac{3}{4}$ mph

(B) $4\frac{1}{4}$ mph

(C) $3\frac{3}{4}$ mph

(D) $5\frac{1}{4}$ mph

3. Shirley walked from her school to a nearby store at an average rate of 60 yards per minute. The distance between the two locations is 600 yards. Shirley made the return trip at an average rate of 30 yards per minute. What was her average rate for the round short trip?

4. Frank drives a car from place P to place Q. From P to Q, the average rate of the car is 50 mph; from Q to P, the average rate is 40 mph. Which of the following statements is true?

(A) The average rate for the entire trip is 45 mph.

(B) The average rate for the entire trip is 40 mph shorter.

(C) The average rate for the entire trip is 50 mph longer.

(D) The average rate for the entire trip is less than 45 mph.

1. If the sum of 5 numbers is between 42 and 47, then the average (arithmetic mean) of the five numbers could be which of the following?

(A) 9.4

(B) 8.4

(C) 8.3

(D) 9.3

Solution: Answer: (D)

$$\underbrace{\frac{42}{5}=8.4}_{\text{The least average}} \quad , \quad \underbrace{\frac{47}{5}=9.4}_{\text{The greatest average}}$$

$$\downarrow$$

$$8.4 < x < 9.4$$

$$\Downarrow$$

$$8.4 < \boxed{9.3} < 9.4$$

2. If the sum of 3 numbers is between 62 and 65, then the average (arithmetic mean) of the three numbers could be which of the following?

(A) $22\frac{1}{3}$

(B) $21\frac{1}{5}$

(C) $21\frac{2}{3}$

(D) $20\frac{1}{3}$

Solution: Answer: (B)

$$\underbrace{\frac{62}{3}=20\frac{2}{3}}_{\text{The least average}} \quad , \quad \underbrace{\frac{65}{3}=21\frac{2}{3}}_{\text{The greatest average}}$$

$$\downarrow$$

$$20\frac{2}{3} < x < 21\frac{2}{3}$$

$$\Downarrow$$

$$20\frac{2}{3} < \boxed{21\frac{1}{5}} < 21\frac{2}{3}$$

3. If the sum of 4 numbers is between 42 and 46, then the average (arithmetic mean) of the four numbers could be which of the following?

(A) 10.5

(B) 10.4

(C) 11.4

(D) 11.5

Solution: Answer: (C)

$$\underbrace{\frac{42}{4}=10.5}_{\text{The least average}} \quad , \quad \underbrace{\frac{46}{4}=11.5}_{\text{The greatest average}}$$

$$\downarrow$$

$$10.5 < x < 11.5$$

$$\Downarrow$$

$$10.5 < \boxed{11.4} < 11.5$$

Possible Average

1. If the sum of 5 numbers is between 42 and 47, then the average (arithmetic mean) of the five numbers could be which of the following?

(A) 9.4

(B) 8.4

(C) 8.3

(D) 9.3

2. If the sum of 3 numbers is between 62 and 65, then the average (arithmetic mean) of the three numbers could be which of the following?

(A) $22\frac{1}{3}$

(B) $21\frac{1}{5}$

(C) $21\frac{2}{3}$

(D) $20\frac{1}{3}$

3. If the sum of 4 numbers is between 42 and 46, then the average (arithmetic mean) of the four numbers could be which of the following?

(A) 10.5

(B) 10.4

(C) 11.4

(D) 11.5

1. Robert buys 100 stock shares of RBM, Inc at \$32 each, 120 shares at \$40 each and 80 shares at \$20 each. The average per share cost is?

(A) \$28
(B) \$30
(C) \$32
(D) \$34

Solution: Answer: (C)

$$\text{Average cost} = \frac{\text{Total Cost}}{\text{Total Shares}}$$

$$\begin{cases} 100 \times 32 = 3200 \\ 120 \times 40 = 4800 \\ 80 \times 20 = 1600 \end{cases}$$

$$\downarrow$$

$$\text{Average cost} = \frac{3200 + 4800 + 1600}{100 + 120 + 80}$$

$$\Downarrow$$

$$\frac{96\cancel{00}}{3\cancel{00}} = 32$$

2. Jack buys 120 stock shares of RBM, Inc at \$30 each, 100 shares at \$40 each and 85 shares at \$20 each. The average per share cost is most close to?

(A) \$26.3
(B) \$30.5
(C) \$34.5
(D) \$37.2

Solution: Answer: (B)

$$\text{Average cost} = \frac{\text{Total Cost}}{\text{Total Shares}}$$

$$\begin{cases} 120 \times 30 = 3600 \\ 100 \times 40 = 4000 \\ 85 \times 20 = 1700 \end{cases}$$

$$\downarrow$$

$$\text{Average cost} = \frac{3600 + 4000 + 1700}{120 + 100 + 85}$$

$$\Downarrow$$

$$\frac{9300}{305} \approx 30.5$$

3. Ten students of a class had an average English reading score of 560. The remaining twenty students of the class had an average of 530. What is the average score of the whole class?

(A) 560
(B) 530
(C) 540
(D) 520

Solution: Answer: (C)

$$\frac{10 \times 560 + 20 \times 530}{10 + 20} = 540$$

$$\Downarrow$$

Answer is (C).

4. Shirley, Kathie and Cynthia are on a trip. Shirley drives during the first three hours at an average rate of 48 miles per hour. Kathie drives during the next one hour at an average rate of 52 miles per hour. Cynthia drives for the next four hours at an average rate of 51 miles per hour. They reach their destination after exactly eight hours. Their average rate of the entire trip, in miles per hour, was

Solution: Answer: 50

$$\underbrace{\text{Average speed}}_{\text{for entire trip}} = \frac{\text{Total Distance}}{\text{Total Time}}$$

$$\Downarrow$$

$$\frac{3 \times 48 + 1 \times 52 + 4 \times 51}{3 + 1 + 4} = 50$$

Weighted Average and Average (Arithmetic Mean)

5. The average (arithmetic mean) of the exam scores of a class of m students is 60, and the average of the exam score of the class of n students is 95. When the scores of both classes are combined, the average score is 87. What is the value of $\frac{n}{m}$?

Solution Answer: 27/8, 3.38 or 3.37

$$\frac{60m+95n}{m+n}=87$$

$$\downarrow$$

$$60m+95n=87(m+n)$$

$$\downarrow$$

$$95n-87n=87m-60m$$

$$\Downarrow$$

$$\frac{n}{m}=\frac{27}{8}$$

6. Leslie solved 10 math problems on Tuesday, four times as many math problems on Thursday as on Tuesday, and four less than twice as many math problems on Friday as on Tuesday. What is the average number of math problems Leslie solved per day over the 3 days?

Solution: Answer: 22

$$\frac{10+4\times10+2\times10-4}{3}$$

$$\Downarrow$$

$$\frac{66}{3}=22$$

7. Jennifer solved r Biology problems on Tuesday, four times as many Biology problems on Thursday as on Tuesday, and r less than twice as many Biology problems on Friday as on Tuesday. What is the average number of Biology problems Jennifer solved per day over the 3 days?

(A) $6r$
(B) $3r$
(C) $2r$
(D) $5r$

Solution: Answer: (C)

$$\frac{r+4r+2r-r}{3}$$

$$\Downarrow$$

$$\frac{6r}{3}=2r$$

8. Q is a set of numbers. The average (arithmetic mean) of Q is k. If R is a set that is produced by 10 times each number in Q, what is the average of the numbers in R?

(A) $3k$
(B) $5k$
(C) $8k$
(D) $10k$

Solution: Answer: (D)

$$\begin{cases}\dfrac{q_1+q_2+\ldots+q_n}{n}=k\\ \dfrac{10q_1+10q_2+\ldots+10q_n}{n}=\dfrac{10(q_1+q_2+\ldots+q_n)}{n}\end{cases}$$

$$\Downarrow$$

$$10\cdot\frac{(q_1+q_2+\ldots+q_n)}{n}=10k$$

1. Robert buys 100 stock shares of RBM, Inc at $32 each, 120 shares at $40 each and 80 shares at $20 each. The average per share cost is?

(A) $28
(B) $30
(C) $32
(D) $34

2. Jack buys 120 stock shares of RBM, Inc at $30 each, 100 shares at $40 each and 85 shares at $20 each. The average per share cost is most close to?

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(B) $30.5
(C) $34.5
(D) $37.2

3. Ten students of a class had an average English reading score of 560. The remaining twenty students of the class had an average of 530. What is the average score of the whole class?

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(B) 530
(C) 540
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Weighted Average and Average (Arithmetic Mean)

5. The average (arithmetic mean) of the exam scores of a class of m students is 60, and the average of the exam score of the class of n students is 95. When the scores of both classes are combined, the average score is 87. What is the value of $\frac{n}{m}$?

6. Leslie solved 10 math problems on Tuesday, four times as many math problems on Thursday as on Tuesday, and four less than twice as many math problems on Friday as on Tuesday. What is the average number of math problems Leslie solved per day over the 3 days?

7. Jennifer solved r Biology problems on Tuesday, four times as many Biology problems on Thursday as on Tuesday, and r less than twice as many Biology problems on Friday as on Tuesday. What is the average number of Biology problems Jennifer solved per day over the 3 days?

(A) $6r$
(B) $3r$
(C) $2r$
(D) $5r$

8. Q is a set of numbers. The average (arithmetic mean) of Q is k. If R is a set that is produced by 10 times each number in Q, what is the average of the numbers in R?

(A) $3k$
(B) $5k$
(C) $8k$
(D) $10k$

Using Mean Concept to Find Numbers

1. If the average of three distinct positive integers is 80, what is the greatest possible value of one of the integers?

Solution: Answer: 237

$$x+y+z=3\times 80=240$$
$$\downarrow$$
$$1+2+z=240$$
$$\Downarrow$$
$$z=237$$

2. The average (arithmetic mean) of r and s is a, and the average of r, s and q is b. What is the value of q?

(A) $2a-3b$

(B) $3b-2a$

(C) $2b-3a$

(D) $3b+2a$

Solution: Answer: (B)

$$\begin{cases} \dfrac{r+s+q}{3}=b \\ \downarrow \\ q=3b-(r+s) \end{cases}$$

$$\begin{cases} \dfrac{r+s}{2}=a \\ r+s=2a \end{cases}$$
$$\Downarrow$$
$$q=3b-2a$$

3. The average (arithmetic mean) of a list of 15 scores is 79. If one of the scores is removed, the average of the remaining scores is 81. What is the score that was removed?

Solution: Answer: 51

Create an equation.

$$\downarrow$$
$$\frac{15\times 79-x}{14}=81$$
$$\downarrow$$
$$1185-1134=x$$
$$\Downarrow$$
$$x=51$$

4. The average (arithmetic mean) of a list of 25 scores is 89. If one of the scores is removed, the average of the remaining scores is 88. What is the score that was removed?

Solution: Answer: 113

Create an equation.

$$\downarrow$$
$$\frac{25\times 89-x}{24}=88$$
$$\downarrow$$
$$2225-x=88\times 24$$
$$\downarrow$$
$$2225-2112=x$$
$$\Downarrow$$
$$x=113$$

Using Mean Concept to Find Numbers

5. If the average of three different positive integers is 80, what is the smallest possible difference of two of the integers?

Solution: Answer: 1

$$\underbrace{\frac{x+y+z}{3}=80}_{\text{Superfluous information}}$$

$$\Downarrow$$

$$x-y=2-1=1$$

6. The average (arithmetic mean) of r and s is π, and the average of r, s and q is $\sqrt{\pi}$. What is the value of q?

(A) $2\pi-3\sqrt{\pi}$
(B) $3\sqrt{\pi}-2\pi$
(C) $2\pi+3\sqrt{\pi}$
(D) $3\sqrt{\pi}+2\pi$

Solution: Answer: (B)

$$\begin{cases}\dfrac{r+s+q}{3}=\sqrt{\pi}\\ \downarrow\\ q=3\sqrt{\pi}-(r+s)\end{cases}$$

$$\begin{cases}\dfrac{r+s}{2}=\pi\\ r+s=2\pi\end{cases}$$

$$\Downarrow$$

$$q=3\sqrt{\pi}-2\pi$$

7. A class of 20 college students had an average (arithmetic mean) of 90 points on a mathematics exam out of a possible 100. If 10 of the students had a perfect score, what was the average score for the remaining students?

(A) 80
(B) 82
(C) 73
(D) 78

Solution: Answer: (A)

$$\frac{20\times 90-10\times 100}{10}$$

$$\Downarrow$$

$$\frac{800}{10}=80$$

8. A class of 22 students in a high school had an average (arithmetic mean) of 85 points on a physics exam out of a possible 100. If 12 of the students had a 95 point score, what was the average score for the remaining students?

(A) 80
(B) 82
(C) 73
(D) 78

Solution: Answer: (C)

Create an expression.

$$\downarrow$$

$$\frac{22\times 85-12\times 95}{10}=\frac{1870-1140}{10}$$

$$\Downarrow$$

$$\frac{730}{10}=73$$

9. The average (arithmetic mean) of two numbers is MY. If the first number is Y, what is the other number?

(A) $MY - Y$

(B) $2M\left(\frac{1-Y}{Y}\right)$

(C) $M + 2Y$

(D) $Y(2M - 1)$

Solution: Answer: (D)

$$\frac{Y + x}{2} = MY$$

$$\downarrow$$

$$x = 2MY - Y$$

$$\Downarrow$$

$$x = Y(2M - 1)$$

10. The average (arithmetic mean) of two numbers is $M + Y$. If the first number is M, what is the other number?

(A) $MY - Y$

(B) $2MY - M$

(C) $M + 2Y$

(D) $2M\left(\frac{1-Y}{Y}\right)$

Solution: Answer: (C)

$$\frac{M + x}{2} = M + Y$$

$$\downarrow$$

$$x = 2(M + Y) - M$$

$$\downarrow$$

$$x = 2M + 2Y - M$$

$$\Downarrow$$

$$M + 2Y$$

11. Since beginning of this quarter, Jack has taken 6 math exams . If he earns a 92 on the next exam, his exam average (arithmetic mean) will be increased by one point. What is his current exam average?

(A) 85.7

(B) 85.0

(C) 91.4

(D) 92.0

Solution: Answer: (B)

$$\frac{6x + 92}{6 + 1} = x + 1$$

$$\downarrow$$

$$6x + 92 = 7x + 7$$

$$\downarrow$$

$$92 - 7 = 7x - 6x$$

$$\Downarrow$$

$$x = 85$$

12. If the average (arithmetic mean) of r, s, and p is a, which of the following is the average of r, s, p, and q?

(A) $\frac{3a + 3q}{4}$

(B) $\frac{3a + q}{4}$

(C) $\frac{3a - q}{4}$

(D) $\frac{a + q}{4}$

Solution: Answer: (B)

$$\frac{r + s + p}{3} = a$$

$$\downarrow$$

$$r + s + p = 3a$$

$$\Downarrow$$

$$\frac{(r + s + p) + q}{4} = \frac{3a + q}{4}$$

Using Mean Concept to Find Numbers

13. Since beginning of this quarter, Mary has taken 6 history exams. If she earns a 75 on the next exam, her exam average (arithmetic mean) will be decreased by three points. which of the following is her current exam average?

(A) 85.7
(B) 88.6
(C) 96.4
(D) 96.0

Solution: Answer: (D)

$$\frac{6x+75}{6+1}=x-3$$
$$\downarrow$$
$$6x+75=7x-21$$
$$\downarrow$$
$$21+75=7x-6x$$
$$\Downarrow$$
$$x=96$$

14. The average (arithmetic mean) of two numbers is $\frac{M}{Y}$. If the first number is $2M$, what is the other number?

(A) $2M\left(\frac{1-Y}{Y}\right)$
(B) $2MY-M$
(C) $M+2Y$
(D) $Y(2M-1)$

Solution: Answer: (A)

$$\frac{2M+x}{2}=\frac{M}{Y}$$
$$\downarrow$$
$$x=2\frac{M}{Y}-2M$$
$$\downarrow$$
$$x=2M\left(\frac{1}{Y}-1\right)$$
$$\downarrow$$
$$x=2M\left(\frac{1}{Y}-\frac{Y}{Y}\right)$$
$$\Downarrow$$
$$2M\left(\frac{1-Y}{Y}\right)$$

1. If the average of three distinct positive integers is 80, what is the greatest possible value of one of the integers?

2. The average (arithmetic mean) of r and s is a, and the average of r, s and q is b. What is the value of q?

(A) $2a-3b$

(B) $3b-2a$

(C) $2b-3a$

(D) $3b+2a$

3. The average (arithmetic mean) of a list of 15 scores is 79. If one of the scores is removed, the average of the remaining scores is 81. What is the score that was removed?

4. The average (arithmetic mean) of a list of 25 scores is 89. If one of the scores is removed, the average of the remaining scores is 88. What is the score that was removed?

Using Mean Concept to Find Numbers

5. If the average of three different positive integers is 80, what is the smallest possible difference of two of the integers?

6. The average (arithmetic mean) of r and s is π, and the average of r, s and q is $\sqrt{\pi}$. What is the value of q?

(A) $2\pi - 3\sqrt{\pi}$
(B) $3\sqrt{\pi} - 2\pi$
(C) $2\pi + 3\sqrt{\pi}$
(D) $3\sqrt{\pi} + 2\pi$

7. A class of 20 college students had an average (arithmetic mean) of 90 points on a mathematics exam out of a possible 100. If 10 of the students had a perfect score, what was the average score for the remaining students?

(A) 80
(B) 82
(C) 73
(D) 78

8. A class of 22 students in a high school had an average (arithmetic mean) of 85 points on a physics exam out of a possible 100. If 12 of the students had a 95 point score, what was the average score for the remaining students?

(A) 80
(B) 82
(C) 73
(D) 78

9. The average (arithmetic mean) of two numbers is MY. If the first number is Y, what is the other number?

(A) $MY - Y$

(B) $2M\left(\frac{1-Y}{Y}\right)$

(C) $M + 2Y$

(D) $Y(2M - 1)$

10. The average (arithmetic mean) of two numbers is $M + Y$. If the first number is M, what is the other number?

(A) $MY - Y$

(B) $2MY - M$

(C) $M + 2Y$

(D) $2M\left(\frac{1-Y}{Y}\right)$

11. Since beginning of this quarter, Jack has taken 6 math exams . If he earns a 92 on the next exam, his exam average (arithmetic mean) will be increased by one point. What is his current exam average?

(A) 85.7

(B) 85.0

(C) 91.4

(D) 92.0

12. If the average (arithmetic mean) of r, s, and p is a, which of the following is the average of r, s, p, and q?

(A) $\frac{3a+3q}{4}$

(B) $\frac{3a+q}{4}$

(C) $\frac{3a-q}{4}$

(D) $\frac{a+q}{4}$

Using Mean Concept to Find Numbers

13. Since beginning of this quarter, Mary has taken 6 history exams. If she earns a 75 on the next exam, her exam average (arithmetic mean) will be decreased by three points. which of the following is her current exam average?

(A) 85.7

(B) 88.6

(C) 96.4

(D) 96.0

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(A) $2M\left(\frac{1-Y}{Y}\right)$

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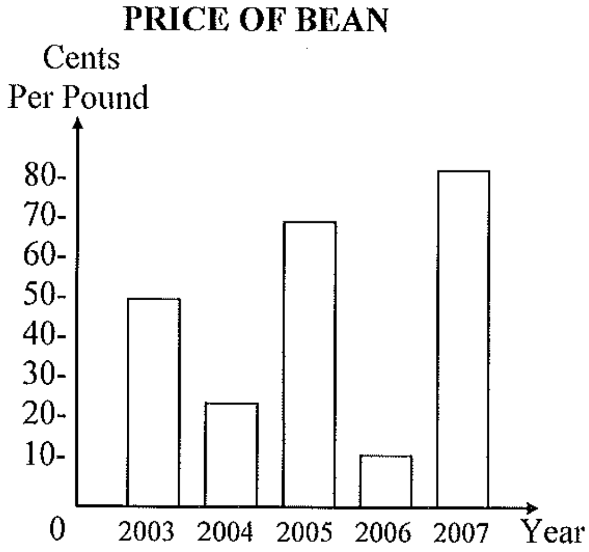
(C) $M + 2Y$

(D) $Y(2M - 1)$



1. According to the graph above, between which two consecutive years was there the greatest change in the price of bean?

(A) 2005 and 2006
(B) 2003 and 2004
(C) 2004 and 2005
(D) 2006 and 2007

Solution: Answer: (D)

Pay attention to the word "change".

⇓

Answer is (D).

2. According to the graph above, between which two consecutive years was there the second greatest change in the price of bean?

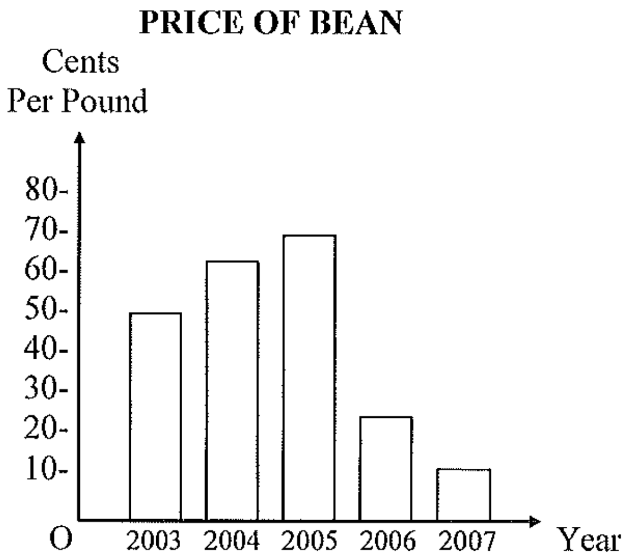
(A) 2005 and 2006
(B) 2003 and 2004
(C) 2004 and 2005
(D) 2006 and 2007

Solution: Answer: (A)

Pay attention to the word "change".

⇓

Answer is (A).



3. According to the graph above, between which two consecutive years was there the least change in the price of bean?

(A) 2005 and 2006
(B) 2003 and 2004
(C) 2005 and 2004
(D) 2006 and 2007

Solution: Answer: (C)

Pay attention to the word "change".

⇓

Answer is (C).

4. According to the graph above, between which two consecutive years was there the greatest change in the price of bean?

(A) 2005 and 2007
(B) 2006 and 2005
(C) 2005 and 2004
(D) 2006 and 2007

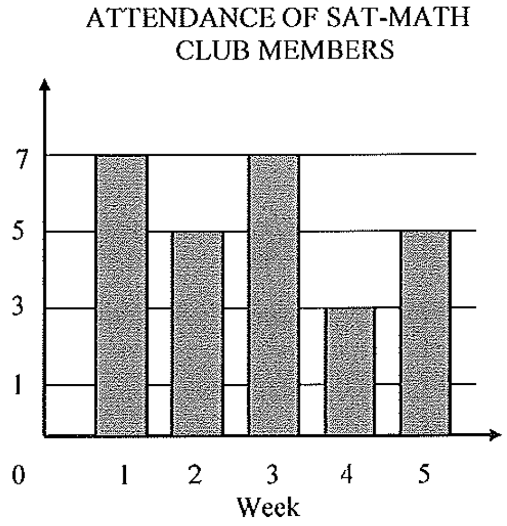
Solution: Answer: (B)

Pay attention to the word "change".

⇓

Answer is (B).

Note: Years of choice (A) are not consecutive.



5. The bar graph above shows attendance of members at meetings of an SAT-math club over 5 weeks. All but four members of the club attended the meeting in the 4th week. No members left or joined the SAT-math club through the 5-week period. What fraction of the members attended the meeting in the 5th week?

(A) $\frac{1}{3}$

(B) $\frac{5}{6}$

(C) $\frac{5}{7}$

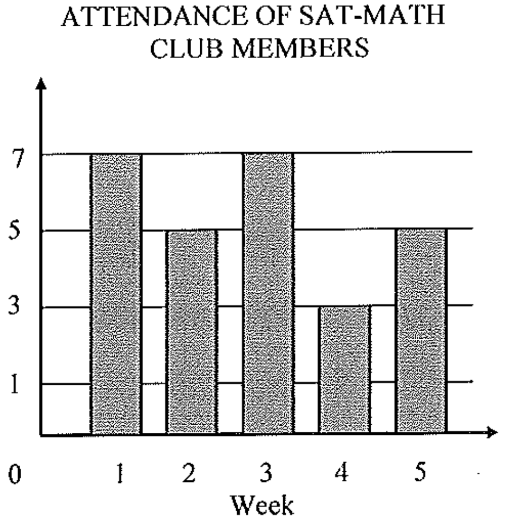
(D) $\frac{1}{2}$

Solution: Answer: (C)

Total members $= 7$

$\Downarrow$

$\text{Fraction}_{5^{th}} = \frac{5}{7}$



6. The bar graph above shows attendance of members at meetings of an SAT-math club over 5 weeks. All but four members of the club attended the meeting in the 4th week. No members left or joined the SAT-math club through the 5-week period. What fraction of the members attended the meeting in the second week?

(A) $\frac{1}{3}$

(B) $\frac{5}{7}$

(C) $\frac{5}{6}$

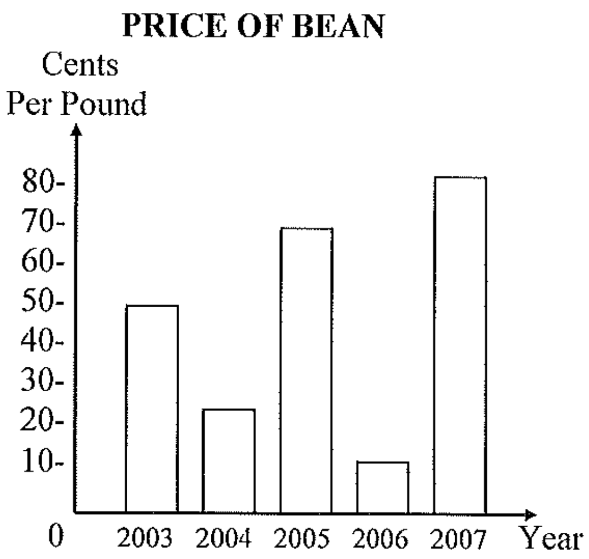
(D) $\frac{1}{2}$

Solution: Answer: (B)

Total members $= 7$

$\Downarrow$

$\text{Fraction}_{2^{nd}} = \frac{5}{7}$



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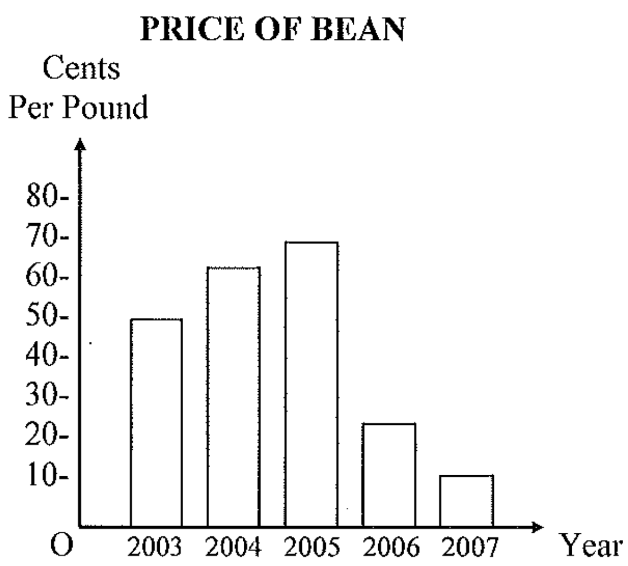

3. According to the graph above, between which two consecutive years was there the least change in the price of bean?

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(D) 2006 and 2007

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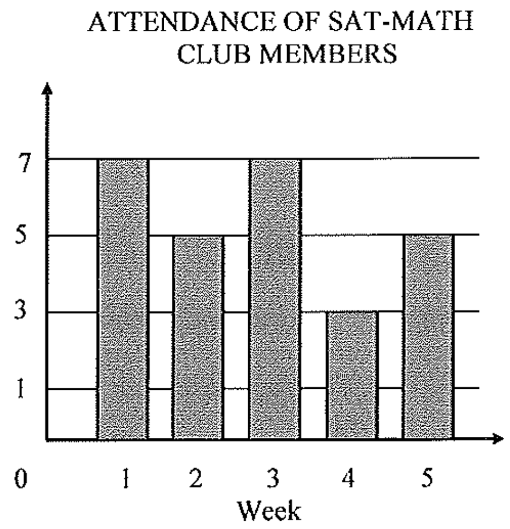
(D) 2006 and 2007



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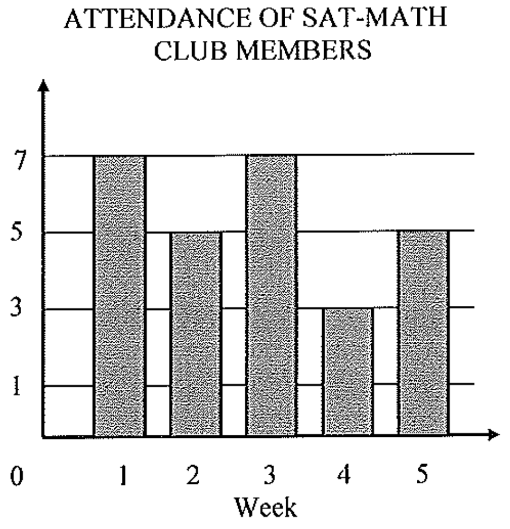
(B) $\frac{5}{6}$

(C) $\frac{5}{7}$

(D) $\frac{1}{2}$



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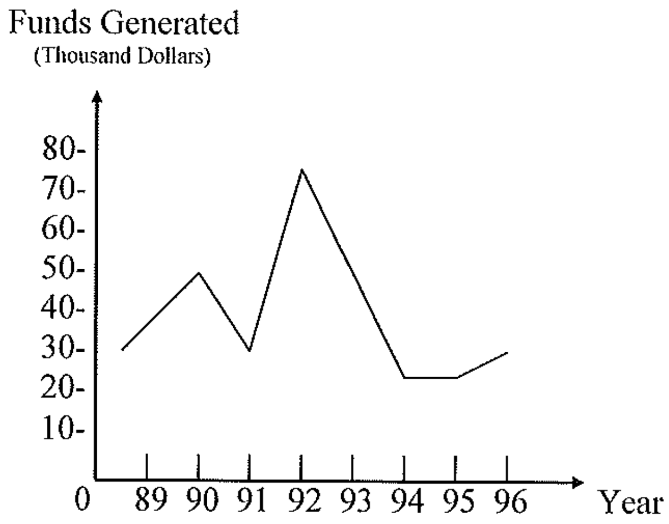
(A) $\frac{1}{3}$

(B) $\frac{5}{7}$

(C) $\frac{5}{6}$

(D) $\frac{1}{2}$

Funds Generated By Glob Disaster Foundation



1. According to the graph above, the greatest increase in funds occurred between which two consecutive years?

(A) 94 and 95
(B) 91 and 92
(C) 92 and 94
(D) 89 and 90

Solution: Answer: (B)

Pay attention to the word "increase".

⇓

Answer is (B).

2. According to the graph above, the least change in funds occurred between which two consecutive years?

(A) 95 and 96
(B) 91 and 92
(C) 92 and 94
(D) 94 and 95

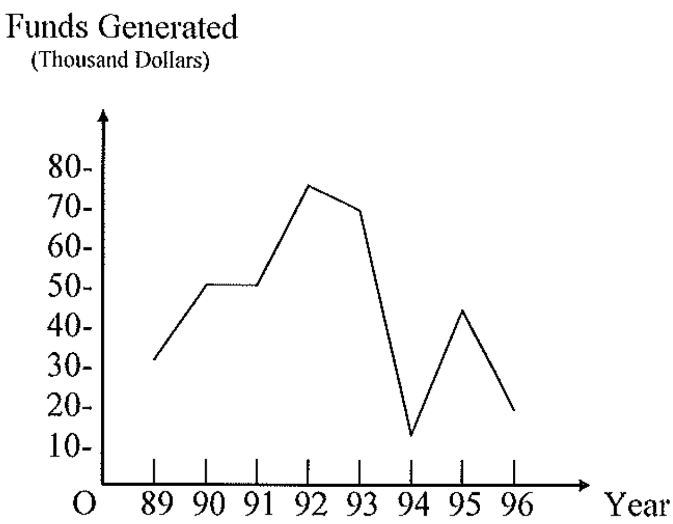
Solution: Answer: (D)

Pay attention to the word "change".

⇓

Answer is (D).

Funds Generated By Glob Disaster Foundation



3. According to the graph above, the greatest decrease in funds occurred between which two consecutive years?

(A) 94 and 95
(B) 91 and 92
(C) 93 and 94
(D) 89 and 90

Solution: Answer: (C)

Pay attention to the word "decrease".

⇓

Answer is (C).

4. According to the graph above, the greatest increase in funds occurred between which two consecutive years?

(A) 94 and 95
(B) 91 and 92
(C) 93 and 94
(D) 89 and 90

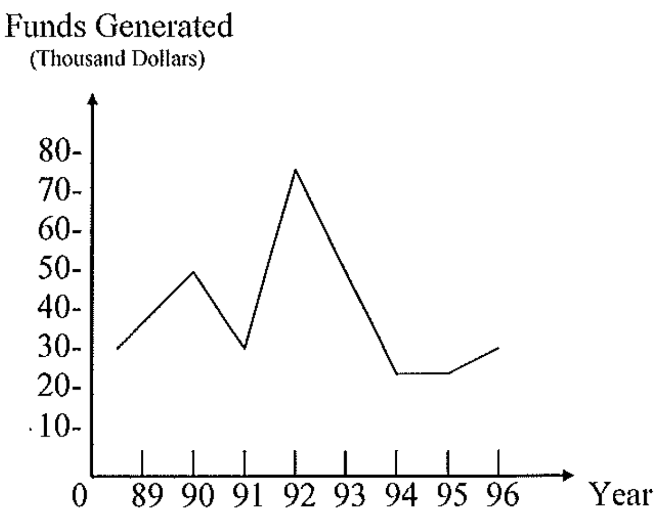
Solution: Answer: (A)

Pay attention to the word "increase".

⇓

Answer is (A).

Funds Generated By Glob Disaster Foundation



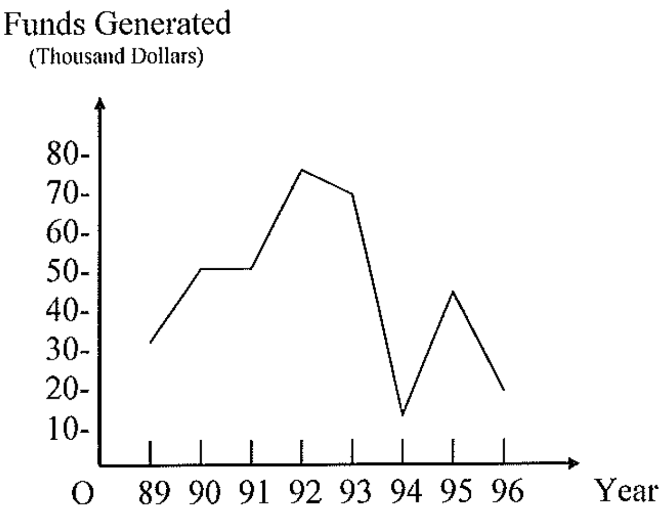
1. According to the graph above, the greatest increase in funds occurred between which two consecutive years?

(A) 94 and 95
(B) 91 and 92
(C) 92 and 94
(D) 89 and 90

2. According to the graph above, the least change in funds occurred between which two consecutive years?

(A) 95 and 96
(B) 91 and 92
(C) 92 and 94
(D) 94 and 95

Funds Generated By Glob Disaster Foundation



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(A) 94 and 95
(B) 91 and 92
(C) 93 and 94
(D) 89 and 90

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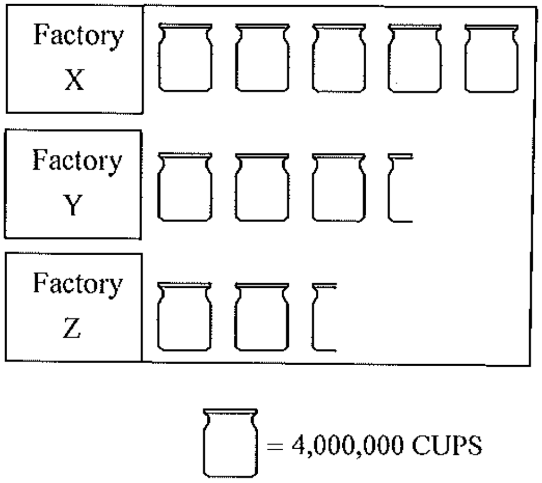
(A) 94 and 95
(B) 91 and 92
(C) 93 and 94
(D) 89 and 90

CUP PRODUCTION IN YEAR 2008



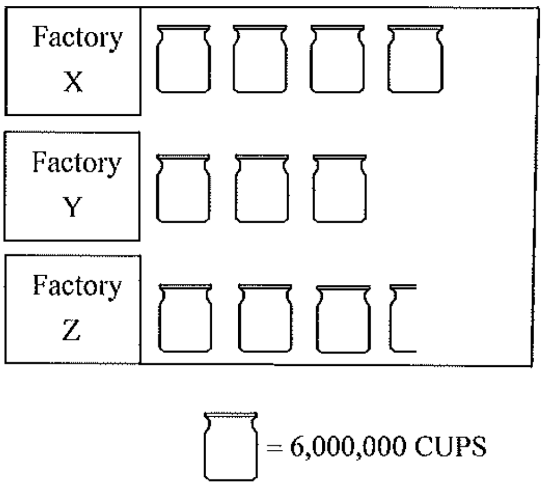
1. According to the chart above, Factory Z produced how many less cups in year 2008 than Factory Y did?

(A) 4,600,000
(B) 8,000,000
(C) 3,000,000
(D) 4,000,000

Solution: Answer: (D)

$$Y = 3.5 \times 4 = 14$$
$$Z = 2.5 \times 4 = 10$$
$$\downarrow$$
$$Y - Z = 14 - 10 = 4$$
$$\Downarrow$$
$$4 \times 10^6 = 4,000,000$$

CUP PRODUCTION IN YEAR 2008



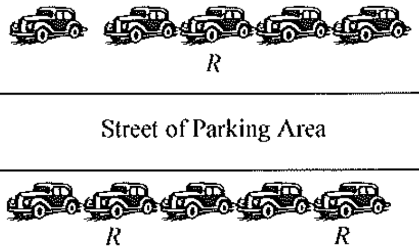
2. According to the chart above, what is the ratio of the cups produced by Factory X in year 2008 to the cups by Factory Y?

(A) $\frac{3}{4}$
(B) $\frac{4}{5}$
(C) $\frac{4}{3}$
(D) $\frac{5}{4}$

Solution: Answer: (C)

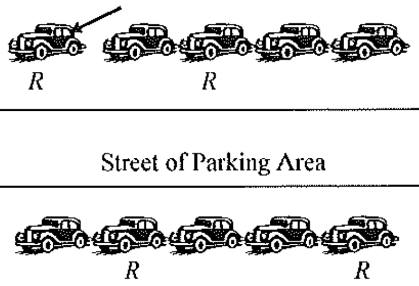
$$\frac{X}{Y} = \frac{4}{3}$$

Pictographs



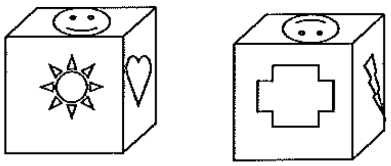
3. The figure above shows five cars on each side of a street of a parking area. No two cars directly across from each other on opposite side of the street are the same color and no two cars next to each other on the same side of the street are the same color. The cars labeled *R* represent red cars. How many of the rest cars can be red cars?

Solution: Only this car Answer: 1



⇓

Answer is 1.



4. The figure above shows two views of the same cube. If the 6th surface of the cube contains a ▽, what is on the surface opposite the surface with the ☼?

(A) ⚡

(B) ☺

(C) ▽

(D) ⬡

Solution: Answer: (D)

(1) The shape ▽ must be on the bottom surface.

(2) Try to rotate one view of two views of the cube.

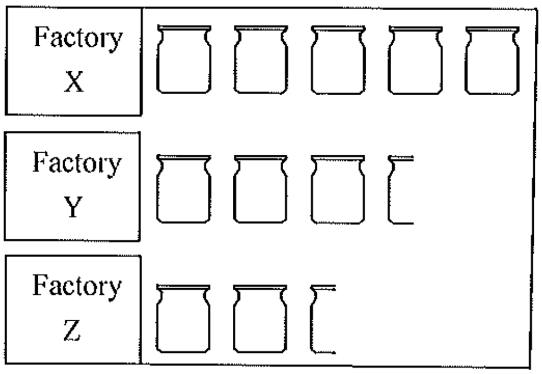
↓

⬡ is on the surface opposite the surface with the ☼.

⇓

Answer is (D).

CUP PRODUCTION IN YEAR 2008

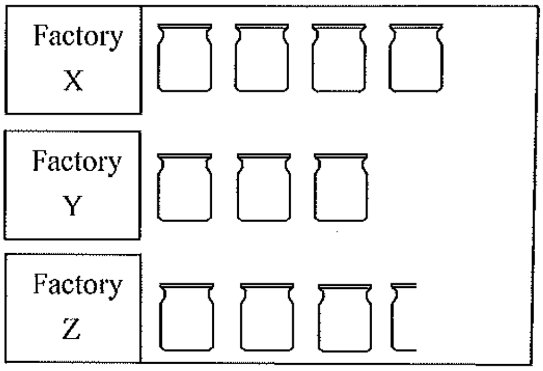


= 4,000,000 CUPS

1. According to the chart above, Factory Z produced how many less cups in year 2008 than Factory Y did?

(A) 4,600,000
(B) 8,000,000
(C) 3,000,000
(D) 4,000,000

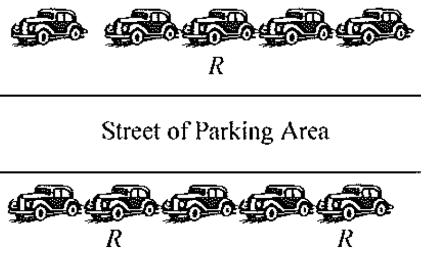
CUP PRODUCTION IN YEAR 2008



= 6,000,000 CUPS

2. According to the chart above, what is the ratio of the cups produced by Factory X in year 2008 to the cups by Factory Y?

(A) $\frac{3}{4}$
(B) $\frac{4}{5}$
(C) $\frac{4}{3}$
(D) $\frac{5}{4}$



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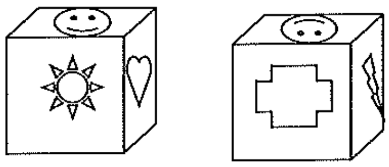

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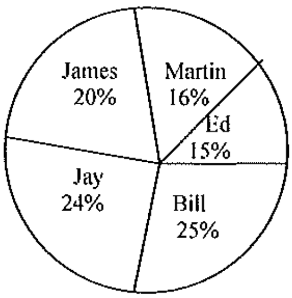
(A) 

(B) 

(C) 

(D) 

PERCENTS OF TOTAL DOLLAR SALES



1. The pie graph above shows each salesperson's sales as a percent of the total dollar sales. Five salespersons sold cars for a total of \$3,600,000 over 6-month period. What is the sum of Ed and Bill sales?

(A) 900,000
(B) 540,000
(C) 1,440,000
(D) 720,000

Solution: Answer: (C)

$$15\% + 25\% = 40\%$$

$$\downarrow$$

$$36 \times 4 = 144 = 1.44m$$

$$\downarrow$$

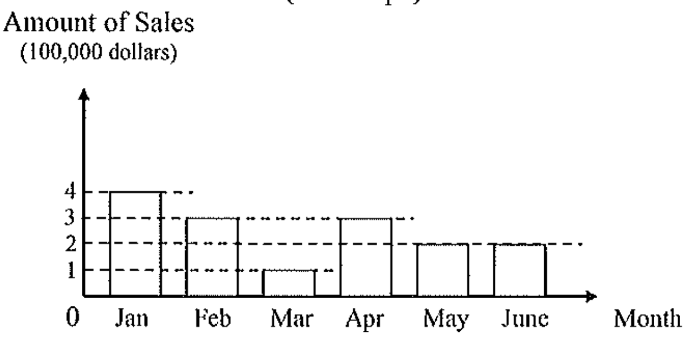
$$1.44m = 1.44 \times 10^6$$

$$\Downarrow$$

$$1,440,000$$

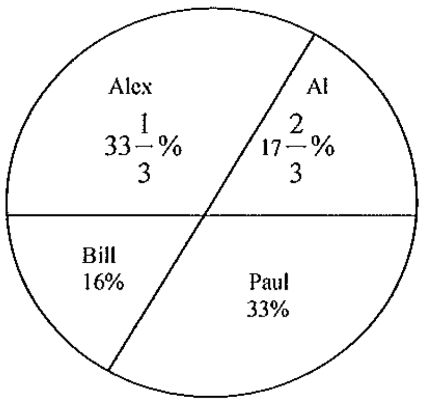
ONE SALESPERSON'S SALES

(Bar Graph)



PERCENTS OF TOTAL DOLLAR SALES

(Pie Graph)



2. In the two graphs above, the bar graph shows the sales of one salesperson over the 6 months and the pie graph shows each salesperson's sales as a percent of the total dollar sales. Four salespersons sold cars for a total of \$4,500,000 over 6-month period. The sales of which salesperson are represented by the bar graph?

(A) Paul
(B) Alex
(C) Bill
(D) Al

Solution: Answer: (B)

(1) $4+3+1+3+2+2=15$

(2) $$\frac{15 \cdot \cancel{10^5}}{45 \cdot \cancel{10^5}} = \frac{1}{3}$$

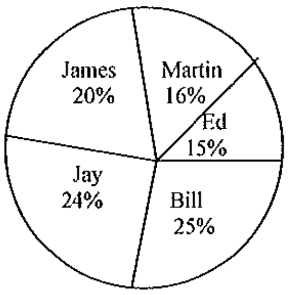
$$\downarrow$$

$$\frac{1}{3} \times 100\% = \frac{100}{3}\% = 33\frac{1}{3}\%$$

$$\Downarrow$$

Answer is (B) Alex.

PERCENTS OF TOTAL DOLLAR SALES



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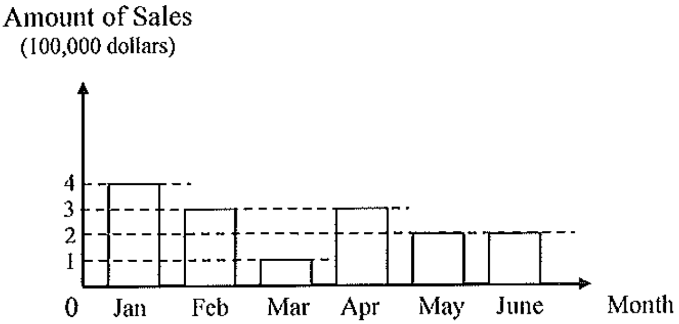
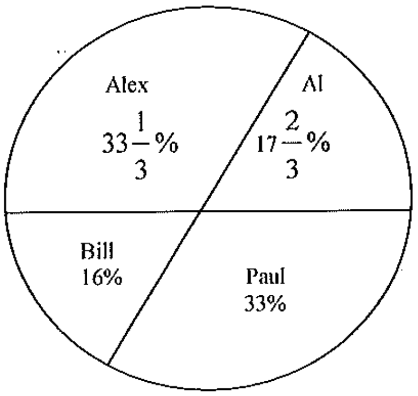
ONE SALESPERSON'S SALES
(Bar Graph)

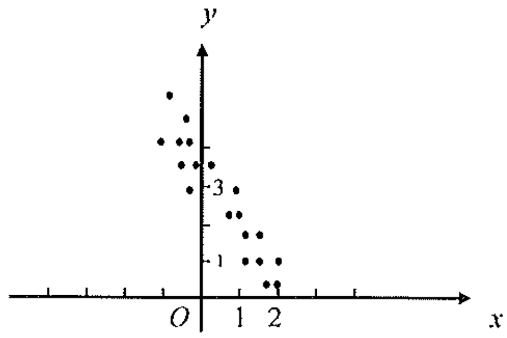


PERCENTS OF TOTAL DOLLAR SALES
(Pie Graph)



2. In the two graphs above, the bar graph shows the sales of one salesperson over the 6 months and the pie graph shows each salesperson's sales as a percent of the total dollar sales. Four salespersons sold cars for a total of $4,500,000 over 6-month period. The sales of which salesperson are represented by the bar graph?

(A) Paul
(B) Alex
(C) Bill
(D) Al



1. Which of the lines described by the following equations best fits these points?

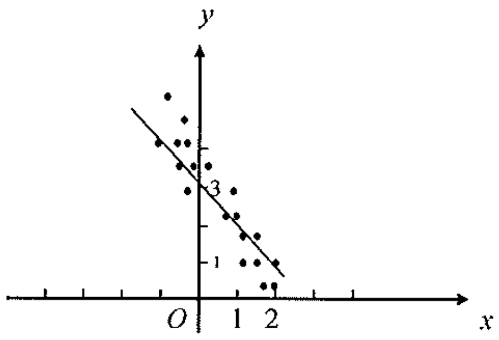
(A) $y = -x + 5$

(B) $y = x + 3$

(C) $y = x + 5$

(D) $y = -x + 3$

Solution: Answer: (D)



Draw a line roughly. Because of the direction of the line, the slope of the line is negative. So omit the equations with positive slope. Then (A), (D) and (E) are left.

$$\begin{cases} (1)\ x = 0 \rightarrow\ y = 3 \\ (2)\ x = 2 \rightarrow y = 1 \end{cases}$$

Use plug-in method.

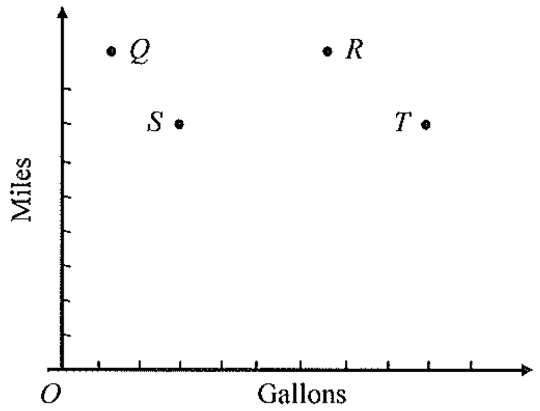
$\downarrow$

$y = -x + 3$ best fits these points.

$\Downarrow$

Answer is (D)

Note: This is not a scatterplot type question. But the figure is like one.

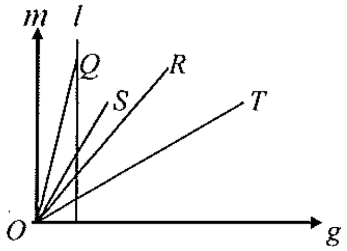


2. In the scatter plot above, Brian took 4 distinct motorcycles to test. The motorcycles are identified as Q, R, S, and T as shown. If the motorcycles are arranged in order from best mileage per gallon to worst mileage per gallon, which of the following should the correct order be?

(A) Q, S, R, T (B) T, R, S, Q (C) Q, T, R, S

(D) Q, R, S, T

Solution: Answer: (A)



Draw 4 line segments from point O to the 4 points and draw line l crossing the 4 segments as shown. The 4 intersections have 4 pairs of g and m coordiates. The 4 g values are the same and the m values are different. The g values represent the amount of gallons and m values represent the amount of miles.

$\downarrow$

$$\text{Slope} = \frac{\text{Change of miles}}{\text{Change of gallons}} = \text{miles per gallon}$$

$$\text{Slope} = \frac{m}{g}$$

$\downarrow$

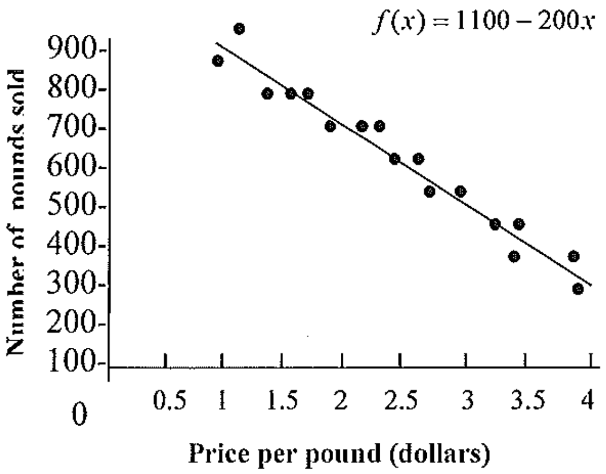
Comparing the 4 slopes

The order from the greatest to the least:

Q, S, R, T

$\Downarrow$

Answer is (A).



A supermarket sells pounds of peanutbutters and decides the price per pound every month. The scatterplot above shows the price and the number of pounds of peanutbutters sold for 17 months with the line of best fit and the equation for the line.

3. For how many of the 17 months shown above was the number of pounds of peanutbutters sold less than the amount predicted by the line of best fit?

(A) 11
(B) 9
(C) 6
(D) 8

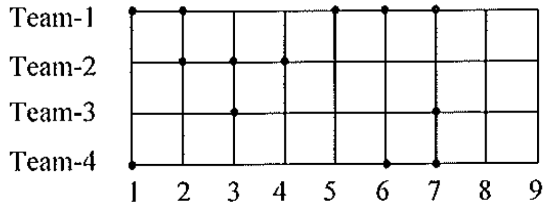
Solution: Answer: (C)

Of the 17 points, 6 lie below the line of best fit.

$\Downarrow$

$N = 6$

Hours Played By Athletes



4. In the figure above, the grid shows the number of hours spent playing by thirteen athletes. Which of the following is true, according to this grid?

(A) Seven athletes played for less than 4 hours each.
(B) Four athletes played 7 hours.
(C) More Team-1 than Team-2 played more than one hour each.
(D) Most athletes were Team-2.

Solution: Answer: (C)

Note: The difference between scatter-plot plane and xy-plane: For scatter-plot plane, each point may have 3 elements. For instance, the elements of the point at left up corner are (value of hours, Team-n, an athlete); for xy-plane, each point has exact 2 elements.

Use plug-in method.

$\Downarrow$

Answer is (C).

5. In the figure above, the grid shows the number of hours spent playing by thirteen athletes. Which of the following is true, according to this grid?

(A) Seven athletes played for at most 3 hours each.
(B) More Team-2 than Team-3 played more than two hours each.
(C) One Team-4 played for 2 hours.
(D) Three athletes played seven hours.

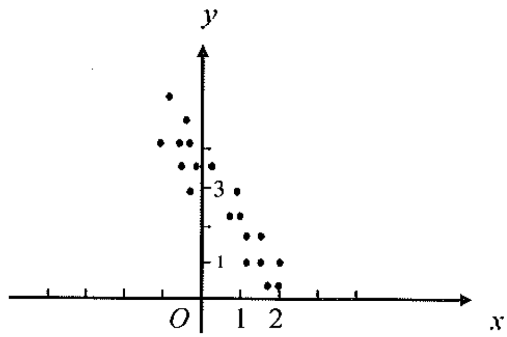
Solution: Answer: (D)

Note: The difference between scatter-plot plane and xy-plane: For scatter-plot plane, each point may have 3 elements. For instance, the elements of the point at left up corner are (value of hours, Team-n, an athlete); for xy-plane, each point has exact 2 elements.

Use plug-in method.

$\Downarrow$

Answer = (D)



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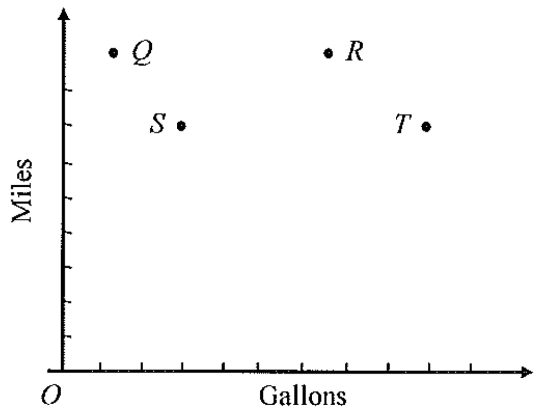
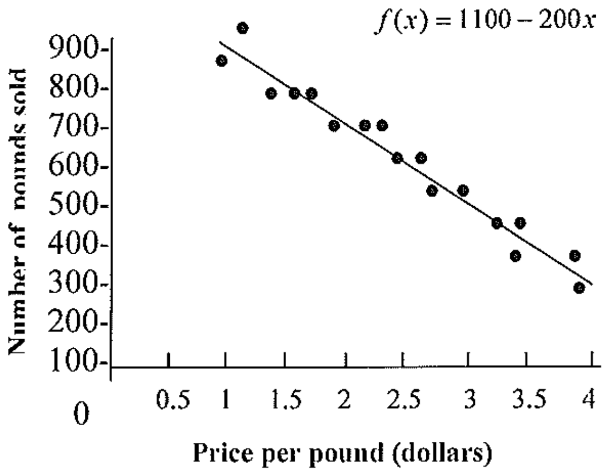


2. In the scatter plot above, Brian took 4 distinct motorcycles to test. The motorcycles are identified as *Q*, *R*, *S*, and *T* as shown. If the motorcycles are arranged in order from best mileage per gallon to worst mileage per gallon, which of the following should the correct order be?

(A) *Q, S, R, T* (B) *T, R, S, Q* (C) *Q, T, R, S*

(D) *Q, R, S, T*



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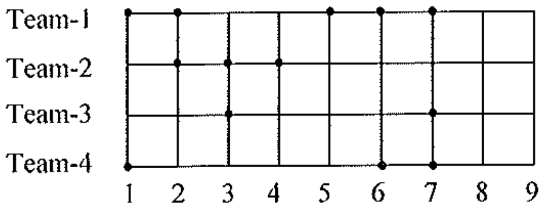

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(C) One Team-4 played for 2 hours.
(D) Three athletes played seven hours.

1. The mean of 3 consecutive odd integers x, y, and z is 9. What is the median of the set $[x, y, z, 13]$?

(A) 11
(B) 8
(C) 7
(D) 10

Solution: Answer: (D)

$$x+y+z=9\times 3=27$$
$$x+(x+2)+(x+4)=27,$$
$$x=7$$
$$x=7, x+2=9, x+4=11$$
$$\text{Set}=[7,\ 9,\ 11,\ 13]$$
$$\Downarrow$$
$$\text{Median}=\frac{9+11}{2}=10$$

2. The mean of 3 consecutive even integers x, y, and z is 8. What is the median of the set $[x, y, z, 6]$?

(A) 11
(B) 7
(C) 8
(D) 10

Solution: Answer: (B)

$$x+y+z=8\times 3=24$$
$$x+(x+2)+(x+4)=24$$
$$x=6$$
$$x=6, x+2=8, x+4=10$$
$$\text{Set}=[6,\ 8,\ 10,\ 6]$$
$$\downarrow$$
$$\text{Reorder: } 6,\ 6,\ 8,\ 10$$
$$\Downarrow$$
$$\text{Median}=\frac{6+8}{2}=7$$

3. The mean of 3 consecutive odd integers x, y, and z is 9. What is the mode of the set $[x, y, z, 7]$?

(A) 11
(B) 8
(C) 7
(D) 10

Solution: Answer: (C)

$$x+y+z=9\times 3=27$$
$$x+(x+2)+(x+4)=27$$
$$x=7$$
$$x=7, x+2=9, x+4=11$$
$$\text{Set}=[7,\ 9,\ 11,\ 7]$$
$$\Downarrow$$
$$\text{Mode}=7$$

4. What is the mean of 4^4, 4^5, 4^6 and 4^7 ?

(A) $4^3(21+4^2)$
(B) 85×4^3
(C) 1360
(D) 4444

Solution: Answer: (B)

$$\frac{4^4+4^5+4^6+4^7}{4}$$
$$\downarrow$$
$$4^3+4^4+4^5+4^6$$
4^3 is a common factor.
$$\downarrow \text{ factoring}$$
$$4^3(1+4+4^2+4^3)$$
$$\downarrow$$
$$4^3(5+16+64)$$
$$\Downarrow$$
$$4^3\times 85$$

5. What is the mean of 4^3, 4^4, 4^5 and 4^{15} ?

(A) $4^3(19+4^2)$
(B) 75×4^3
(C) 1360
(D) $4^2+4^3+4^4+4^{14}$

Solution: Answer: (D)

$$\frac{4^3+4^4+4^5+4^{15}}{4}$$
$$\downarrow$$
$$\frac{4^3}{4}+\frac{4^4}{4}+\frac{4^5}{4}+\frac{4^{15}}{4}$$
$$\Downarrow$$
$$4^2+4^3+4^4+4^{14}$$

6. If the mean of seven consecutive odd integers is n, what is the median of these seven integers?

(A) 1
(B) n
(C) 5
(D) n - 1

Solution: Answer: (B)

Directly use the conclusion:

If the number of total terms of odd or even consecutive integers is odd, median = mean.

$\Downarrow$

Mean = Median = n

7. There are integers 1, 3, 6, -1, 2, 5, 11, 9, 10. Which of the following is its median?

(A) 4
(B) 4.5
(C) 5
(D) 5.5

Solution: Answer: (C)

Increasing order:

-1, 1, 2, 3, 5, 6, 9, 10, 11

$\Downarrow$

Median = 5

8. If the mean of seventy-one consecutive odd integers is $n - 1$, what is the median of these seventy-one integers?

(A) 1
(B) n
(C) 5
(D) $n - 1$

Solution: Answer: (D)

Directly use the conclusion:

If the number of total terms of odd or even consecutive integers is odd, then median = mean.

$\Downarrow$

Mean = Median = $n - 1$

9. Let M1 be the median and M2 be the mode of the following set of numbers, 2, 5, 6, 4, 1, 2, 9, and 2. What is the mean of M1 and M2?

(A) 3
(B) 2.5
(C) 2
(D) 4

Solution: Answer: (B)

Order : 1, 2, 2, 2, 4, 5, 6, 9

$\downarrow$

$$\begin{cases} \text{Median} = \dfrac{2+4}{2} = 3 \\ \text{Mode} = 2 \end{cases}$$

$\Downarrow$

$$\text{Mean} = \frac{\text{M1} + \text{M2}}{2} = \frac{3+2}{2} = 2.5$$

Chart is shown below for the questions 10, 11, and 12.

Data Value	1	2	3	4	5
Frequency Value	1	2	2	3	7

10. What is the mode of the frequency distribution shown above?

Solution: Answer: 5

The value of mode or median always comes from data values, not from frequency values.

$\Downarrow$

Mode = 5

11. What is the median of the frequency distribution shown above?

Solution: Answer: 4

Data value: 1, 2, 2, 3, 3, 4, 4, 4 5, 5, 5, 5, 5, 5, 5

Data Frequency: 1 2 2 3 7

$\downarrow$

There are 15 values at total.

8^{th} number is at the middle position.

$\Downarrow$

The value of 8th term is median = 4

12. What is the average of the median and mode of the frequency distribution shown above?

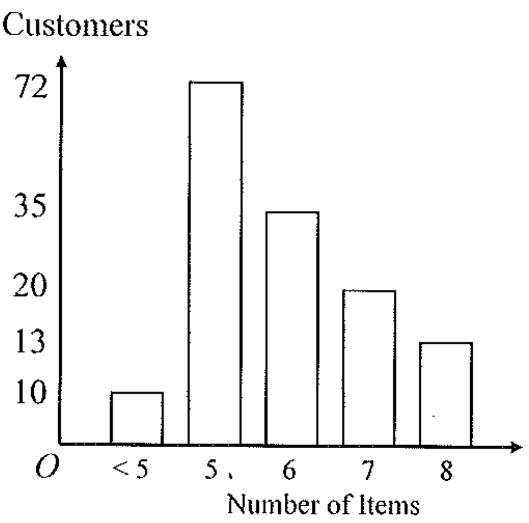
Solution: Answer: 4.5

$$\text{Average} = \frac{\text{Median} + \text{Mode}}{2}$$

$\Downarrow$

$$\frac{5+4}{2} = 4.5$$

ITEM AND CUSTOMER INFORMATION



13. The bar graph above shows the number of items that 150 customers purchased from a department store. What is the median of items purchased per customer?

(A) 4
(B) 5
(C) 6
(D) 7

Solution: Answer: (B)

The questioh asks "median of items", not "median of customers".

↓

Customers is frequency.

↓

Data value:	< 5	5	6	7	8
Frequency:	10	72	35	20	13
	82		68		

150 is an even number of values. (82+68)

↓

The values of 75th and 76th positions are 5.

⇓

$$\text{Median} = \frac{5+5}{2} = 5$$

14. If six numers are 128, 121, 123, 121, 127, 133, which of the following is their correct relationship?

(A) mode > median < mean
(B) mode > median > mean
(C) mode < median > mean
(D) mode < median < mean

Solution: Answer: (D)

Reorder the numbers first.

121, 121, 123, 127, 128, 133

↓

$$\begin{cases} \text{median} = \dfrac{123+127}{2} = 125 \\ \text{mean} = \dfrac{121+121+123+127+128+133}{6} = 125.5 \\ \text{mode} = 121 \end{cases}$$

⇓

Their relationship: mode < median < mean

15. The sides of a quadrilateral have lengths of $3x+5$, $6x-12$, $9x+7$, and $2x-8$. The mean of the lengths of sides is 23. What is the greatest absolute value of the difference between the lengths of any two sides?

Solution: Answer: 50

$$\frac{3x+5+6x-12+9x+7+2x-8}{4} = 23$$

↓

$$20x-8=92,\ 20x=100,\ x=5$$

↓

$$\begin{cases} 9x+7 = 9\times5+7 = 52 \text{ (greatest)} \\ 2x-8 = 2\times5-8 = 2 \text{ (smallest)} \end{cases}$$

⇓

The greatest difference = $52-2=50$

16. For which of the following sets of numbers is the median greater than the mean?

(A) [1, 2, 3, 4, 4]
(B) [1, 2, 3, 4, 5]
(C) [2, 3, 4, 5, 6]
(D) [3, 4, 5, 6, 8]

Solution: Answer: (A)

If the number of total terms of consecutive integers is odd, then its median equals its average.

$$\text{Median of } [1,\ 2,\ 3,\ 4,\ 5] = 3$$

$$\Downarrow$$

$$\text{Average of } \left[1,\ 2,\ 3,\ 4,\ \underset{\neq 5 \text{ and } < 5}{4}\right] < 3$$

17. For which of the following sets of numbers is the median less than the mean?

(A) [1, 2, 3, 4, 4]
(B) [1, 2, 3, 4, 5]
(C) [3, 4, 5, 6, 8]
(D) [3, 4, 5, 6, 7]

Solution: Answer: (C)

If the number of total terms of consecutive integers is odd, then its median is equal to its average.

$$\text{Median of } [3,\ 4,\ 5,\ 6,\ 7] = 5$$

$$\Downarrow$$

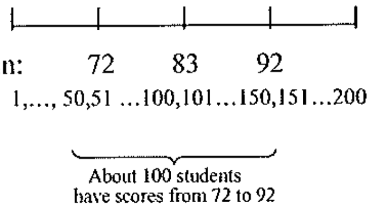
$$\text{Average of } \left[3,\ 4,\ 5,\ 6,\ \underset{\neq 7 \text{ and } > 7}{8}\right] > 5$$

Mean	78
Median	83
Lower quartile	72
Upper quartile	92
Standard Deviation	9

18. The statistical information above provides a summary of chemistry scores of 200 students at North Ridge High School. Approximate 100 of the students in the statistical information have chemistry scores

(A) from 68 to 91.
(B) less than 72.
(C) less than 99.
(D) from 72 to 92.

Solution: Answer: (D)



Note: 72 represents lower quartile;
92 represents upper quartile.

19. Which of the following lists of numbers has the least standard deviation?

(A) 4, 4, 13 (B) 14, 14, 14 (C) 6, 4, 8 (D) 5, 7, 9

Solution: Answer: (B)

Let x_a = average of numbers in a number set.

$$x_a = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

A deviation is the difference between a data value and the average (mean) the data set.

$$x_n - x_a$$

Standard deviation definition:

$$\sigma = \sqrt{\frac{(x_1 - x_a)^2 + (x_2 - x_a)^2 + \cdots (x_n - x_a)^2}{n}}$$

↓

If a choice has the least sum of squares of deviations, that choice has the least standard deviation.

↓

$$x_a = \frac{14 + 14 + 14}{3} = 14$$

$$(14-14)^2 + (14-14)^2 + (14-14)^2 = 0$$

⇓

Answer = (B)

20. The median of a list of 139 consecutive integers is 73. What is the greatest integer in the list?

Solution: Answer: 142

$139 - 1 = 138$

(One number position for 73)

$138 \div 2 = 69$

…, 73, …

↑ ↑

69 numbers 69 numbers

⇓

$73 + 69 = \boxed{142}$

21. In a set of 5 numbers, which of the following must not affect the median?

(A) Increasing each number by 5
(B) Decreasing each number by 5
(C) Tripling each number
(D) Decreasing the least number only

Solution: Answer: (D)

Directly use the conclusion: If the number of values is odd, then decreasing the least value or increasing the greatest value does not affect median.

22. The median of a list of 39 consecutive integers is 23. What is the greatest integer in the list?

Solution: Answer: 42

$39 - 1 = 38$

(One number position for 23)

$38 \div 2 = 19$

…, 23, …

↑ ↑

19 numbers 19 numbers

⇓

$23 + 19 = 42$

23. In a set of 5 integers, which of the following must not affect the median?

(A) Increasing the smallest integer only
(B) Decreasing the largest integer only
(C) Increasing the second smallest integer only
(D) Increasing the largest integer only

Solution: Answer: (D)

Directly use the conclusion: If the number of values is odd, then decreasing the least value or increasing the greatest value does not affect median.

ITEM AND CUSTOMER INFORMATION

Number of Items	Number of Customers
8	13
7	20
6	35
5	72
Less than 5	10

24. The chart above shows the number of items that 150 customers purchased from a department store. What is the median of items purchased per customer?

Solution: Answer: 5

$\underbrace{\text{Less than 5}}_{\text{10 customers}}$ $\underbrace{5\cdots5}_{\text{72 customers}}$ $6\cdots6$ $7\cdots7$ $8\cdots8$

35 customers 20 customers 13 customers

75^{th} and 76^{th} among 5's

$$\Downarrow$$

$$\text{median} = \frac{5+5}{2} = 5$$

25. In chart above, what is the arithmetic mean of items purchased per customer?

(A) The mean cannot be determined.

(B) 5

(C) 11

(D) 3

Solution: Answer: (A)

$$\frac{72\cdot5+35\cdot6+20\cdot7+13\cdot8+10\cdot x}{150}$$

$$\Downarrow$$

The mean cannot be determined.

26. In the chart above, what is the mode of items purchased per customer?

(A) 8

(B) 72

(C) 13

(D) 5

Solution: Answer: (D)

$\underbrace{5\cdots5}_{\text{72 customers}}$

5s appare most times.

$$\Downarrow$$

mode = 5

Note: 72 is frequency, not a mode.

ITEM AND CUSTOMER INFORMATION

Number of Items	Number of Customers
8	13
7	20
6	35
5	72
Less than 5	10

27. The chart above shows the number of items that 150 customers purchased from a department store. Which of the following can be determined from the information in the chart?

I. The median of items purchased each customer

II. The mean of items purchased each customer

III. The mode of the number of items purchased each customer

(A) I only

(B) I and III only

(C) III only

(D) II and III only

Solution: Answer: (B)

$\underbrace{\text{Less than 5}}_{\text{10 customers}}$ $\underbrace{5\cdots5}_{\text{72 customers}}$ $6\cdots6$ $7\cdots7$ $8\cdots8$

35 customers 20 customers 13 customers

$$\uparrow$$

$$\text{Median} = \frac{75^{th}\text{ value} + 76^{th}\text{ value}}{2} = \frac{5+5}{2} = 5$$

Mean cannot be determined from the information.

Mode = 5

$$\Downarrow$$

Answer is (B).

Note: Determining who is frequency is important. For this question, frequency is the number of customers.

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(A) 11
(B) 8
(C) 7
(D) 10

2. The mean of 3 consecutive even integers x, y, and z is 8. What is the median of the set $[x, y, z, 6]$?

(A) 11
(B) 7
(C) 8
(D) 10

3. The mean of 3 consecutive odd integers x, y, and z is 9. What is the mode of the set $[x, y, z, 7]$?

(A) 11
(B) 8
(C) 7
(D) 10

4. What is the mean of 4^4, 4^5, 4^6 and 4^7 ?

(A) $4^3(21+4^2)$

(B) 85×4^3

(C) 1360

(D) 4444

5. What is the mean of 4^3, 4^4, 4^5 and 4^{15} ?

(A) $4^3(19+4^2)$

(B) 75×4^3

(C) 1360

(D) $4^2+4^3+4^4+4^{14}$

6. If the mean of seven consecutive odd integers is n, what is the median of these seven integers?

(A) 1
(B) n
(C) 5
(D) n - 1

7. There are integers 1, 3, 6, -1, 2, 5, 11, 9, 10. Which of the following is its median?

(A) 4
(B) 4.5
(C) 5
(D) 5.5

8. If the mean of seventy-one consecutive odd integers is $n - 1$, what is the median of these seventy-one integers?

(A) 1
(B) n
(C) 5
(D) $n - 1$

9. Let M1 be the median and M2 be the mode of the following set of numbers, 2, 5, 6, 4, 1, 2, 9, and 2. What is the mean of M1 and M2?

(A) 3
(B) 2.5
(C) 2
(D) 4

Chart is shown below for the questions 10, 11, and 12.

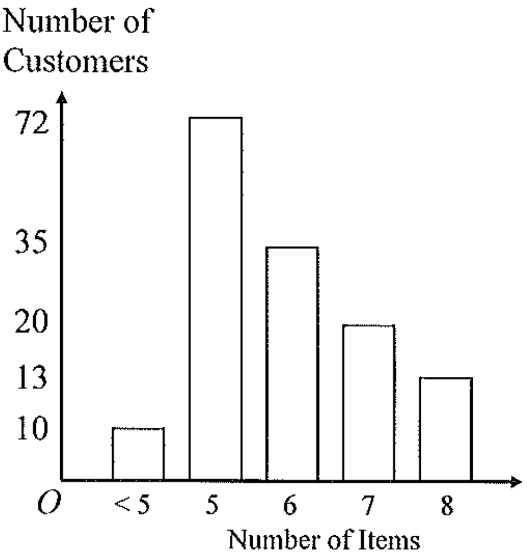
Data Value	1	2	3	4	5
Frequency Value	1	2	2	3	7

10. What is the mode of the frequency distribution shown above?

11. What is the median of the frequency distribution shown above?

12. What is the average of the median and mode of the frequency distribution shown above?

ITEM AND CUSTOMER INFORMATION



13. The bar graph above shows the number of items that 150 customers purchased from a department store. What is the median of items purchased per customer?

(A) 4

(B) 5

(C) 6

(D) 7

14. If six numers are 128, 121, 123, 121, 127, 133, which of the following is their correct relationship?

(A) mode > median < mean

(B) mode > median > mean

(C) mode < median > mean

(D) mode < median < mean

15. The sides of a quadrilateral have lengths of $3x+5$, $6x-12$, $9x+7$, and $2x-8$. The mean of the lengths of sides is 23. What is the greatest absolute value of the difference between the lengths of any two sides?

Median-Mode-Mean-Range- Standard Deviation

16. For which of the following sets of numbers is the median greater than the mean?

(A) [1, 2, 3, 4, 4]
(B) [1, 2, 3, 4, 5]
(C) [2, 3, 4, 5, 6]
(D) [3, 4, 5, 6, 8]

17. For which of the following sets of numbers is the median less than the mean?

(A) [1, 2, 3, 4, 4]
(B) [1, 2, 3, 4, 5]
(C) [3, 4, 5, 6, 8]
(D) [3, 4, 5, 6, 7]

Mean	78
Median	83
Lower quartile	72
Upper quartile	92
Standard Deviation	9

18. The statistical information above provides a summary of chemistry scores of 200 students at North Ridge High School. Approximate 100 of the students in the statistical information have chemistry scores

(A) from 68 to 91.
(B) less than 72.
(C) less than 99.
(D) from 72 to 92.

19. Which of the following lists of numbers has the least standard deviation?

(A) 4, 4, 13 (B) 14, 14, 14 (C) 6, 4, 8 (D) 5, 7, 9

20. The median of a list of 139 consecutive integers is 73. What is the greatest integer in the list?

21. In a set of 5 numbers, which of the following must not affect the median?

(A) Increasing each number by 5
(B) Decreasing each number by 5
(C) Tripling each number
(D) Decreasing the least number only

22. The median of a list of 39 consecutive integers is 23. What is the greatest integer in the list?

23. In a set of 5 integers, which of the following must not affect the median?

(A) Increasing the smallest integer only
(B) Decreasing the largest integer only
(C) Increasing the second smallest integer only
(D) Increasing the largest integer only

ITEM AND CUSTOMER INFORMATION

Number of Items	Number of Customers
8	13
7	20
6	35
5	72
Less than 5	10

24. The chart above shows the number of items that 150 customers purchased from a department store. What is the median of items purchased per customer?

25. In chart above, what is the arithmetic mean of items purchased per customer?

(A) The mean cannot be determined.
(B) 5
(C) 11
(D) 3

26. In the chart above, what is the mode of items purchased per customer?

(A) 8
(B) 72
(C) 13
(D) 5

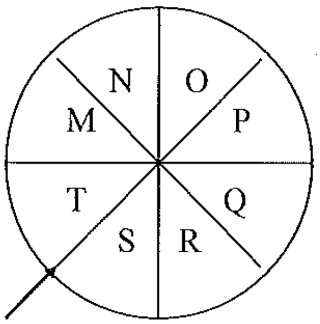
ITEM AND CUSTOMER INFORMATION

Number of Items	Number of Customers
8	13
7	20
6	35
5	72
Less than 5	10

27. The chart above shows the number of items that 150 customers purchased from a department store. Which of the following can be determined from the information in the chart?

I. The median of items purchased each customer
II. The mean of items purchased each customer
III. The mode of the number of items purchased each customer

(A) I only
(B) I and III only
(C) III only
(D) II and III only



1. In the figure above, using the equally spaced spinner, what is the probability of spinning either a P or a Q in one spin?

(A) $\frac{1}{64}$

(B) $\frac{1}{16}$

(C) $\frac{3}{8}$

(D) $\frac{1}{4}$

Solution: Answer: (D)

$$\left[\begin{array}{c} P_P = \frac{1}{8},\ P_Q = \frac{1}{8} \\ \text{"or"} \end{array}\right]$$

$$\Downarrow$$

$$P_{P \text{ or } Q} = P_P + P_Q = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

2. In the figure above, using the equally spaced spinner, what is the probability of spinning an S, an M or an O in one spin?

(A) $\frac{1}{64}$

(B) $\frac{1}{16}$

(C) $\frac{3}{8}$

(D) $\frac{1}{4}$

Solution: Answer: (C)

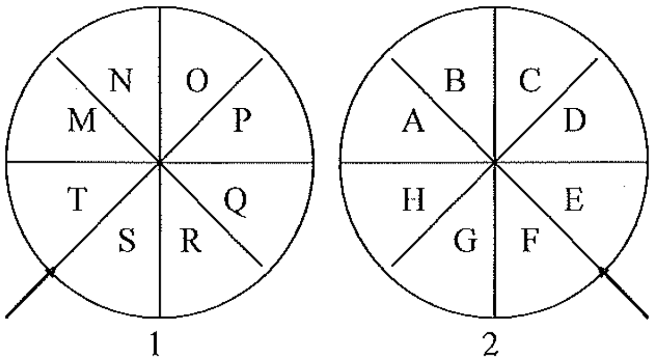
$$\left[\begin{array}{c} P_S = \frac{1}{8},\ P_M = \frac{1}{8},\ P_O = \frac{1}{8} \\ \text{"or"} \end{array}\right]$$

$$\downarrow$$

$$P_{S,\ M \text{ or } O} = P_S + P_M + P_O$$

$$\Downarrow$$

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$



3. In the figure above, what is the probability that equally spaced spinner 1 will stop on N and equally spaced spinner 2 will stop on C if each spinner is given one spin?

(A) $\frac{1}{64}$

(B) $\frac{1}{32}$

(C) $\frac{3}{8}$

(D) $\frac{1}{4}$

Solution: Answer: (A)

$$\left[\begin{array}{c} P_N = \frac{1}{8},\ P_C = \frac{1}{8} \\ \text{"and"} \end{array}\right]$$

$$\Downarrow$$

$$P_{N \text{ and } C} = P_N \times P_C = \frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$$

4. In the figure above, what is the probability that equally spaced spinner 1 will stop on either N or R and equally spaced spinner 2 will stop on C if each spinner is given one spin?

(A) $\frac{1}{64}$

(B) $\frac{1}{32}$

(C) $\frac{3}{8}$

(D) $\frac{1}{4}$

Solution: Answer: (B)

$$\left[\begin{array}{c} P_N = \frac{1}{8},\ P_R = \frac{1}{8},\ P_C = \frac{1}{8} \\ \text{"or"/"and"} \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{c} P_{N \text{ or } R} = P_N + P_R \\ P_{(N \text{ or } R) \text{ and } C} = P_{N \text{ or } R} \times P_C \end{array}\right]$$

$$\Downarrow$$

$$P_{N \text{ or } R} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

$$P_{(N \text{ or } R) \text{ and } C} = \frac{1}{4} \times \frac{1}{8} = \frac{1}{32}$$

5. A green box contains 5 items, of which four are good and the rest are defective, and a yellow box contains 6 items, of which two are good and the rest are defective. An item is drawn randomly from each box. What is the probability that one item is good and one is defective?

(A) $\frac{2}{225}$

(B) $\frac{3}{5}$

(C) 0.65

(D) 0.30

Solution: Answer: (B)

"and" = "×"

↓

$$P_{\text{1 green good and 1 yellow defective}} = \frac{4}{5}\times\frac{4}{6} = \frac{8}{15}$$

$$P_{\text{1 yellow good and 1 green defective}} = \frac{2}{6}\times\frac{1}{5} = \frac{1}{15}$$

Hidden meaning "or" = "+"

↓

$$P_{\text{1 green good and 1 yellow defective}} + P_{\text{1 yellow good and 1 green defective}}$$

⇓

$$\frac{8}{15}+\frac{1}{15}=\frac{3}{5}$$

6. What is the probability of choosing a prime number at random from the set = [0, 2, 4, 9].

(A) 0

(B) 1

(C) $\frac{1}{4}$

(D) $\frac{2}{3}$

Solution: Answer: (C)

Only number 2 is a prime number.

⇓

$$P=\frac{1}{4}$$

7. What is the probability that a prime number is less than 10, given that it is at most 19?

(A) $\frac{5}{9}$

(B) $\frac{5}{8}$

(C) $\frac{1}{2}$

(D) $\frac{2}{8}$

Solution: Answer: (C)

8 prime numbers at most 19:

2, 3, 5, 7, 11, 13, 17, 19

4 of these above less than 10:

2, 3, 5, 7

⇓

$$P=\frac{4}{8}=\frac{1}{2}$$

	Algebra II	Pre-calculus	Calculus	Total
Male	25	35	40	100
Female	35	42	23	100
Total	60	77	63	200

8. The chart above shows the information about math course a group of junior students were enrolled in. If a student is chosen randomly from those who take Calculus or Algebra II, what is the probability that the student belonged to female?

(A) $\frac{58}{100}$

(B) $\frac{65}{100}$

(C) $\frac{77}{100}$

(D) $\frac{75}{100}$

Solution: Answer: (A)

No joint

↓

Type: Conditional probability

⇓

$$P(A/B)=\frac{23}{100}+\frac{35}{100}==\frac{58}{100}$$

9. What is the probability of choosing a prime number at random from the set = [1, 2, 4, 9].

(A) 50%
(B) 25%
(C) $\frac{1}{3}$
(D) 0.6

Solution: Answer: (B)

Only number 2 is a prime number.
Number 1 is not a prime number.

$$\Downarrow$$

$$P = \frac{1}{4} = 25\%$$

10. A pair of dice is tossed 2 times. What is the probability that no 5s or 9s appear as the sum of the sides facing up?

(A) $\frac{7}{9}$
(B) 30%
(C) $\frac{49}{81}$
(D) 0.5

Solution: Answer: (C)

A pair of dice = 2 element sets

$$\downarrow$$

Use Fundamental Counting Principle.

$$\underbrace{6 \times 6 = 36}_{\text{Total passibilities}}$$

1, 2, 3, 4, 5, 6	1,2,3,4,5,6
1 + 4	3+6
4 + 1	6+3
2 + 3	4+5
3 + 2	5+4

4 ways to get 5 and 4 ways to get 9

$$4 + 4 = 8$$

$36 - 8 = 28$ ways to get something else

$$P_{\text{something else}} = \frac{28}{36} = \frac{7}{9}$$

The 2 events occur independently.

$$\Downarrow$$

$$P_{\text{something else}} \times P_{\text{something else}} = \frac{7}{9} \times \frac{7}{9} = \frac{49}{81}$$

11. A pair of dice is tossed 3 times. What is the probability that 5s or 3s appear as the sum of the sides facing up?

(A) 50%
(B) $\frac{1}{6}$
(C) $\left(\frac{5}{6}\right)^3$
(D) $\frac{1}{216}$

Solution: Answer: (D)

A pair of dice = 2 element sets

$$\downarrow$$

Use Fundamental Counting Principle.

$$\underbrace{6 \times 6 = 36}_{\text{Total passibilities}}$$

1, 2, 3, 4, 5, 6	1,2,3,4,5,6
1 + 4	1+2
4 + 1	2+1
2 + 3	
3 + 2	

4 ways to get 5 and 2 ways to get 3

$$4 + 2 = 6$$

$$P_{\text{5s or 3s as sum}} = \frac{6}{36} = \frac{1}{6}$$

The 3 events occur independently.

$$\Downarrow$$

$$P^3_{\text{5s or 3s as sum}} = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

12. Three teams, *A*, *B* and *C*, will play volleyball games. On Tuesday, the team *A* will play the team *B*. On Thursday, the team *B* will play the team *C*. On Saturday, the team *C* will play the team *A*. In each game, every team has a $\frac{1}{3}$ chance to win. What is the probability that the team *A* will win both its games and the team *B* will lose both its games?

(A) $\frac{1}{3}$
(B) $\frac{1}{9}$
(C) 0.31
(D) $\frac{1}{27}$

Solution: Answer: (D)

The question doesn't provide the information about loss and tie. We only can use the information about win.

Team *A* wins its games with team *B* and team *C*. Team *C* wins its game with team *B*.

$$\downarrow$$

Team *A* wins both its games and team *B* loses both its games.

$$\Downarrow$$

$$P = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$$

13. There are 3 teams, A, B and C, will play volleyball games. On Tuesday, the team A will play the team B. On Thursday, the team B will play the team C. On Saturday, the team C will play the team A. In each game, every team has a 50 percent chance to win. What is the probability that team B will lose both its games?

(A) 1

(B) 25%

(C) $\frac{1}{2}$

(D) 0.35

Solution: Answer: (B)

The question doesn't provide the information about loss and tie. We only can use the information about win.

$$\downarrow$$

$$P_{A \text{ to } B,\ B \text{ loses}} = \frac{1}{2},\ P_{B \text{ to } C,\ B \text{ loses}} = \frac{1}{2}$$

"both" = "and"

$$\Downarrow$$

$$P_{(A \text{ to } B,\ B \text{ loses}) \text{ and } (B \text{ to } C,\ B \text{ loses})} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4} = 25\%$$

14. A bag contains 3 yellow balls and 6 balls that are not yellow. What is the probability that the first three balls drawn at random from this bag will not be yellow?

Solution: Answer: 5/21 or .238

The question doesn't point out "with return" or "with replace".

$$\downarrow$$

"without return" or "without replace"

$$\Downarrow$$

$$P = P_1 \times P_2 \times P_3 = \frac{6}{9}\times\frac{5}{8}\times\frac{4}{7} = \frac{5}{21}$$

15. A bag contains 3 green balls and 7 balls that are not green. What is the probability that the first three balls drawn at random from this bag will be green?

Solution: Answer: .008

The question doesn't point out "with return" or "with replace".

$$\downarrow$$

"without return" or "without replace"

$$\Downarrow$$

$$P = \frac{3}{10}\times\frac{2}{9}\times\frac{1}{8} = \frac{1}{120} \approx 0.008$$

16. A circle has diameter 12 and a rectangle that is inside the circle has length 6 and width 1. If one point is chosen at random from the circle, what is the probability that one point is chosen from the rectangle?

(A) 6π

(B) $\frac{5}{6\pi}$

(C) $\frac{1}{6\pi}$

(D) $\frac{6\pi}{5}$

Solution: Answer: (C)

$$A_1 = r^2\pi = 6^2\pi = 36\pi$$

$$A_2 = lw = 1\times 6 = 6$$

$$\Downarrow$$

$$P = \frac{A_1}{A_2} = \frac{6}{36\pi} = \frac{1}{6\pi}$$

x 4 8 5 9 12

17. When a number is selected randomly from the 6 numbers shown above, the probability that the number will be less than 8 is 0.5. Which of the following could be the value of x?

(A) 9
(B) 0
(C) 8
(D) 10

Solution: Answer: (B)

$$\text{probability} = 0.5 = \frac{1}{2} = \frac{3}{6}$$

numbers at least 8

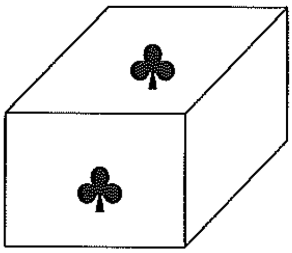
8, 9, and 12

numbers less than 8

$\boxed{0}$ (could be), 4, and 5

$$\Downarrow$$

Answer is (B).



18. In the figure above, a rectangular box is marked with ♣ on exactly two adjacent surfaces. If the box lands so that a ♣ appears on the top surface, what is the probability that the bottom surface does not have a ♣ on it?

(A) 0

(B) $\frac{1}{3}$

(C) 1

(D) 0.5

Solution: Answer: (C)

When ♣ shows on the top, ♣ will never show on the bottom. So the probability that ♣ will not show on the bottom is 1.

$\Downarrow$

Answer is (C).

19. In the figure above, suppose the rectangular box is a cube. It is marked with ♣ on exactly two adjacent surfaces. If the cube lands so that a ♣ appears on the top surface, what is the probability that the bottom surface has a ♣ on it?

(A) 0.5

(B) 0

(C) 1

(D) $\frac{1}{3}$

Solution: Answer: (B)

When ♣ shows on the top, ♣ will never show on the bottom. So the probability that ♣ will not show on the bottom is 0.

$\Downarrow$

Answer is (B).

20. There are 12 cards that are numbered in the order from 0 to 11. If two of the cards selected at random, what is the probability that the two cards have distinct prime numbers?

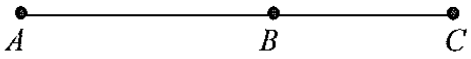
(A) $\frac{15}{66}$ (B) $\frac{5}{33}$ (C) $\frac{2}{3}$ (D) $\frac{1}{9}$

Solution: Answer: (B)

Prime numbers: 2, 3, 5, 7, 11

$\Downarrow$

$$P_{\text{Two-distict prime}} = \frac{5}{\cancel{12}^{3}} \cdot \frac{\cancel{4}}{11} = \frac{5}{33}$$



Note: Figure above not drawn to scale.

21. In the figure above, the length of segment AB is $4x$ and the length of segment BC is $3x$. If a point is chosen at random from segment AC, what is the probability that point will lie on AB?

(A) $\frac{3}{4}$

(B) $\frac{4}{7}$

(C) $\frac{2}{3}$

(D) $\frac{3}{5}$

Solution: Answer: (B)

Total = $4x + 3x = 7x$

$\Downarrow$

$$P_{AB} = \frac{4x}{7x} = \frac{4}{7}$$

22. At a supermarket, $\frac{2}{7}$ of the items are discounted by 70 percent, $\frac{1}{7}$ of the items are discounted by 40 percent, $\frac{3}{14}$ of the items are discounted by 50 percent, and rest items are not discounted. If one item is to be selected randomly, which of the following is the probability that the item is discounted by 40 percent or 50 percent?

(A) $\frac{3}{7}$

(B) $\frac{1}{2}$

(C) $\frac{3}{98}$

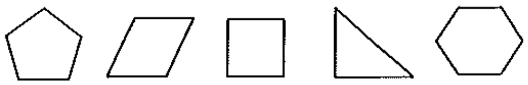
(D) $\frac{3}{49}$

Solution: Answer: (A)

"or" = "+"

$\Downarrow$

$$\frac{1}{7} + \frac{3}{14} = \frac{2+3}{14} = \frac{5}{14}$$



23. In the figures above, if one of them is selected randomly, what is the probability that the number of vertices in the figure will be at least 4 ?

(A) 0.8
(B) $\frac{1}{5}$
(C) $\frac{2}{5}$
(D) 60%

Solution: Answer: (A)

"at least 4" = 4, 5 and 6

4 figures have vertices equal or more than 4.

$\Downarrow$

$$P = \frac{4}{5} = \frac{8}{10} = 0.8$$

24. At a supermarket, $\frac{2}{7}$ of the items are discounted by 70 percent, $\frac{1}{7}$ of the items are discounted by 40 percent, $\frac{3}{14}$ of the items are discounted by 50 percent, and rest items are not discounted. If one item is to be selected randomly, which of the following is the probability that the item is not discounted?

(A) $\frac{3}{7}$ (B) $\frac{1}{2}$ (C) $\frac{5}{14}$ (D) $\frac{3}{49}$

Solution: Answer: (C)

The sum of probabilities of all items = 1.

$\Downarrow$

$$1-\left(\frac{2}{7}+\frac{1}{7}+\frac{3}{14}\right)=1-\frac{9}{14}=\frac{5}{14}$$

25. There are 8 cards that are numbered in the order from 0 to 7. If three of the cards selected at random with return, what is the probability that the three cards are even?

(A) 1 (B) $\frac{1}{8}$ (C) $\frac{3}{2}$ (D) $\frac{1}{4}$

Solution: Answer: (B)

Even numbers: 0, 2, 4, 6

$\Downarrow$

$$P_{\text{Three-even}} = \frac{4}{8}\cdot\frac{4}{8}\cdot\frac{4}{8} = \frac{1}{8}$$

	Algebra II	Pre-calculus	Calculus	Total
Male	25	35	40	100
Female	35	42	23	100
Total	60	77	63	200

26. The chart above shows the information about math course a group of junior students were enrolled in. If a student is chosen randomly from those who take Algebra II, what is the probability that the student chosen is male?

(A) $\frac{35}{100}$
(B) $\frac{25}{100}$
(C) $\frac{25}{60}$
(D) $\frac{75}{100}$

Solution: Answer: (C)

No joint

$\downarrow$

Type: Conditional probability

$\Downarrow$

$$P(A/B) = \frac{25}{60}$$

27. The chart above shows the information about math course a group of junior students were enrolled in. If a student is chosen randomly, what is the probability that the student is male and he takes Pre-calculus ?

(A) $\frac{35}{100}$
(B) $\frac{65}{100}$
(C) $\frac{7}{40}$
(D) $\frac{40}{100}$

Solution: Answer: (C)

Male and Pre-calculus occur together.

A joint exists.

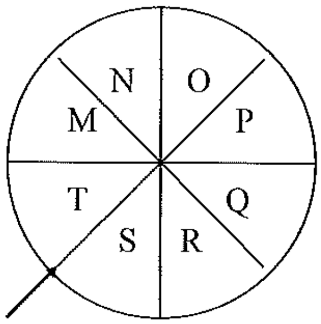
$\downarrow$

Type: Joint probabilty

$\Downarrow$

$$P(A\cap B) = \frac{35}{200} = \frac{7}{40}$$

Grand Total



1. In the figure above, using the equally spaced spinner, what is the probability of spinning either a P or a Q in one spin?

(A) $\frac{1}{64}$

(B) $\frac{1}{16}$

(C) $\frac{3}{8}$

(D) $\frac{1}{4}$

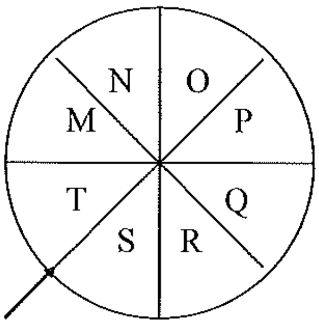
2. In the figure above, using the equally spaced spinner, what is the probability of spinning an S, an M or an O in one spin?

(A) $\frac{1}{64}$

(B) $\frac{1}{16}$

(C) $\frac{3}{8}$

(D) $\frac{1}{4}$



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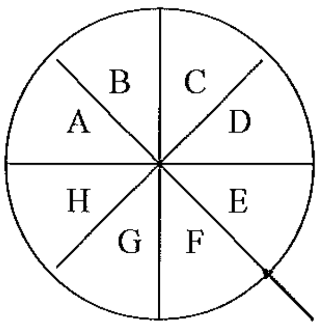


2

3. In the figure above, what is the probability that equally spaced spinner 1 will stop on N and equally spaced spinner 2 will stop on C if each spinner is given one spin?

(A) $\frac{1}{64}$

(B) $\frac{1}{32}$

(C) $\frac{3}{8}$

(D) $\frac{1}{4}$

4. In the figure above, what is the probability that equally spaced spinner 1 will stop on either N or R and equally spaced spinner 2 will stop on C if each spinner is given one spin?

(A) $\frac{1}{64}$

(B) $\frac{1}{32}$

(C) $\frac{3}{8}$

(D) $\frac{1}{4}$

5. A green box contains 5 items, of which four are good and the rest are defective, and a yellow box contains 6 items, of which two are good and the rest are defective. An item is drawn randomly from each box. What is the probability that one item is good and one is defective?

(A) $\frac{2}{225}$

(B) $\frac{3}{5}$

(C) 0.65

(D) 0.30

6. What is the probability of choosing a prime number at random from the set = [0, 2, 4, 9].

(A) 0

(B) 1

(C) $\frac{1}{4}$

(D) $\frac{2}{3}$

7. What is the probability that a prime number is less than 10, given that it is at most 19?

(A) $\frac{5}{9}$

(B) $\frac{5}{8}$

(C) $\frac{1}{2}$

(D) $\frac{2}{8}$

	Algebra II	Pre-calculus	Calculus	Total
Male	25	35	40	100
Female	35	42	23	100
Total	60	77	63	200

8. The chart above shows the information about math course a group of junior students were enrolled in. If a student is chosen randomly from those who take Calculus or Algebra II, what is the probability that the student belonged to female?

(A) $\frac{58}{100}$

(B) $\frac{65}{100}$

(C) $\frac{77}{100}$

(D) $\frac{75}{100}$

9. What is the probability of choosing a prime number at random from the set = [1, 2, 4, 9].

(A) 50%

(B) 25%

(C) $\frac{1}{3}$

(D) 0.6

10. A pair of dice is tossed 2 times. What is the probability that no 5s or 9s appear as the sum of the sides facing up?

(A) $\frac{7}{9}$

(B) 30%

(C) $\frac{49}{81}$

(D) 0.5

11. A pair of dice is tossed 3 times. What is the probability that 5s or 3s appear as the sum of the sides facing up?

(A) 50%

(B) $\frac{1}{6}$

(C) $\left(\frac{5}{6}\right)^3$

(D) $\frac{1}{216}$

12. Three teams, *A, B* and *C*, will play volleyball games. On Tuesday, the team *A* will play the team *B*. On Thursday, the team *B* will play the team *C*. On Saturday, the team *C* will play the team *A*. In each game, every team has a $\frac{1}{3}$ chance to win. What is the probability that the team *A* will win both its games and the team *B* will lose both its games?

(A) $\frac{1}{3}$

(B) $\frac{1}{9}$

(C) 0.31

(D) $\frac{1}{27}$

13. There are 3 teams, *A, B* and *C*, will play volleyball games. On Tuesday, the team *A* will play the team *B*. On Thursday, the team *B* will play the team *C*. On Saturday, the team *C* will play the team *A*. In each game, every team has a 50 percent chance to win. What is the probability that team *B* will lose both its games?

(A) 1

(B) 25%

(C) $\frac{1}{2}$

(D) 0.35

14. A bag contains 3 yellow balls and 6 balls that are not yellow. What is the probability that the first three balls drawn at random from this bag will not be yellow?

15. A bag contains 3 green balls and 7 balls that are not green. What is the probability that the first three balls drawn at random from this bag will be green?

16. A circle has diameter 12 and a rectangle that is inside the circle has length 6 and width 1. If one point is chosen at random from the circle, what is the probability that one point is chosen from the rectangle?

(A) 6π

(B) $\frac{5}{6\pi}$

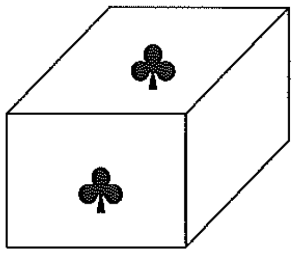
(C) $\frac{1}{6\pi}$

(D) $\frac{6\pi}{5}$

x 4 8 5 9 12

17. When a number is selected randomly from the 6 numbers shown above, the probability that the number will be less than 8 is 0.5. Which of the following could be the value of x?

(A) 9
(B) 0
(C) 8
(D) 10



18. In the figure above, a rectangular box is marked with ♣ on exactly two adjacent surfaces. If the box lands so that a ♣ appears on the top surface, what is the probability that the bottom surface does not have a ♣ on it?

(A) 0

(B) $\frac{1}{3}$

(C) 1

(D) 0.5

19. In the figure above, suppose the rectangular box is a cube. It is marked with ♣ on exactly two adjacent surfaces. If the cube lands so that a ♣ appears on the top surface, what is the probability that the bottom surface has a ♣ on it?

(A) 0.5

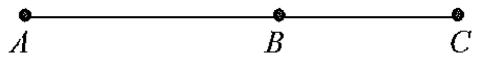
(B) 0

(C) 1

(D) $\frac{1}{3}$

20. There are 12 cards that are numbered in the order from 0 to 11. If two of the cards selected at random, what is the probability that the two cards have distinct prime numbers?

(A) $\frac{15}{66}$ (B) $\frac{5}{33}$ (C) $\frac{2}{3}$ (D) $\frac{1}{9}$



Note: Figure above not drawn to scale.

21. In the figure above, the length of segment AB is $4x$ and the length of segment BC is $3x$. If a point is chosen at random from segment AC, what is the probability that point will lie on AB?

(A) $\frac{3}{4}$

(B) $\frac{4}{7}$

(C) $\frac{2}{3}$

(D) $\frac{3}{5}$

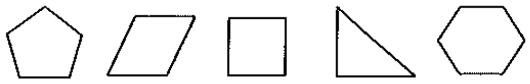
22. At a supermarket, $\frac{2}{7}$ of the items are discounted by 70 percent, $\frac{1}{7}$ of the items are discounted by 40 percent, $\frac{3}{14}$ of the items are discounted by 50 percent, and rest items are not discounted. If one item is to be selected randomly, which of the following is the probability that the item is discounted by 40 percent or 50 percent?

(A) $\frac{3}{7}$

(B) $\frac{1}{2}$

(C) $\frac{3}{98}$

(D) $\frac{3}{49}$



23. In the figures above, if one of them is selected randomly, what is the probability that the number of vertices in the figure will be at least 4 ?

(A) 0.8

(B) $\frac{1}{5}$

(C) $\frac{2}{5}$

(D) 60%

24. At a supermarket, $\frac{2}{7}$ of the items are discounted by 70 percent, $\frac{1}{7}$ of the items are discounted by 40 percent, $\frac{3}{14}$ of the items are discounted by 50 percent, and rest items are not discounted. If one item is to be selected randomly, which of the following is the probability that the item is not discounted?

(A) $\frac{3}{7}$ (B) $\frac{1}{2}$ (C) $\frac{5}{14}$ (D) $\frac{3}{49}$

25. There are 8 cards that are numbered in the order from 0 to 7. If three of the cards selected at random with return, what is the probability that the three cards are even?

(A) 1 (B) $\frac{1}{8}$ (C) $\frac{3}{2}$ (D) $\frac{1}{4}$

	Algebra II	Pre-calculus	Calculus	Total
Male	25	35	40	100
Female	35	42	23	100
Total	60	77	63	200

26. The chart above shows the information about math course a group of junior students were enrolled in. If a student is chosen randomly from those who take Algebra II, what is the probability that the student chosen is male?

(A) $\frac{35}{100}$

(B) $\frac{25}{100}$

(C) $\frac{25}{60}$

(D) $\frac{75}{100}$

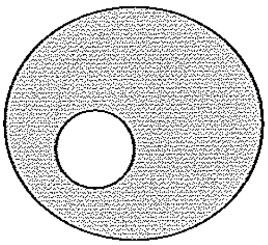
27. The chart above shows the information about math course a group of junior students were enrolled in. If a student is chosen randomly, what is the probability that the student is male and he takes Pre-calculus ?

(A) $\frac{35}{100}$

(B) $\frac{65}{100}$

(C) $\frac{7}{40}$

(D) $\frac{40}{100}$



Note: Figure above not drawn to scale.

1. In the figure above, the large circle has diameter 16 and the small circle has radius 2. If a point is chosen randomly from the large circle, what is the probability that the point chosen will be in the shaded region?

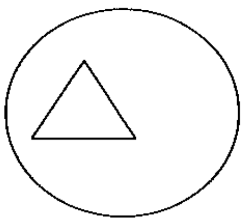
(A) $\frac{1}{16}$
(B) $\frac{1}{8}$
(C) $\frac{2}{9}$
(D) $\frac{15}{16}$

Solution: Answer: (D)

$$\begin{bmatrix} A_{\text{large}} = 8^2\pi = 64\pi \\ A_{\text{small}} = 2^2\pi = 4\pi \end{bmatrix}$$

$$\Downarrow$$

$$P = \frac{A_{\text{large}} - A_{\text{small}}}{A_{\text{large}}} = \frac{64\pi - 4\pi}{64\pi} = \frac{15}{16}.$$



Note: Figure above not drawn to scale.

2. In the figure above, the circle has radius 10 and the triangle that is an equilateral one has length of side 2. If a point is chosen at random in the circle, what is the probability that the point chosen will be in the triangle?

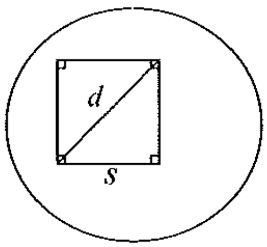
(A) $\frac{\sqrt{3}}{100\pi}$
(B) $\frac{\sqrt{3}}{\pi}$
(C) $\sqrt{3}$
(D) π

Solution: Answer: (A)

$$\begin{bmatrix} A_{\text{circle}} = 10^2\pi = 100\pi \\ A_{\text{triangle}} = \frac{s^2\sqrt{3}}{4} = \frac{2^2\sqrt{3}}{4} = \sqrt{3} \end{bmatrix}$$

$$\Downarrow$$

$$P = \frac{A_{\text{triangle}}}{A_{\text{circle}}} = \frac{\sqrt{3}}{100\pi}$$



Note: Figure above not drawn to scale.

3. In the figure above, the circle has radius $r = 8$ and the square has diagonal $d = 4\sqrt{2}$. If a point is chosen randomly in the circle, what is the probability that the point chosen will be in the square?

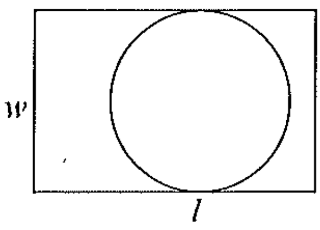
(A) 4π
(B) π
(C) $\frac{1}{4\pi}$
(D) 2π

Solution: Answer: (C)

$$\begin{bmatrix} A_{\text{circle}} = 8^2\pi = 64\pi \\ A_{\text{square}} = s^2 = 4^2 = 16 \end{bmatrix}$$

$$\Downarrow$$

$$P = \frac{A_{\text{square}}}{A_{\text{circle}}} = \frac{16}{64\pi} = \frac{1}{4\pi}$$



Note: Figure above not drawn to scale.

4. In the figure above, the rectangle has length $l = 9$ and width $w = 3$ and the circle has diameter 6. If a point is chosen at random in the rectangle, what is the probability that the point chosen will be from the interior of the rectangle and exterior of the circle?

(A) $\frac{\pi}{3}$
(B) $1 - \frac{\pi}{3}$
(C) 3π
(D) 2π

Solution: Answer: (B)

$$\begin{bmatrix} A_{\text{rectangle}} = l \times w = 9 \times 3 = 27 \\ A_{\text{circle}} = r^2\pi = 3^2\pi = 9\pi \end{bmatrix}$$

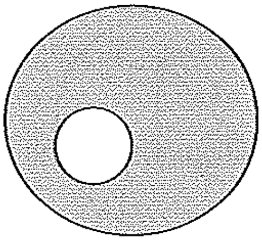
$$\downarrow$$

$$P = \frac{A_{\text{rectangle}} - A_{\text{circle}}}{A_{\text{rectangle}}} = \frac{27 - 9\pi}{27}$$

$$\Downarrow$$

$$\frac{27}{27} - \frac{9\pi}{27} = 1 - \frac{\pi}{3}$$

Geometric Probabilities



Note: Figure above not drawn to scale.

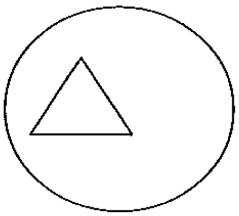
1. In the figure above, the large circle has diameter 16 and the small circle has radius 2. If a point is chosen randomly from the large circle, what is the probability that the point chosen will be in the shaded region?

(A) $\frac{1}{16}$

(B) $\frac{1}{8}$

(C) $\frac{2}{9}$

(D) $\frac{15}{16}$



Note: Figure above not drawn to scale.

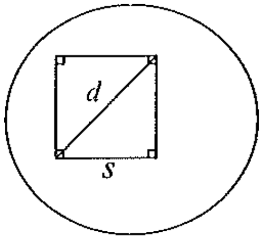
2. In the figure above, the circle has radius 10 and the triangle that is an equilateral one has length of side 2. If a point is chosen at random in the circle, what is the probability that the point chosen will be in the triangle?

(A) $\frac{\sqrt{3}}{100\pi}$

(B) $\frac{\sqrt{3}}{\pi}$

(C) $\sqrt{3}$

(D) π



Note: Figure above not drawn to scale.

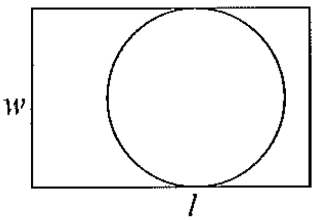
3. In the figure above, the circle has radius $r = 8$ and the square has diagonal $d = 4\sqrt{2}$. If a point is chosen randomly in the circle, what is the probability that the point chosen will be in the square?

(A) 4π

(B) π

(C) $\frac{1}{4\pi}$

(D) 2π



Note: Figure above not drawn to scale.

4. In the figure above, the rectangle has length $l = 9$ and width $w = 3$ and the circle has diameter 6. If a point is chosen at random in the rectangle, what is the probability that the point chosen will be from the interior of the rectangle and exterior of the circle?

(A) $\frac{\pi}{3}$

(B) $1 - \frac{\pi}{3}$

(C) 3π

(D) 2π

1. If Mr. Smith tutors three students, what is the probability that at least one is a girl?

Solution: Answer: 7/8 or .875

Only two states: girls and boys.

Use $T = 2^n$ to get the total possibilities, where n is the number of students.

$$T = 2^3 = 8$$

Method 1

Only one pattern has no girl.

$P = 1 - 1/8 = 7/8 = 0.875$

Method 2

Order doesn't matter.
There are 3 elements.

For "at least one is a girl":

1 position for 1 girl
2 positions for 2 girls
3 positions for 3 girls

Possibilities of "at least one is a girl":

$$\frac{3}{1!} + \frac{3\times 2}{2!} + \frac{3\times 2\times 1}{3!} = 3 + 3 + 1 = 7$$

$$\Downarrow$$

$$P_{\text{At least one girl}} = \frac{7}{8} = 0.875$$

2. If 3 coins are to be flipped, what is the probability that exactly 2 tails will be flipped?

(A) $\frac{3}{8}$

(B) $\frac{5}{8}$

(C) $\frac{7}{8}$

(D) $\frac{1}{4}$

Solution: Answer: (A)

Only two states: tails and heads.

Use $T = 2^n$ to get the total possibilities, where n is the number of coins.

$$T = 2^3 = 8$$

Order doesn't matter.
3 elements, 2 positions

$$\downarrow$$

$$\frac{3\times 2}{2!} = 3$$

$$\Downarrow$$

$$P_{\text{Exact 2 tails}} = \frac{3}{8}$$

3. If 4 coins are to be flipped, what is the probability that exactly 3 heads will be flipped?

(A) $\frac{3}{4}$

(B) $\frac{5}{8}$

(C) $\frac{7}{8}$

(D) $\frac{1}{4}$

Solution: Answer: (D)

Only two states: tails and heads.

Use $T = 2^n$ to get the total possibilities, where n is the number of coins.

$$T = 2^4 = 16$$

Order doesn't matter.
4 elements, 3 positions

$$\downarrow$$

$$\frac{4\times 3\times 2}{3!} = 4$$

$$\Downarrow$$

$$P_{\text{Exact 3 heads}} = \frac{4}{16} = \frac{1}{4}$$

4. If Mr. Bush has three children, what is the probability that at least two are boys?

Solution: Answer: 1/2 or .5

Only two states: girls and boys.

Use $T = 2^n$ to get the total possibilities, where n is the number of children.

$$T = 2^3 = 8$$

Order doesn't matter.
There are 3 elements.

For "at least two are boys":

2 positions for 2 boys
3 positions for 3 boys

Possibilities of "at least two are boys":

$$\frac{3\times 2}{2!} + \frac{3\times 2\times 1}{3!} = 3 + 1 = 4$$

$$\Downarrow$$

$$P_{\text{At least 2 boys}} = \frac{4}{8} = \frac{1}{2}$$

5. What is the probability of getting exact three heads when flipping four coins?

Solution: Answer: 1/4 or .25

Only two states: tails and heads.

Use $T = 2^n$ to get the total possibilities, where n is the number of coins.

$$T = 2^4 = 16$$

Order doesn't matter.

There are 4 elements.

For "exact three heads":

3 positions for 3 heads

Possibilities of "exact three heads":

$$\frac{4 \times 3 \times 2}{3!} = 4$$

$$\Downarrow$$

$$P_{\text{Exact 3 heads}} = \frac{4}{16} = \frac{1}{4}$$

6. What is the probability of getting exact two tails when flipping three coins?

Solution: Answer: 3/8 or .375

Only two states: tails and heads.

Use $T = 2^n$ to get the total possibilities, where n is the number of coins.

$$T = 2^3 = 8$$

Order doesn't matter.

There are 3 elements.

For "exact two tails":

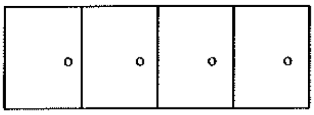
2 positions for 2 tails

Possibilities of "exact two tails":

$$\frac{3 \times 2}{2!} = 3$$

$$\Downarrow$$

$$P_{\text{Exact 2 tails}} = \frac{3}{8}$$



7. The figure above represents four offices, which will be arranged at random to 4 teachers, one teacher per office. If Dr. Smith and Ms. Bush are 2 of the 4 teachers, what is the probability that each of the two teachers will be arranged an office at either end?

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$

Solution: Answer: (B)

(1) Total Possibilities

4 positions, 4 elements

Order does matter.

$$4 \cdot 3 \cdot 2 \cdot 1 = 24$$

(2) Possibilities for the 2 teachers

2 ends 2 middles

$$\boxed{2} \cdot \boxed{1} \cdot \boxed{2} \cdot \boxed{1} = 4$$

or

$$\begin{bmatrix} 1 & 3 & 4 & 2 \\ 1 & 4 & 3 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 4 & 1 \\ 2 & 4 & 3 & 1 \end{bmatrix} \rightarrow 4$$

(1) and (2)

$$\Downarrow$$

(3) $$P = \frac{4}{24} = \frac{1}{6}$$

1. If Mr. Smith tutors three students, what is the probability that at least one is a girl?

2. If 3 coins are to be flipped, what is the probability that exactly 2 tails will be flipped?

(A) $\frac{3}{8}$

(B) $\frac{5}{8}$

(C) $\frac{7}{8}$

(D) $\frac{1}{4}$

3. If 4 coins are to be flipped, what is the probability that exactly 3 heads will be flipped?

(A) $\frac{3}{4}$

(B) $\frac{5}{8}$

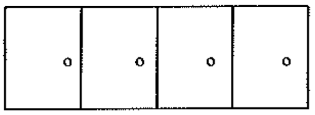
(C) $\frac{7}{8}$

(D) $\frac{1}{4}$

4. If Mr. Bush has three children, what is the probability that at least two are boys?

5. What is the probability of getting exact three heads when flipping four coins?

6. What is the probability of getting exact two tails when flipping three coins?



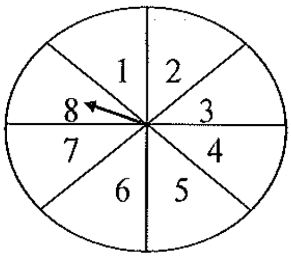
7. The figure above represents four offices, which will be arranged at random to 4 teachers, one teacher per office. If Dr. Smith and Ms. Bush are 2 of the 4 teachers, what is the probability that each of the two teachers will be arranged an office at either end?

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{8}$

(D) $\frac{1}{4}$



Note: Figure above not drawn to scale.

1. In the figure above, Jennifer spins the arrow twice. The fraction m/n is formed, where m is the number of the sector where the arrow stops after the first spin and n is the number of the sector where the arrow stops after the second spin. On every spin, if each of the numbered sectors has an equal chance that the arrow stops on the sector, what is the probability that the different fraction m/n is less than 1?

(A) $\frac{21}{64}$

(B) $\frac{15}{64}$

(C) $\frac{19}{64}$

(D) $\frac{5}{64}$

Solution: Answer: (A)

(1). 2 element sets

↓

Use Fundamental Counting Principle.

↓

Total possibilities $= 8 \times 8 = 64$

(2). $m/n < 1$

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$

$\frac{2}{3}, \left(\frac{2}{4} = \frac{1}{2}\right), \frac{2}{5}, \left(\frac{2}{6} = \frac{1}{3}\right), \frac{2}{7}, \left(\frac{2}{8} = \frac{1}{4}\right)$

$\frac{3}{4}, \frac{3}{5}, \left(\frac{3}{6} = \frac{1}{2}\right), \frac{3}{7}, \frac{3}{8}$

$\frac{4}{5}, \left(\frac{4}{6} = \frac{2}{3}\right), \frac{4}{7}, \left(\frac{4}{8} = \frac{1}{2}\right)$

$\frac{5}{6}, \frac{5}{7}, \frac{5}{8}$

$\frac{6}{7}, \left(\frac{6}{8} = \frac{3}{4}\right)$

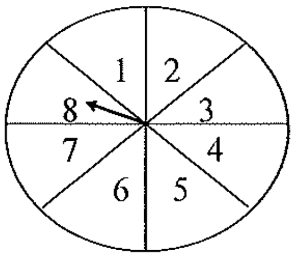
$\frac{7}{8}$

↓

Restricted possibilities for $m/n < 0 = 21$

⇓

(3). $p_{\text{fraction } \frac{m}{n} < 1} = \frac{21}{64}$



Note: Figure above not drawn to scale.

2. In the figure above, Stephanie spins the arrow twice. The fraction m/n is formed, where m is the number of the sector where the arrow stops after the first spin and n is the number of the sector where the arrow stops after the second spin. On every spin, if each of the numbered sectors has an equal chance that the arrow stops on the sector, what is the probability that the fraction m/n is less than $\frac{1}{2}$?

(A) $\frac{17}{64}$

(B) $\frac{15}{64}$

(C) $\frac{3}{16}$

(D) $\frac{5}{64}$

Solution: Answer: (C)

(1) 2 element sets

↓

Use Fundamental Counting Principle.

Total possibilities $= 8 \times 8 = 64$

(2) $\frac{m}{n} < \frac{1}{2}$

$\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$

$\frac{2}{5}, \left(\frac{2}{6} = \frac{1}{3}\right), \frac{2}{7}, \left(\frac{2}{8} = \frac{1}{4}\right)$

$\frac{3}{7}, \frac{3}{8}$

↓

Restricted possibilities = 12

⇓

(3) $p_{\text{fraction } \frac{m}{n} < \frac{1}{2}} = \frac{12}{64} = \frac{3}{16}$

M, N, O, P, Q, R, S

3. A list is composed of all possible 3-letter combinations formed by using the letters above such that the last letter is R and one of the remaining letters is S. No letter is used more than once in a combination in the list and one 3-letter combination is at random chosen from the list. What is the probability that the combination chosen will be SNR?

(A) $\frac{5}{49}$

(B) $\frac{6}{49}$

(C) $\frac{5}{10}$

(D) $\frac{1}{10}$

Solution: Answer: (D)

(a)

(1). $\boxed{1}_{S} \times \boxed{5}_{MNOPQ} \times \boxed{1}_{R} = 5$

(2). $\boxed{5}_{MNOPQ} \times \boxed{1}_{S} \times \boxed{1}_{R} = 5$

$\downarrow$

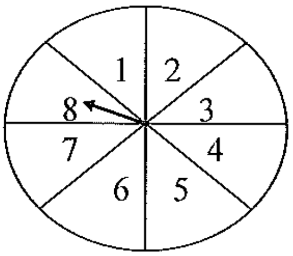
All possible combinations $= 5 + 5 = 10$

(b). By (1), only one combination is SNR.

By (2), no any combination is SNR.

$\Downarrow$

$P = 1/10$



Note: Figure above not drawn to scale.

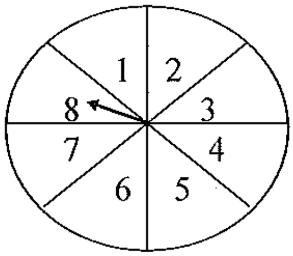
1. In the figure above, Jennifer spins the arrow twice. The fraction m/n is formed, where m is the number of the sector where the arrow stops after the first spin and n is the number of the sector where the arrow stops after the second spin. On every spin, if each of the numbered sectors has an equal chance that the arrow stops on the sector, what is the probability that the different fraction m/n is less than 1?

(A) $\frac{21}{64}$

(B) $\frac{15}{64}$

(C) $\frac{19}{64}$

(D) $\frac{5}{64}$



Note: Figure above not drawn to scale.

2. In the figure above, Stephanie spins the arrow twice. The fraction m/n is formed, where m is the number of the sector where the arrow stops after the first spin and n is the number of the sector where the arrow stops after the second spin. On every spin, if each of the numbered sectors has an equal chance that the arrow stops on the sector, what is the probability that the fraction m/n is less than $\frac{1}{2}$?

(A) $\frac{17}{64}$

(B) $\frac{15}{64}$

(C) $\frac{3}{16}$

(D) $\frac{5}{64}$

M, N, O, P, Q, R, S

3. A list is composed of all possible 3-letter combinations formed by using the letters above such that the last letter is R and one of the remaining letters is S. No letter is used more than once in a combination in the list and one 3-letter combination is at random chosen from the list. What is the probability that the combination chosen will be SNR?

(A) $\frac{5}{49}$

(B) $\frac{6}{49}$

(C) $\frac{5}{10}$

(D) $\frac{1}{10}$

Trigonometry

Trigonometry

1. If $\cos 54^\circ = \tan x^\circ$, what does x equal?

(A) 0.53

(B) 40

(C)-0.53

(D) 30.45

Solution: Answer: (D)

Use a TI graphing calculator.

2nd ANGLE 1:° ENTER

$$\downarrow$$

$$\cos 54^\circ = \tan x$$

$$\downarrow$$

$$\tan^{-1}\left(\cos 54^\circ\right) = x$$

$$\downarrow$$

$$x = \tan^{-1}\left(0.588\right)$$

Degree Mode

$$\Downarrow$$

$$x \approx 30.45$$

2. Convert the degrees to radians or the radians to degrees for $\frac{\pi}{5}$ and 160°. Which of the following a pair of results is true?

(Radians are assumed if no unit of measrement is indicated.)

(A) 36° and 2.79

(B) 62° and 4.13

(C) 20° and 2π

(D) 26° and 3.89

Solution: Answer: (A)

$$\frac{\pi}{5}\cdot\frac{180}{\pi} = 36^\circ$$

and

$$160\cdot\frac{\pi}{180} = 2.79^R$$

3. If $0 \le x \le \frac{\pi}{2}$ and $\cos x = 4\sin x$, what is the value of x ?

(A) .43

(B) 0.55

(C) 14.04°

(D) 25.61°

Solution: Answer: (C)

$$\cos x = 4\sin x$$

$$\downarrow$$

$$1 = \frac{4\sin x}{\cos x}$$

$$\downarrow$$

$$\frac{1}{4} = \tan x$$

$$\downarrow$$

$$\tan^{-1}\left(\frac{1}{4}\right) = x$$

$$\Downarrow$$

$$x = 14.04^\circ$$

4. If $\arcsin(\sin x) = 0$ and $0 \le x \le \frac{\pi}{2}$, then x could equal

(A) 0

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{4}$

Solution: Answer: (A)

Since $\sin x$ and $\arcsin x$ are inverses of each other on the interval $0 \le x \le \frac{\pi}{2}$, x could be equal to 0 if $\arcsin(\sin x) = 0$.

or

$$\sin 0 = \sin x$$

$$\Downarrow$$

$$x = 0$$

5. If $0 \le x \le \frac{\pi}{2}$ and $\cos x - 2\sin x = 0$, what is the value of x ?

(A) 0.64

(B) 0.59

(C) 26.57°

(D) 25.67°

Solution: Answer: (C)

$$\cos x - 2\sin x = 0$$

$$\cos x = 2\sin x$$

$$\downarrow$$

$$\tan x = \frac{1}{2}$$

$$\Downarrow$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) = x,\ x = 26.57^\circ$$

6. Convert the degrees to radians or the radians to degrees for $\frac{\pi}{5}$ and 260°. Which of the following a pair of results is true?

(Radians are assumed if no unit of measrement is indicated.)

(A) 36° and 5.45

(B) 63° and 5.45

(C) 36° and 4.54

(D) 26° and 5.34

Solution: Answer: (C)

$$\frac{\pi}{5} \cdot \frac{180}{\pi} = 36^\circ$$

and

$$260 \cdot \frac{\pi}{180} \approx 4.54^R$$

7. If $\sin\theta = \frac{1}{\tan\theta}$, which of the following is one possible radian measure of the angle θ ?

(A) 51.83^r

(B) -1.618^r

(C) 0.905^r

(D) 0.618^r

Solution: Answer: (C)

$$\sin\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}}$$

$$\downarrow$$

$$\sin\theta = \frac{\cos\theta}{\sin\theta},\ \sin^2\theta = \cos\theta$$

$$\downarrow$$

$$1 - \cos^2\theta = \cos\theta$$

$$\downarrow$$

$$\cos^2\theta + \cos\theta - 1 = 0$$

$$\downarrow$$

Let $\cos\theta = x$.

$$\downarrow$$

$$x^2 + x - 1 = 0$$

(You may use a calculator to solve it.)

Solve for x.

$$\downarrow$$

$$x_1 = \frac{-1-\sqrt{5}}{2} \approx -1.618 < -1$$

$$x = \cos\theta$$

$$-1 \le \cos\theta \le 1 \rightarrow -1 \le x \le 1$$

$$\downarrow$$

$x_1 < -1$ is wrong.

$$x_2 = \frac{-1+\sqrt{5}}{2} \approx 0.618$$

$$\cos\theta = 0.618$$

$$\Downarrow$$

$$\theta = \cos^{-1}0.618 \approx 51.83^\circ \text{ or } 0.905^r$$

8. For $0 < \theta < \frac{\pi}{2}$, if the slope of line L_1 is $\frac{3}{4}$ and the slope of line L_2 is $\frac{3}{5}$, which line has the bigger measure θ of the angle formed by the line and x-axis?

(A) L_2

(B) No way to know

(C) none

(D) L_1

Solution: Answer: (D)

$$\tan\theta_1 = \frac{3}{4}$$

$$\downarrow$$

$$\theta_1 = \tan^{-1}\frac{3}{4} \approx 36.87^\circ$$

$$\tan\theta_2 = \frac{3}{5}$$

$$\downarrow$$

$$\theta_2 = \tan^{-1}\frac{3}{5} \approx 30.96^\circ$$

$$\Downarrow$$

$$\theta_1 > \theta_2$$

9. If $\sin\theta = \frac{1}{\tan\theta}$, which of the following is one possible degree measure of the angle θ ?

(A) 0.905°

(B) -1.61°

(C) 51.83°

(D) 38.17°

Solution: Answer: (C)

$$\sin\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}}$$

$$\downarrow$$

$$\sin\theta = \frac{\cos\theta}{\sin\theta},\ \sin^2\theta = \cos\theta$$

$$\downarrow$$

$$1-\cos^2\theta = \cos\theta$$

$$\downarrow$$

$$\cos^2\theta + \cos\theta - 1 = 0$$

$$\downarrow$$

Let $\cos\theta = x$.

$$\downarrow$$

$$x^2 + x - 1 = 0$$

(You may use a calculator to solve it.)

Solve for x.

$$\downarrow$$

$$x_1 = \frac{-1-\sqrt{5}}{2} \approx -1.618 < -1$$

$$x = \cos\theta$$

$$-1 \le \cos\theta \le 1 \rightarrow -1 \le x \le 1$$

$$\downarrow$$

$x_1 < -1$ is wrong.

$$x_2 = \frac{-1+\sqrt{5}}{2} \approx 0.618$$

$$\cos x = 0.618$$

$$\Downarrow$$

$$\theta = \cos^{-1}0.618 \approx 51.83^\circ$$

1. If $\cos 54^\circ = \tan x^\circ$, what does x equal?

(A) 0.53

(B) 40

(C)-0.53

(D) 30.45

2. Convert the degrees to radians or the radians to degrees for $\frac{\pi}{5}$ and 160°. Which of the following a pair of results is true?

(Radians are assumed if no unit of measrement is indicated.)

(A) 36° and 2.79

(B) 62° and 4.13

(C) 20° and 2π

(D) 26° and 3.89

3. If $0 \le x \le \frac{\pi}{2}$ and $\cos x = 4 \sin x$, what is the value of x ?

(A) .43

(B) 0.55

(C) 14.04°

(D) 25.61°

4. If $\arcsin(\sin x) = 0$ and $0 \le x \le \frac{\pi}{2}$, then x could equal

(A) 0

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{4}$

5. If $0 \le x \le \frac{\pi}{2}$ and $\cos x - 2\sin x = 0$, what is the value of x ?

(A) 0.64

(B) 0.59

(C) 26.57°

(D) 25.67°

6. Convert the degrees to radians or the radians to degrees for $\frac{\pi}{5}$ and 260°. Which of the following a pair of results is true?

(Radians are assumed if no unit of measrement is indicated.)

(A) 36° and 5.45

(B) 63° and 5.45

(C) 36° and 4.54

(D) 26° and 5.34

7. If $\sin\theta = \frac{1}{\tan\theta}$, which of the following is one possible radian measure of the angle θ ?

(A) 51.83^r

(B) -1.618^r

(C) 0.905^r

(D) 0.618^r

8. For $0 < \theta < \frac{\pi}{2}$, if the slope of line L_1 is $\frac{3}{4}$ and the slope of line L_2 is $\frac{3}{5}$, which line has the bigger measure θ of the angle formed by the line and x-axis?

(A) L_2

(B) No way to know

(C) none

(D) L_1

9. If $\sin\theta = \frac{1}{\tan\theta}$, which of the following is one possible degree measure of the angle θ ?

(A) 0.905°

(B) -1.61°

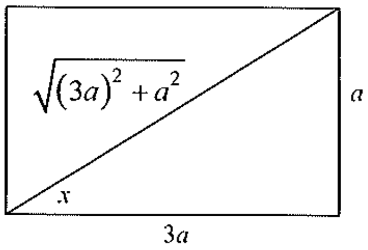
(C) 51.83°

(D) 38.17°

1. If the ratio of length of a rectangle to width of the rectangle is 3, what is the cosine of the angle formed by the diagonal with the longer side?

(A) 0.92
(B) 0.83
(C) 0.91
(D) 0.95

Solution: Answer: (D)



$$\cos x = \frac{3a}{\sqrt{(3a)^2+a^2}} = \frac{3a}{\sqrt{9a^2+a^2}}$$

$$\Downarrow$$

$$\cos x = \frac{3a}{\sqrt{10a^2}} = \frac{3\cancel{a}}{\cancel{a}\sqrt{10}} \approx .95$$

or

$$\cos(\tan^{-1}(1/3)) = .95$$

2. If Q is the angle formed by the line $3y = 6x + 9$ and the x-axis, then $m\angle Q = \theta$ is equal to

(A) 38.33°
(B) 39.40°
(C) 23.32°
(D) 63.43°

Solution: Answer: (D)

$$3y = 6x + 9$$

$$\downarrow$$

$$y = 2x + 3$$

$$\downarrow$$

$$\text{slope} = 2$$

$$\tan\theta = \text{slope}$$

$$\downarrow$$

$$\tan\theta = 2$$

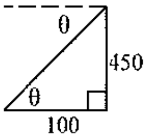
$$\Downarrow$$

$$m\angle Q = \theta = \tan^{-1} 2 = 63.43^\circ$$

3. If a small ball is laid one hundred feet from a building that is 450 feet tall, what is the degree measure of the angle of depression from the upside of the building to the ball?

(A) 22.17°
(B) 77.47°
(C) 45°
(D) 25.67°
(E) 77.83°

Solution: Answer: (B)



$$\tan x = \frac{450}{100} = 4.5$$

$$\Downarrow$$

$$x = \tan^{-1} 4.5 \approx 77.47^\circ$$

4. Which of the following expressions is equal to $\dfrac{\sin^2\theta - \cos^2\theta}{\cos^4\theta - \sin^4\theta}$, where $0 < \theta < \dfrac{\pi}{2}$?

(A) 1
(B) $\dfrac{\cos^2\theta - \sin^2\theta}{2}$
(C) -1
(D) $\cos^2\theta - \sin^2\theta$

Solution: Answer: (C)

$$\frac{\sin^2\theta - \cos^2\theta}{\cos^4\theta - \sin^4\theta}$$

$$\downarrow$$

$$\frac{-\cancel{\left(\cos^2\theta - \sin^2\theta\right)}}{\cancel{\left(\cos^2\theta - \sin^2\theta\right)}\left(\cos^2\theta + \sin^2\theta\right)}$$

$$\Downarrow$$

$$\frac{-1}{\left(\sin^2\theta + \cos^2\theta\right)} = \frac{-1}{1} = -1$$

5. Which of the following values is equivalent to $(2\cos x)(9\cos x)+(3\sin x)(6\sin x)$?

(A) 1

(B) 18

(C) 0

(D) 12

Solution: Answer: (B)

$$(2\cos x)(9\cos x)+(3\sin x)(6\sin x)$$

$$\downarrow$$

$$18\cos^2 x+18\sin^2 x$$

$$\downarrow$$

$$18\left(\cos^2 x+\sin^2 x\right)$$

$$\Downarrow$$

$$18\times 1=18$$

6. Which of the following is equivalent to $(\cos x-\sin x)(\sin x+\cos x)+(\sin x-\cos x)(\cos x+\sin x)$?

(A) $2\sin^2 x$

(B) 2

(C) 0

(D) 1

Solution: Answer: (C)

(1)

$$(\cos x-\sin x)(\sin x+\cos x)$$

$$\downarrow$$

$$(\cos x-\sin x)(\cos x+\sin x)$$

$$\downarrow$$

$$c^2-s^2$$

(2)

$$(\sin x-\cos x)(\cos x+\sin x)$$

$$\downarrow$$

$$(\sin x-\cos x)(\sin x+\cos x)$$

$$\downarrow$$

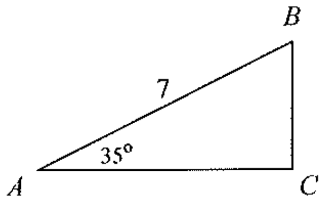
$$s^2-c^2$$

By the results of (1) and (2)

$$\Downarrow$$

$$c^2-s^2+s^2-c^2=0$$

Note: Figure below not drawn to scale.



7. In the figure above, what is the sum of AC and BC?

(A) 9.75

(B) 5.73

(C) 4.02

(D) 11.01

Solution: Answer: (A)

$$\begin{cases}\sin 35^\circ=\dfrac{BC}{7}\\ \downarrow\\ BC=7\sin 35^\circ\approx 4.02\end{cases}$$

and

$$\begin{cases}\cos 35^\circ=\dfrac{AC}{7}\\ \downarrow\\ AC=7\cos 35^\circ\approx 5.73\end{cases}$$

$$\Downarrow$$

$$AC+BC=9.75$$

8. If the degree measure of an angle is 73°, what is the radian measure of the angle?

(A) 0.24
(B) 1.27
(C) 1.44
(D) 1.23

Solution: Answer: (B)

Use a TI graphing calculator.

Enter 73

Press [2nd] and ANGLE.

select 1:° and Press [ENTER].

You will see 73°.

Press [MODE]. Select Radian.

Press [ENTER] and press [CLEAR]

You will see:

73°

Press [ENTER]

You will see:

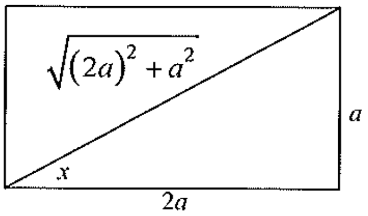
73°

1.2741

9. If the ratio of length of a rectangle to width of the rectangle is 2, what is the cosine of the angle formed by the diagonal with the longer side?

(A) 0.96
(B) 1.89
(C) 1.98
(D) .89

Solution: Answer: (D)



$$\cos x = \frac{2a}{\sqrt{(2a)^2+a^2}} = \frac{2a}{\sqrt{4a^2+a^2}}$$

$$\downarrow$$

$$\cos x = \frac{2a}{\sqrt{5a^2}} = \frac{2\cancel{a}}{\cancel{a}\sqrt{5}} \approx .89$$

or

$$\cos(\tan^{-1}(1/2)) = .89$$

10. Where defined, $\dfrac{\sin x - \cos x}{1 - \tan x}$ is equal to

(A) $\cos x$
(B) $\tan x$
(C) $-\csc x$
(D) $-\cos x$

Solution: Answer: (D)

$$\frac{\sin x - \cos x}{1 - \tan x} = \frac{\sin x - \cos x}{1 - \dfrac{\sin x}{\cos x}}$$

$$\downarrow$$

$$\frac{\sin x - \cos x}{\dfrac{\cos x - \sin x}{\cos x}} = \frac{-(\cos x - \sin x)}{\dfrac{\cos x - \sin x}{\cos x}}$$

$$\downarrow$$

$$\frac{-\cancel{(\cos x - \sin x)}\cos x}{\cancel{\cos x - \sin x}}$$

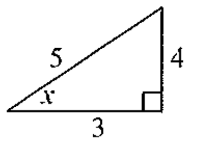
$$\Downarrow$$

$$\boxed{-\cos x}$$

11. What is the measure of the smallest angle in a 3-4-5 right triangle?

(A) 0.64°
(B) 36.87°
(C) 38.67°
(D) 63.82°

Solution: Answer: (B)



$$x = \sin^{-1}\frac{3}{5} \approx 36.87^\circ$$

12. In the xy-plane, if three points are $A(2,\ 3), B(5,\ 3)$ and $C(5,\ 7)$, what is the tangent of $\angle ACB$?

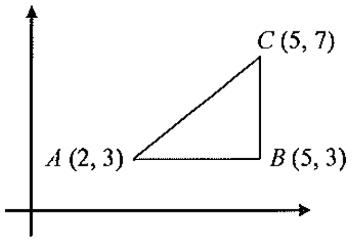
(A) $\frac{4}{3}$

(B) $\frac{7}{5}$

(C) $\frac{2}{4}$

(D) $\frac{3}{4}$

Solution: Answer: (D)



$$\tan C = \frac{5-2}{7-3} = \frac{3}{4}$$

13. For all x, $\sin(2\pi + x) + \sin x + \sin(x + \pi) =$

(A) $\sin x$

(B) $\sin 2x - 2\sin x$

(C) $\sin x + 2\cos x$

(D) $2\sin x + \sin^2 x$

Solution: Answer: (A)

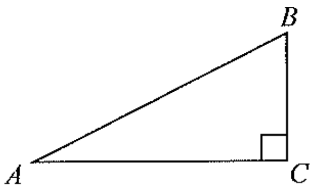
$$\sin(2\pi + x) + \sin x + \sin(x + \pi)$$

$$\sin(2\pi + x) = \sin x$$

$$\sin(x + \pi) = -\sin x$$

$$\Downarrow$$

$$\sin x + \cancel{\sin x} \cancel{-\sin x} = \boxed{\sin x}$$



14. In triangle ABC in the figure above, if $\cos A = \frac{12}{13}$, then which of the following is $\cos B$?

(A) $\frac{13}{5}$

(B) $\frac{13}{12}$

(C) $\frac{12}{5}$

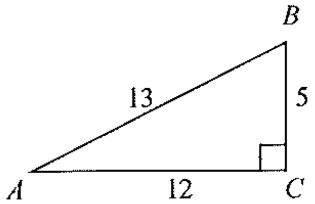
(D) $\frac{5}{13}$

Solution: Answer: (D)

$$\cos A = \frac{12}{13}$$

$$\downarrow$$

5-12-13 right triangle.



$$\Downarrow$$

$$\cos B = \frac{5}{13}$$

15. For all x, $\cos(2\pi + x) + \sin x + \sin(x + \pi) =$

(A) $\cos x + 2\sin x$

(B) $\cos x - 2\sin x$

(C) $\cos x$

(D) $\cos x + \sin^2 x$

Solution: Answer: (C)

$$\cos(2\pi + x) + \sin x + \sin(x + \pi)$$

$$\cos(2\pi + x) = \cos x$$

$$\sin(x + \pi) = -\sin x$$

$$\Downarrow$$

$$\cos x + \cancel{\sin x} \cancel{-\sin x} = \boxed{\cos x}$$

16. Which of the following values is equivalent to $(6\cos x)(4\cos x)+(3\sin x)(8\sin x)$?

(A) 24
(B) 18
(C) 0
(D) 1

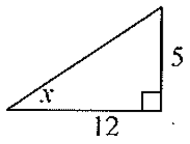
Solution: Answer: (A)

$$(6\cos x)(4\cos x)+(3\sin x)(8\sin x)$$
$$\downarrow$$
$$24\cos^2 x+24\sin^2 x=24\left(\cos^2 x+\sin^2 x\right)$$
$$\Downarrow$$
$$24\times 1=24$$

17. What is the measure of the smallest angle in a 5-12-13 right triangle?

(A) 67.38°
(B) 76.38°
(C) 22.62°
(D) 63.82°

Solution: Answer: (C)



$$x=\tan^{-1}\frac{5}{12}\approx 22.62^\circ$$

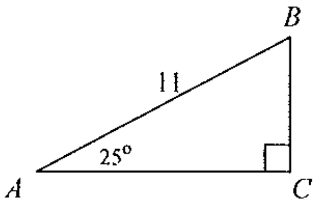
18. If $0<x<\frac{\pi}{2}$, $2\sin^2 x+\sin^2 x\,\frac{1}{\tan^2 x}+\cos^2 x$ equals

(A) 2
(B) $3\sin^2 x+2$
(C) 1
(D) $2\cos^2 x+1$

Solution: Answer: (A)

$$2\sin^2 x+\sin^2 x\,\frac{1}{\tan^2 x}+\cos^2 x$$
$$\downarrow$$
$$2\sin^2 x+\sin^2 x\,\frac{1}{\frac{\cos^2 x}{\sin^2 x}}+\cos^2 x$$
$$2\sin^2 x+\cancel{\sin^2 x}\,\frac{\cos^2 x}{\cancel{\sin^2 x}}+\cos^2 x$$
$$\downarrow$$
$$2\sin^2 x+2\cos^2 x$$
$$\Downarrow$$
$$2(\sin^2 x+\cos^2 x)=2\cdot 1=2$$

Note: Figure below not drawn to scale.



19. In the figure above, what is the sum of AC and BC?

(A) 14.62°
(B) 15.71°
(C) 14.02°
(D) 11.41°

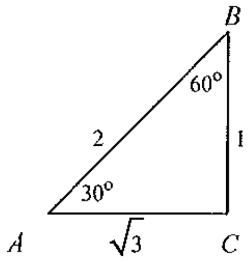
Solution: Answer: (A)

$$\sin 25^\circ=\frac{BC}{11},\ BC=11\sin 25^\circ\approx 4.65$$
$$\cos 25^\circ=\frac{AC}{11},\ AC=11\cos 25^\circ\approx 9.97$$
$$\Downarrow$$
$$AC+BC=14.62$$

20. If, in triangle ABC, $\angle C$ is a right angle and $\tan A=\frac{\sqrt{3}}{3}$, what is the value of $\sin B-\sin A$?

(A) -0.37°
(B) 1.37°
(C) -1.33°
(D) 0.37°

Solution: Answer: (D)



$$\sin B-\sin A=\frac{\sqrt{3}}{2}-\frac{1}{2}$$
$$\Downarrow$$
$$\sin B-\sin A\approx 0.87-0.5\approx 0.37$$

21. For $0 < \theta < \frac{\pi}{2}$, if the slope of line L_1 is $\frac{3}{4}$ and the slope of line L_2 is $\frac{3}{5}$, which line has the greater measure θ of the angle formed by the line and x-axis?

(A) L_2

(B) No way to know

(C) none

(D) L_1

Solution: Answer: (D)

$$\tan\theta_1 = \frac{3}{4}, \rightarrow \theta_1 = \tan^{-1}\frac{3}{4} \approx 36.87^\circ$$

$$\tan\theta_2 = \frac{3}{5}, \rightarrow \theta_2 = \tan^{-1}\frac{3}{5} \approx 30.96^\circ$$

$$\Downarrow$$

$$\theta_1 > \theta_2$$

22. $\dfrac{4\sqrt{\cos^2 16\theta + \sin^2 16\theta}}{\sqrt{16\sin^2\theta + 16\cos^2\theta}} =$

(A) 4

(B) $\frac{1}{16}$

(C) 8

(D) 1

Solution: Answer: (D)

$$\frac{4\sqrt{\cos^2 16\theta + \sin^2 16\theta}}{\sqrt{16\sin^2\theta + 16\cos^2\theta}}$$

$$\downarrow$$

$$\frac{4\cdot 1}{\sqrt{16\left(\sin^2\theta + \cos^2\theta\right)}}$$

$$\Downarrow$$

$$\frac{4\cdot 1}{\sqrt{16\cdot 1}} = \frac{4}{4} = 1$$

23. If the degree measure of an angle is 88°, what is the radian measure of the angle?

(A) 0.23

(B) 1.54

(C) 0.97

(D) 1.11

Solution: Answer: (B)

Use a TI graphing calculator.

Enter 88

Press 2nd and ANGLE.

select 1:° and Press ENTER.

You will see 88°.

Press MODE. Select Radian.

Press ENTER and press CLEAR

You will see:

88°

Press ENTER

You will see:

88°

1.5358

24. $5 - 4\cos^2\theta - 4\sin^2\theta$

(A) 0

(B) 1

(C) 2

(D) 3

Solution: Answer: (B)

$$5 - 4(\cos^2\theta + 4\sin^2\theta)$$

$$\Downarrow$$

$$5 - 4\cdot 1 = 1$$

25. If $k > 0$, $p = k\sin\theta$, and $q = k\cos\theta$, then $\dfrac{\sqrt{p^2+q^2}}{k} =$

(A) $\sqrt{k}$

(B) 1

(C) $2k$

(D) k

Solution: Answer: (B)

$$\sqrt{p^2+q^2} = \sqrt{(k\sin\theta)^2 + (k\cos\theta)^2}$$

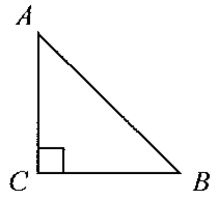
$$\downarrow$$

$$\sqrt{k^2\sin^2\theta + k^2\cos^2\theta}$$

$$\sqrt{k^2(\sin^2\theta + \cos^2\theta)} = \sqrt{k^2\cdot 1} = k$$

$$\Downarrow$$

$$\frac{\sqrt{p^2+q^2}}{k} = \frac{k}{k} = 1$$



26. In right triangle ABC above, if $AB = 15$, $BC = 12$, and $AC = 9$, then the sine of $\angle A$ is

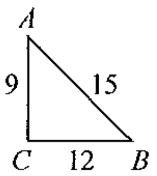
(A) $\dfrac{2}{5}$

(B) $\dfrac{4}{5}$

(C) $\dfrac{3}{5}$

(D) $\dfrac{5}{3}$

Solution: Answer: (B)



$$\sin A = \frac{12}{15} = \frac{4}{5}$$

27. If $(\csc x)(\sec x) < 0$, which of the following must be true?

I. x is in the quadrant II or III.

II. $\csc x < 0$

III. $\sec x < 0$

(A) None

(B) I only

(C) II only

(D) I and II only

Solution: Answer: (A)

	QI	QII	QIII	QIV
$\csc x$	+	+	−	−
$\sec x$	+	−	−	+

$$(\csc x)(\sec x) < 0$$

$$\Downarrow$$

x is in quadrant II or IV.

28. $(1-\cos\theta)(1+\cos\theta) = ?$

(A) $1+\sin^2\theta$

(B) $\cos^2\theta$

(C) $1+\cos^2\theta$

(D) $\sin^2\theta$

Solution: Answer: (D)

Use formula:

$$x^2 - y^2 = (x-y)(x+y)$$

$$\downarrow$$

$$(1-\cos\theta)(1+\cos\theta) = 1^2 - \cos^2\theta$$

Use formula:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Downarrow$$

$$1 - \cos^2\theta = \sin^2\theta$$

29. If $0 < x < \frac{\pi}{2}$, $2\cos^2 x + \cos^2 x \tan^2 x + \sin^2 x$ equals

(A) $3\cos^2 x + 1$

(B) $3\cos^2 x + 2$

(C) 1

(D) 2

Solution: Answer: (D)

$$2\cos^2 x + \cos^2 x \tan^2 x + \sin^2 x$$

$$\downarrow$$

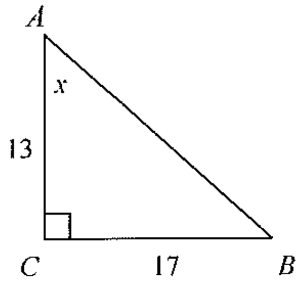
$$2\cos^2 x + \cancel{\cos^2 x}\,\frac{\sin^2 x}{\cancel{\cos^2 x}} + \sin^2 x$$

$$\downarrow$$

$$2\cos^2 x + 2\sin^2 x$$

$$\Downarrow$$

$$2(\cos^2 x + \sin^2 x) = 2 \cdot 1 = 2$$



30. In the figure above, $\frac{AC}{BC} = \frac{13}{17}$, $\cos x =$

(A) -0.61

(B) 0.61

(C) 1.12

(D) 0.72

Solution Answer: (B)

$$\frac{AC}{BC} = \frac{13}{17}$$

$$\downarrow$$

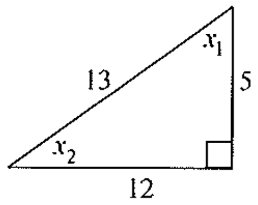
$$\tan x = \frac{17}{13}$$

$$\downarrow$$

$$x = \tan^{-1}\frac{17}{13} \approx 52.5946^\circ$$

$$\Downarrow$$

$$\cos x = \cos 52.5946^\circ \approx 0.6075$$



31. In the figure above, which of the following must be true?

I. $\tan x_1 = \frac{5}{12}$

II. $\sin x_2 = \frac{12}{13}$

III. $\sin x_1 = \cos x_2$

(A) I only

(B) II only

(C) III only

(D) I and II only

Solution: Answer: (C)

I. $\tan x_1 = \frac{12}{5} \neq \frac{5}{12}$

$\Downarrow$

I is false.

II. $\sin x_2 = \frac{5}{13} \neq \frac{12}{13}$

$\Downarrow$

II is false.

III. $\sin x_1 = \cos(90 - x_1) = \cos x_2$

$\Downarrow$

III is true.

32. If $\cos x = \frac{6}{11}$, what could $\tan x$ equal?

(A) .017

(B) 0.98

(C) 1.54

(D) 1.13

Solution: Answer: (C)

$$\cos x = \frac{6}{11}$$

$\downarrow$

$$\cos^{-1}\left(\frac{6}{11}\right) \approx 56.944^\circ$$

$\Downarrow$

$$\tan 56.944^\circ \approx 1.54$$

33. $\sin x + \cos x(\cot x) = ?$

(A) 1

(B) $\sec x$

(C) $\csc x$

(D) $2\sin x$

Solution: Answer: (C)

$$\sin x + \cos x \cdot \frac{\cos x}{\sin x}$$

$\downarrow$

$$\frac{\sin x \cdot \sin x}{\sin x} + \cos x \cdot \frac{\cos x}{\sin x}$$

$\downarrow$

$$\frac{\sin^2 x + \cos^2 x}{\sin x} = \frac{1}{\sin x}$$

$\Downarrow$

$$\frac{1}{\sin x} = \csc x$$

36. In triangle ABC, if $\angle C$ is a right angle, and $\tan A = \sqrt{3}$, what is the value of $\sin B - \sin A$?

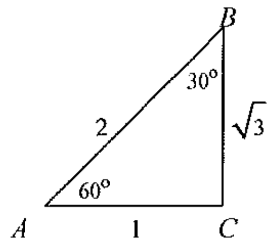
(A)-0.37

(B)-1.23

(C) 1.37

(D)-1.33

Solution: Answer: (A)



$$\frac{1}{2} - \frac{\sqrt{3}}{2} \approx 0.5 - 0.87$$

$$\Downarrow$$

$$\sin B - \sin A \approx -0.37$$

37. In right triangle ABC, if $\sin A = \frac{4}{5}$, what is the value of $\cos B$?

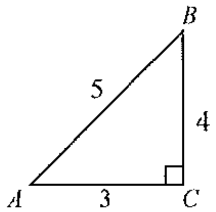
(A) $\frac{3}{5}$

(B) $\frac{5}{3}$

(C) $\frac{5}{4}$

(D) $\frac{4}{5}$

Solution: Answer: (D)



$$A + B = 90^\circ$$

$$\Downarrow$$

$$\sin A = \cos B = \frac{4}{5}$$

38. $\sqrt{1-\sin^2 x} =$

(A) $\cos x$

(B) $\sin x$

(C) $\sin x - 1$

(D) $|\cos x|$

Solution: Answer: (D)

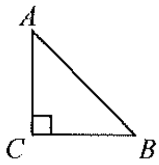
$$\sqrt{1-\sin^2 x}$$

$$\downarrow$$

$$\sqrt{\cos^2 x}$$

$$\Downarrow$$

$$|\cos x|$$



40. In right triangle ABC above, if $AB = 15$, $BC = 9$, and $AC = 12$, then the sine of $\angle A$ is

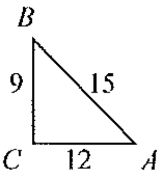
(A) $\frac{2}{5}$

(B) $\frac{4}{5}$

(C) $\frac{3}{5}$

(D) $\frac{5}{3}$

Solution: Answer: (C)



$$\sin A = \frac{9}{15} = \frac{3}{5}$$

41. In a right triangle ABC, if $\sin A = \frac{4}{5}$, what is the value of $\tan B$?

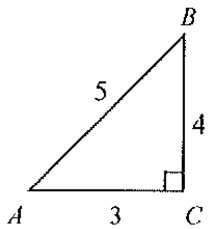
(A) $\frac{4}{5}$

(B) $\frac{5}{3}$

(C) $\frac{5}{4}$

(D) $\frac{3}{4}$

Solution: Answer: (D)

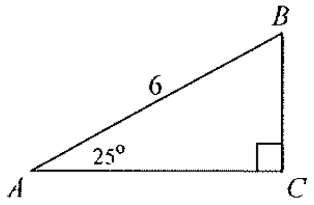


$$A + B = 90^\circ$$

$$\Downarrow$$

$$\tan B = \frac{3}{4}$$

Note: Figure below not drawn to scale.



42. In the figure above, what is the difference of AC and BC?

(A) 2.9

(B) 8.0

(C) 4.0

(D) 10

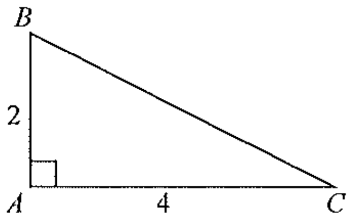
Solution: Answer: (A)

$$\sin 25^\circ = \frac{BC}{6},\ BC = 6\sin 25^\circ \approx 2.54$$

$$\cos 25^\circ = \frac{AC}{6},\ AC = 6\cos 25^\circ \approx 5.44$$

$$\Downarrow$$

$$AC - BC \approx 2.9$$



44. As shown in the figure above, what is the cosine of angle C ?

(A) $\frac{4\sqrt{5}}{5}$

(B) $\frac{2\sqrt{5}}{5}$

(C) $\frac{\sqrt{5}}{5}$

(D) $\frac{\sqrt{3}}{4}$

Solution: Answer: (B)

$$BC = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$$

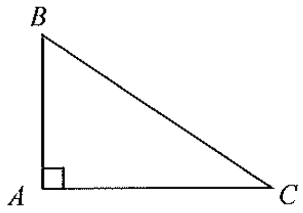
$$\downarrow$$

$$\cos\theta_C = \frac{4}{2\sqrt{5}}$$

$$\Downarrow$$

$$\frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

Note: Figure below not drawn to scale.



46. In the figure above, in the right triangle ABC, if $\sin C = 0.8$, what is $\sin B$?

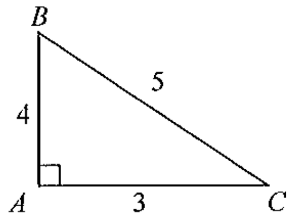
(A) 0.2

(B) 0.4

(C) 0.6

(D) 0.5

Solution: Answer: (C)



$$\sin C = 0.8 = \frac{8}{10} = \frac{4}{5}$$

$$\downarrow$$

A $3-4-5$ righr triangle exist.

$$\Downarrow$$

$$\sin B = \frac{3}{5} = 0.6$$

48. If the degree measure of an angle is 98°, what is the radian measure of the angle?

(A) 0.27

(B) 1.71

(C) 0.71

(D) 1.51

Solution: Answer: (B)

Use a TI graphing calculator.

Enter 98

Press 2nd and ANGLE.

select 1:° and Press ENTER.

You will see 98°.

Press MODE. Select Radian.

Press ENTER and press CLEAR

You will see:

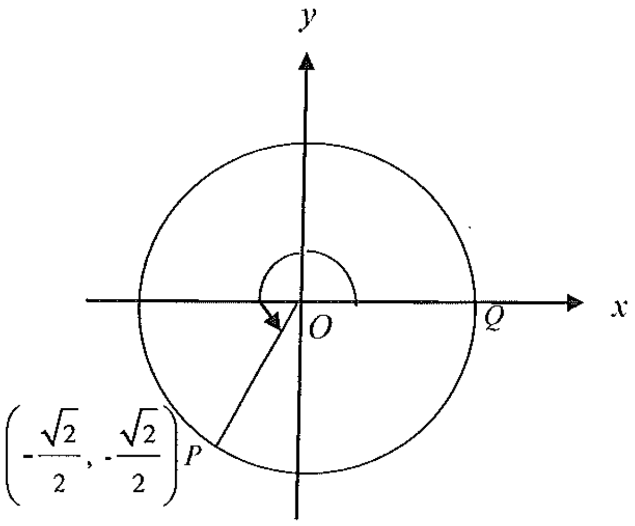
98°

Press ENTER

You will see:

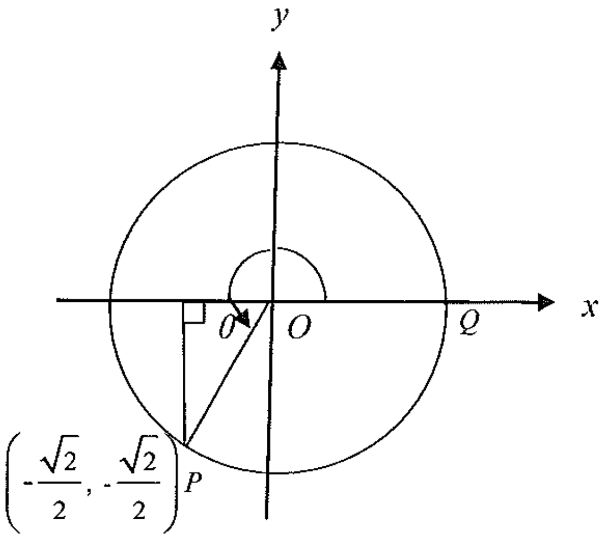
98°

1.7104



49. In the figure above, if the measure of $\angle QOP$ is $\frac{5\pi}{x}$ radians, what is the value of x?

Solution: Answer: 4



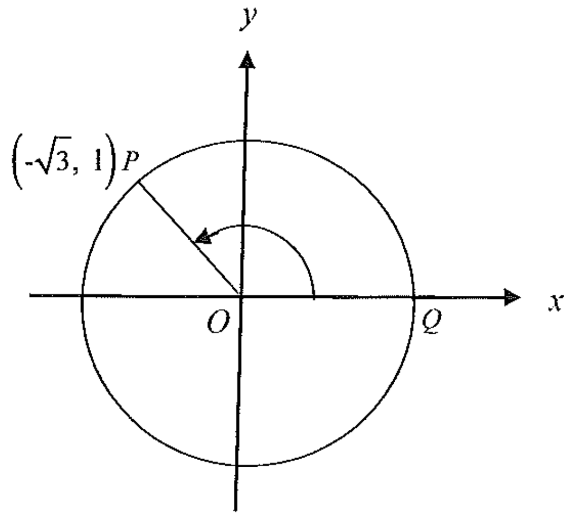
$$\tan\theta = \frac{y}{x} = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1, \quad \theta = 45^\circ \cdot \frac{\pi}{180} = \frac{\pi}{4}$$

$$\downarrow$$

$$\angle QOP = \theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}, \quad \frac{\cancel{5\pi}}{4} = \frac{\cancel{5\pi}}{x}$$

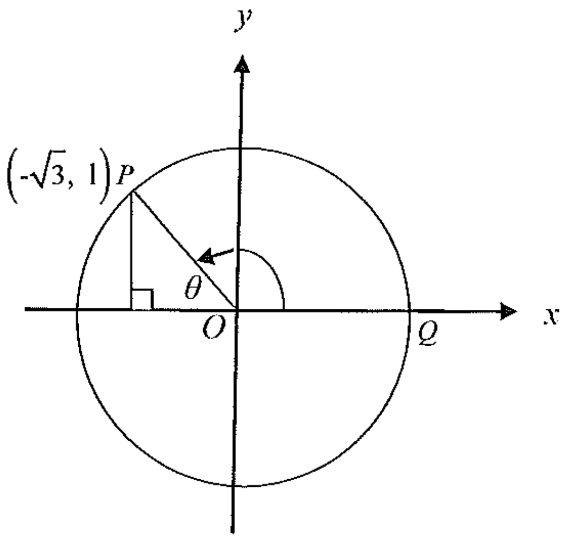
$$\Downarrow$$

$$x = 4$$



50. In the figure above, if the measure of $\angle QOP$ is $15\pi / x$ radians, what is the value of x?

Solution: Answer: 18



$$\tan\theta = \frac{y}{x} = \frac{1}{-\sqrt{3}}$$

$$\downarrow$$

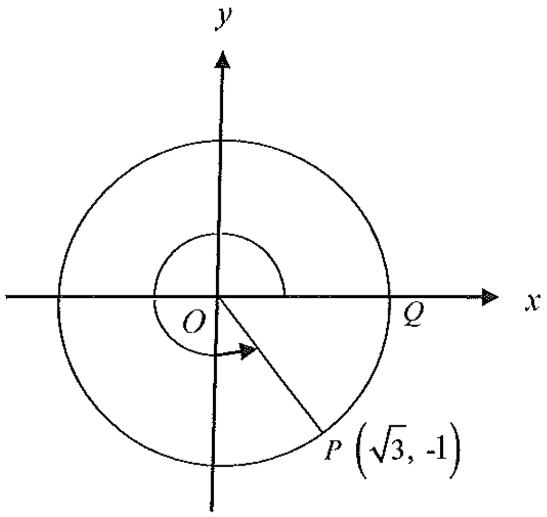
$$\text{Reference angle} = \theta = 30^\circ \cdot \frac{\pi}{180} = \frac{\pi}{6}$$

$$\downarrow$$

$$\angle QOP = \pi - \frac{\pi}{6} = \frac{5\pi}{6}, \quad \frac{\cancel{5\pi}}{6} = \frac{\cancel{15\pi}^{3}}{x}$$

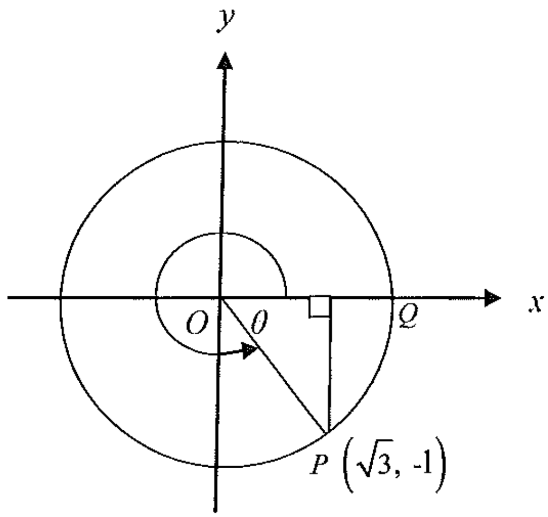
$$\Downarrow$$

$$\frac{6}{1} = \frac{x}{3}, \qquad x = 18$$



51. In the figure above, if the measure of $\angle QOP$ is $22\pi / x$ radians, what is the value of x?

Solution: Answer: 12



$$\tan\theta = \frac{y}{x} = \frac{-1}{\sqrt{3}}$$

$$\downarrow$$

$$\text{Reference angle} = \theta = 30^\circ$$

$$\downarrow$$

$$\angle QOP = 360 - 30 = 330 \cdot \frac{\pi}{180} = \frac{11\pi}{6}$$

$$\Downarrow$$

$$\frac{\cancel{11\pi}}{6} = \frac{\cancel{22\pi}^{2}}{x}, \quad \frac{6}{1} = \frac{x}{2}, \quad \boxed{x = 12}$$

1. If the ratio of length of a rectangle to width of the rectangle is 3, what is the cosine of the angle formed by the diagonal with the longer side?

(A) 0.92

(B) 0.83

(C) 0.91

(D) 0.95

2. If Q is the angle formed by the line $3y = 6x + 9$ and the x-axis, then $m\angle Q = \theta$ is equal to

(A) 38.33°

(B) 39.40°

(C) 23.32°

(D) 63.43°

3. If a small ball is laid one hundred feet from a building that is 450 feet tall, what is the degree measure of the angle of depression from the upside of the building to the ball?

(A) 22.17°

(B) 77.47°

(C) 45°

(D) 25.67°

(E) 77.83°

4. Which of the following expressions is equal to $\dfrac{\sin^2\theta - \cos^2\theta}{\cos^4\theta - \sin^4\theta}$, where $0 < \theta < \dfrac{\pi}{2}$?

(A) 1

(B) $\dfrac{\cos^2\theta - \sin^2\theta}{2}$

(C) -1

(D) $\cos^2\theta - \sin^2\theta$

5. Which of the following values is equivalent to $(2\cos x)(9\cos x)+(3\sin x)(6\sin x)$?

(A) 1

(B) 18

(C) 0

(D) 12

6. Which of the following is equivalent to $(\cos x-\sin x)(\sin x+\cos x)+(\sin x-\cos x)(\cos x+\sin x)$?

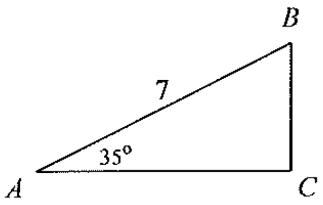
(A) $2\sin^2 x$

(B) 2

(C) 0

(D) 1

Note: Figure below not drawn to scale.



7. In the figure above, what is the sum of AC and BC?

(A) 9.75

(B) 5.73

(C) 4.02

(D) 11.01

8. If the degree measure of an angle is 73°, what is the radian measure of the angle?

(A) 0.24

(B) 1.27

(C) 1.44

(D) 1.23

9. If the ratio of length of a rectangle to width of the rectangle is 2, what is the cosine of the angle formed by the diagonal with the longer side?

(A) 0.96

(B) 1.89

(C) 1.98

(D) .89

10. Where defined, $\frac{\sin x - \cos x}{1 - \tan x}$ is equal to

(A) $\cos x$

(B) $\tan x$

(C) $-\csc x$

(D) $-\cos x$

11. What is the measure of the smallest angle in a 3-4-5 right triangle?

(A) 0.64°

(B) 36.87°

(C) 38.67°

(D) 63.82°

12. In the xy-plane, if three points are $A(2,\ 3)$, $B(5,\ 3)$ and $C(5,\ 7)$, what is the tangent of $\angle ACB$?

(A) $\frac{4}{3}$

(B) $\frac{7}{5}$

(C) $\frac{2}{4}$

(D) $\frac{3}{4}$

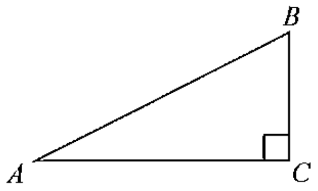
13. For all x, $\sin(2\pi + x) + \sin x + \sin(x + \pi) =$

(A) $\sin x$

(B) $\sin 2x - 2\sin x$

(C) $\sin x + 2\cos x$

(D) $2\sin x + \sin^2 x$



14. In triangle ABC in the figure above, if $\cos A = \frac{12}{13}$, then which of the following is $\cos B$?

(A) $\frac{13}{5}$

(B) $\frac{13}{12}$

(C) $\frac{12}{5}$

(D) $\frac{5}{13}$

15. For all x, $\cos(2\pi + x) + \sin x + \sin(x + \pi) =$

(A) $\cos x + 2\sin x$

(B) $\cos x - 2\sin x$

(C) $\cos x$

(D) $\cos x + \sin^2 x$

16. Which of the following values is equivalent to $(6\cos x)(4\cos x)+(3\sin x)(8\sin x)$?

(A) 24

(B) 18

(C) 0

(D) 1

17. What is the measure of the smallest angle in a 5-12-13 right triangle?

(A) 67.38°

(B) 76.38°

(C) 22.62°

(D) 63.82°

18. If $0<x<\frac{\pi}{2}$, $2\sin^2 x+\sin^2 x\ \frac{1}{\tan^2 x}+\cos^2 x$ equals

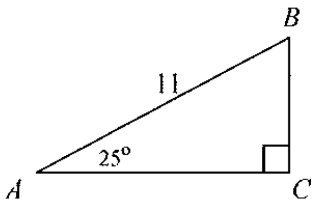
(A) 2

(B) $3\sin^2 x+2$

(C) 1

(D) $2\cos^2 x+1$

Note: Figure below not drawn to scale.



19. In the figure above, what is the sum of AC and BC?

(A) 14.62°

(B) 15.71°

(C) 14.02°

(D) 11.41°

20. If, in triangle ABC, $\angle C$ is a right angle and $\tan A=\frac{\sqrt{3}}{3}$, what is the value of $\sin B-\sin A$?

(A) -0.37°

(B) 1.37°

(C) -1.33°

(D) 0.37°

21. For $0 < \theta < \frac{\pi}{2}$, if the slope of line L_1 is $\frac{3}{4}$ and the slope of line L_2 is $\frac{3}{5}$, which line has the greater measure θ of the angle formed by the line and x-axis?

(A) L_2

(B) No way to know

(C) none

(D) L_1

22. $\frac{4\sqrt{\cos^2 16\theta + \sin^2 16\theta}}{\sqrt{16\sin^2\theta + 16\cos^2\theta}} =$

(A) 4

(B) $\frac{1}{16}$

(C) 8

(D) 1

23. If the degree measure of an angle is 88°, what is the radian measure of the angle?

(A) 0.23

(B) 1.54

(C) 0.97

(D) 1.11

24. $5 - 4\cos^2\theta - 4\sin^2\theta$

(A) 0

(B) 1

(C) 2

(D) 3

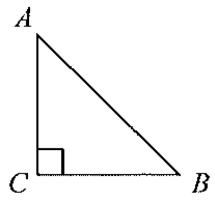
25. If $k > 0$, $p = k\sin\theta$, and $q = k\cos\theta$, then $\dfrac{\sqrt{p^2+q^2}}{k} =$

(A) $\sqrt{k}$

(B) 1

(C) $2k$

(D) k



26. In right triangle ABC above, if $AB = 15$, $BC = 12$, and $AC = 9$, then the sine of $\angle A$ is

(A) $\dfrac{2}{5}$

(B) $\dfrac{4}{5}$

(C) $\dfrac{3}{5}$

(D) $\dfrac{5}{3}$

27. If $(\csc x)(\sec x) < 0$, which of the following must be true?

I. x is in the quadrant II or III.

II. $\csc x < 0$

III. $\sec x < 0$

(A) None

(B) I only

(C) II only

(D) I and II only

28. $(1-\cos\theta)(1+\cos\theta) = ?$

(A) $1+\sin^2\theta$

(B) $\cos^2\theta$

(C) $1+\cos^2\theta$

(D) $\sin^2\theta$

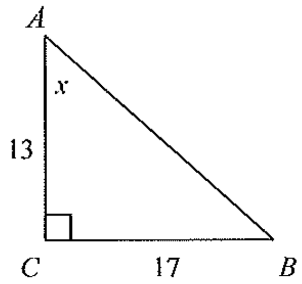
29. If $0 < x < \frac{\pi}{2}$, $2\cos^2 x + \cos^2 x \tan^2 x + \sin^2 x$ equals

(A) $3\cos^2 x + 1$

(B) $3\cos^2 x + 2$

(C) 1

(D) 2



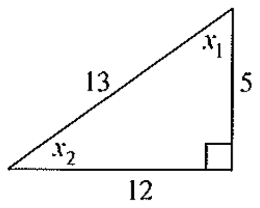
30. In the figure above, $\frac{AC}{BC} = \frac{13}{17}$, $\cos x =$

(A) -0.61

(B) 0.61

(C) 1.12

(D) 0.72



31. In the figure above, which of the following must be true?

I. $\tan x_1 = \frac{5}{12}$

II. $\sin x_2 = \frac{12}{13}$

III. $\sin x_1 = \cos x_2$

(A) I only

(B) II only

(C) III only

(D) I and II only

32. If $\cos x = \frac{6}{11}$, what could $\tan x$ equal?

(A) .017

(B) 0.98

(C) 1.54

(D) 1.13

33. $\sin x + \cos x(\cot x) = ?$

(A) 1

(B) $\sec x$

(C) $\csc x$

(D) $2\sin x$

36. In triangle ABC, if $\angle C$ is a right angle, and $\tan A = \sqrt{3}$, what is the value of $\sin B - \sin A$?

(A)-0.37

(B)-1.23

(C) 1.37

(D)-1.33

37. In right triangle ABC, if $\sin A = \frac{4}{5}$, what is the value of $\cos B$?

(A) $\frac{3}{5}$

(B) $\frac{5}{3}$

(C) $\frac{5}{4}$

(D) $\frac{4}{5}$

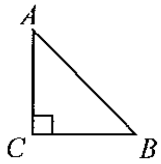
38. $\sqrt{1-\sin^2 x} =$

(A) $\cos x$

(B) $\sin x$

(C) $\sin x - 1$

(D) $|\cos x|$



40. In right triangle ABC above, if $AB = 15$, $BC = 9$, and $AC = 12$, then the sine of $\angle A$ is

(A) $\frac{2}{5}$

(B) $\frac{4}{5}$

(C) $\frac{3}{5}$

(D) $\frac{5}{3}$

41. In a right triangle ABC, if $\sin A = \frac{4}{5}$, what is the value of $\tan B$?

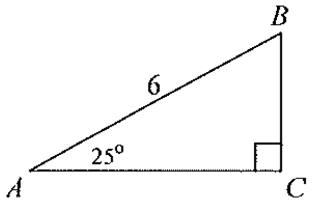
(A) $\frac{4}{5}$

(B) $\frac{5}{3}$

(C) $\frac{5}{4}$

(D) $\frac{3}{4}$

Note: Figure below not drawn to scale.



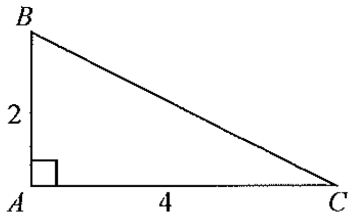
42. In the figure above, what is the difference of AC and BC?

(A) 2.9

(B) 8.0

(C) 4.0

(D) 10



44. As shown in the figure above, what is the cosine of angle C ?

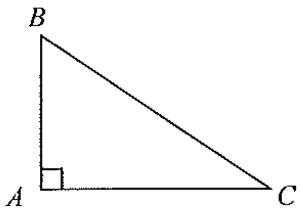
(A) $\frac{4\sqrt{5}}{5}$

(B) $\frac{2\sqrt{5}}{5}$

(C) $\frac{\sqrt{5}}{5}$

(D) $\frac{\sqrt{3}}{4}$

Note: Figure below not drawn to scale.



46. In the figure above, in the right triangle ABC, if $\sin C = 0.8$, what is $\sin B$?

(A) 0.2

(B) 0.4

(C) 0.6

(D) 0.5

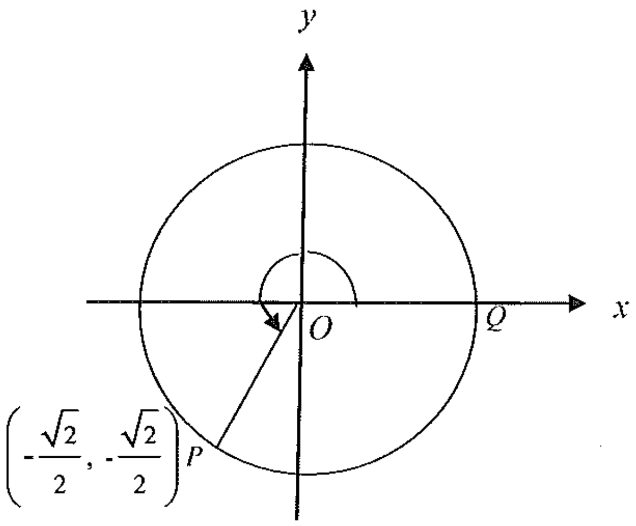
48. If the degree measure of an angle is 98°, what is the radian measure of the angle?

(A) 0.27

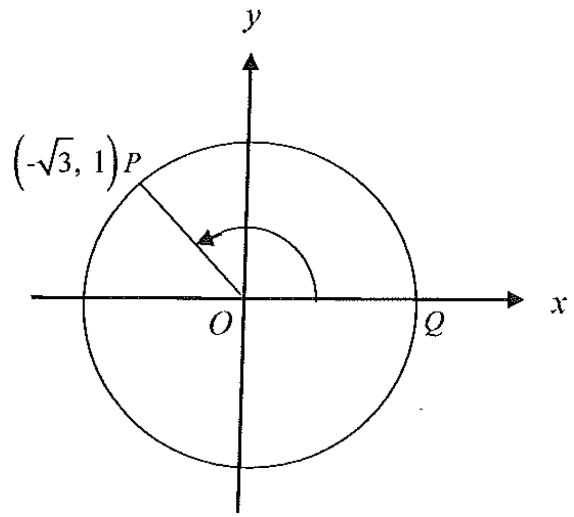
(B) 1.71

(C) 0.71

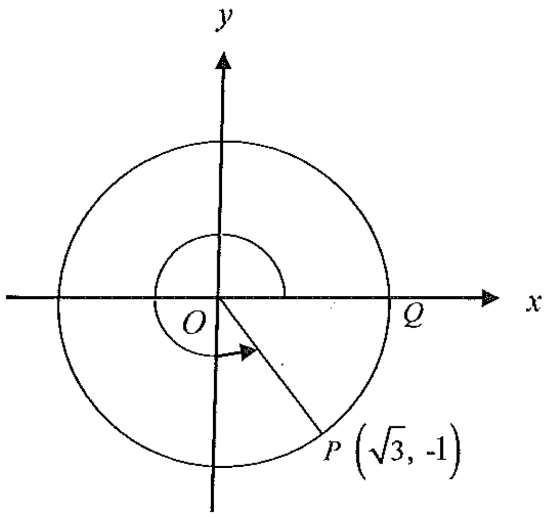
(D) 1.51



49. In the figure above, if the measure of $\angle QOP$ is $\frac{5\pi}{x}$ radians, what is the value of x?



50. In the figure above, if the measure of $\angle QOP$ is $15\pi / x$ radians, what is the value of x?



51. In the figure above, if the measure of $\angle QOP$ is $22\pi / x$ radians, what is the value of x?

1. In right triangle ABC, if $AB = 15$, $BC = 12$, and $AC = 9$, then the sine of $\angle A$ is

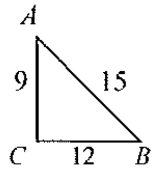
(A) $\frac{1}{5}$

(B) $\frac{4}{5}$

(C) $\frac{3}{5}$

(D) $\frac{5}{3}$

Solution: Answer: (B)



$$\sin A = \frac{12}{15} = \frac{4}{5}$$

2. In a triangle ABC, if $\sin B = \frac{1}{4}$ and $\sin C = \frac{2}{5}$, around what is $\sin A$ equal to?

(A) 0.32

(B) 0.57

(C) 0.62

(D) 0.21

Solution: Answer: (C)

$$\begin{cases} \theta_B = \sin^{-1}\frac{1}{4} = 14.4775^\circ \\ \theta_C = \sin^{-1}\frac{2}{5} = 23.5781^\circ \end{cases}$$

$$\downarrow$$

$$\theta_A = 180 - \theta_B - \theta_C \approx 141.9444^\circ$$

$$\Downarrow$$

$$\sin A = \sin 141.9444^\circ \approx 0.62$$

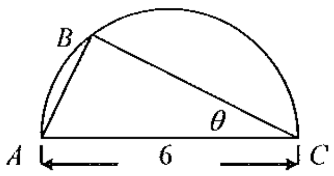


Figure 2

3. In Figure 2, triangle ABC is inscribed in a semicircle. What is the sum of AB and BC in terms of θ ?

(A) 3

(B) $6\sin\theta\cos\theta$

(C) $2\sin(2\theta)$

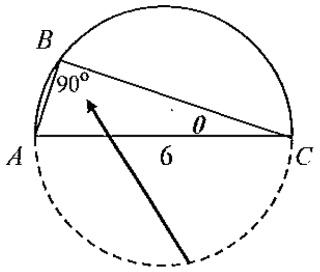
(D) $6(\sin\theta + \cos\theta)$

Solution: Answer: (D)

semicircle

$$\downarrow$$

$\overline{AC}$ = diameter



The value of $AC = 180^\circ$

$$\downarrow$$

$$\angle ABC = \underbrace{\frac{\text{The value of } AC}{2}}_{\text{A rule}} = \frac{180^\circ}{2} = 90^\circ$$

$$\downarrow$$

$$\sin\theta = \frac{AB}{6}, \quad AB = 6\sin\theta$$

$$\cos\theta = \frac{BC}{6}, \quad BC = 6\cos\theta$$

$$\downarrow$$

$$AB + BC = 6\sin\theta + 6\cos\theta$$

$$\Downarrow$$

$$6(\sin\theta + \cos\theta)$$

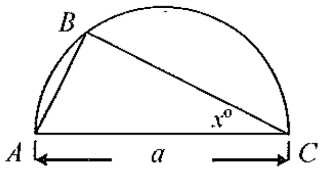


Figure 3

4. In Figure 3, a triangle is inscribed in a semicircle. what is the perimeter of the triangle in terms of x ?

(A) $3a\sin x\cos x$

(B) $a(\sin x+\cos x+1)$

(C) $a\sin x+2a\cos x$

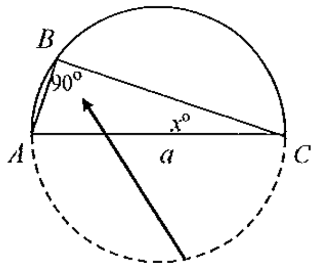
(D) $a(\sin x+\cos x+a)$

Solution: Answer: (B)

semicircle

$\downarrow$

$\overline{AC}$ = diameter



Value of arc $AC = 180^\circ$

$\downarrow$

$$\angle ABC = \underbrace{\frac{\text{Value of arc } AC}{2}}_{\text{A rule}} = \frac{180^\circ}{2} = 90^\circ$$

$\downarrow$

$$\sin x = \frac{AB}{a},\ AB = a\sin x$$

$$\cos x = \frac{BC}{a},\ BC = a\cos x$$

$\downarrow$

Perimeter $= AB + BC + AC$

$\downarrow$

$a\sin x + a\cos x + a$

$\Downarrow$

$a(\sin x+\cos x+1)$

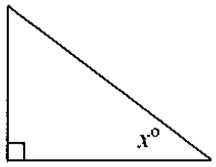


Figure 4

5. In Figure 4, if $\sin x = 0.85$, $\cot x =$

(A) 0.33

(B) 2.31

(C) 1.16

(D) 0.62

Solution: Answer: (D)

ABC is a right triangle.

$\downarrow$

$0 < x < 90^\circ$

$\sin^{-1} 0.85 = x$

$\downarrow$

$x \approx 58.2117^\circ$, $\tan 58.2117^\circ \approx 1.614$

$\downarrow$

$$\cot 58.2117^\circ = \frac{1}{\tan 58.2117^\circ}$$

$\Downarrow$

$$\frac{1}{1.614} \approx 0.62$$

6. In right triangle ABC, if $AB = 13$ and $BC = 12$, what is sine of $\angle B$?

(A) $\frac{14}{5}$

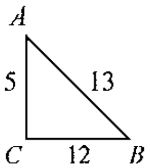
(B) $\frac{4}{5}$

(C) $\frac{5}{13}$

(D) $\frac{5}{3}$

Solution: Answer: (C)

5-12-13 right triangle



$\Downarrow$

$$\sin B = \frac{5}{13}$$

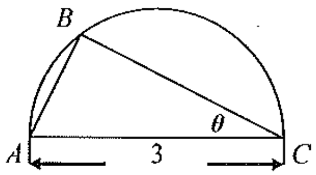


Figure 5

7. In Figure 5, a triangle is inscribed in a semicircle with diameter 3. What is the perimeter of the triangle in terms of θ?

(A) $\dfrac{9\sin\theta\cdot\cos\theta}{2}$ (B) $\dfrac{6\sin\theta\cdot\cos\theta}{2}$

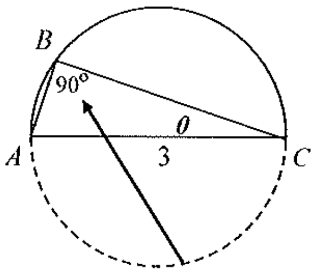
(C) $3\sin\theta+\cos\theta+3$ (D) $3(\sin\theta+\cos\theta+1)$

Solution: Answer: (D)

semicircle

$\downarrow$

$\overline{AC}$ = diameter



Value of $AC = 180^\circ$

$\downarrow$

$$\angle ABC = \underbrace{\frac{\text{Value of } AC}{2}}_{\text{A rule}} = \frac{180^\circ}{2} = 90^\circ$$

$\downarrow$

$$\sin\theta = \frac{AB}{3},\quad AB = 3\sin\theta$$

$$\cos\theta = \frac{BC}{3},\quad BC = 3\cos\theta$$

$\downarrow$

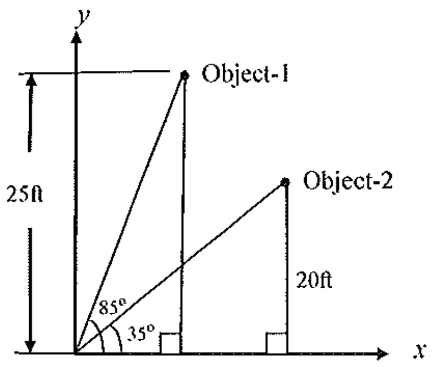
Perimeter $= AB + BC + AC$

$\downarrow$

$3\sin\theta + 3\cos\theta + 3$

$\Downarrow$

$3(\sin\theta+\cos\theta+1)$



8. In the figure above, what is the distance d, in feet, between Object-1 and Object-2 ?

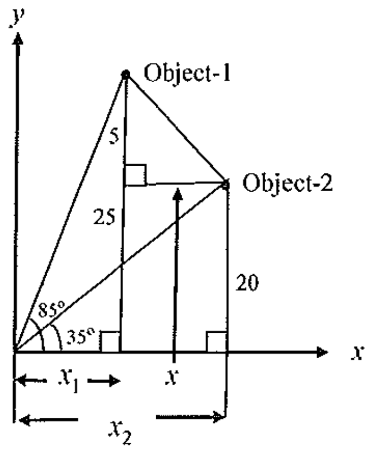
(A) 26.84

(B) 28.64

(C) 24.64

(D) 26.48

Solution: Answer: (A)



$$d = \sqrt{x^2 + 5^2}$$

$$\begin{cases} \tan 85^\circ = \dfrac{25}{x_1}, & x_1 = \dfrac{25}{\tan 85^\circ} = 2.19 \\ \tan 35^\circ = \dfrac{20}{x_2}, & x_2 = \dfrac{20}{\tan 35^\circ} = 28.56 \end{cases}$$

$\downarrow$

$$x = x_2 - x_1 = 28.56 - 2.19 = 26.37$$

$\Downarrow$

$$d = \sqrt{26.37^2 + 5^2} = 26.84$$

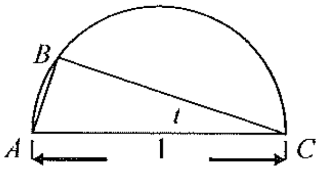


Figure 7

9. In Figure 7, a triangle is inscribed in a semicircle. what is the area of the triangle in terms of t ?

(A) $\sin t \cos t$

(B) $\dfrac{\sin t \cos t}{2}$

(C) $2 \sin t \cos t$

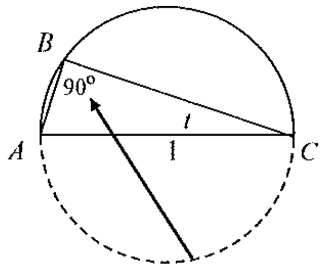
(D) $\dfrac{\sin t \cos t}{4}$

Solution: Answer: (B)

semicircle

$\downarrow$

$\overline{AC}$ = diameter



Value of $AC = 180^\circ$

$\downarrow$

$$\angle ABC = \underbrace{\frac{\text{Value of } AC}{2}}_{\text{A rule}} = \frac{180^\circ}{2} = 90^\circ$$

$\downarrow$

$$\sin t = \frac{AB}{1},\ AB = \sin t$$

$$\cos t = \frac{BC}{1},\ BC = \cos t$$

$\Downarrow$

$$\text{Area} = \frac{AB \cdot BC}{2} = \frac{\sin t \cdot \cos t}{2}$$

10. If $\sin x = -u$, $u > 0$, in $\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$ and $\dfrac{3\pi}{2} < x < 2\pi$, what is $\tan x$?

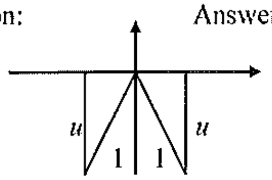
(A) $\dfrac{u}{\sqrt{1^2 - u^2}}$

(B) $\dfrac{\sqrt{1^2 - u^2}}{u}$

(C) $\sqrt{1^2 - u^2}$

(D) $\pm\dfrac{u}{\sqrt{1^2 - u^2}}$

Solution: Answer: (D)



$$\sin x = -\frac{u}{1} = -u$$

(1) When $\sin x$ is in quadrant III,

$$\cos x = -\frac{\sqrt{1^2 - u^2}}{1} = -\sqrt{1^2 - u^2}$$

$$\tan x = \frac{-u}{-\sqrt{1^2 - u^2}} = +\frac{u}{\sqrt{1^2 - u^2}}$$

(2) When $\sin x$ is in quadrant IV,

$$\cos x = \frac{\sqrt{1^2 - u^2}}{1} = \sqrt{1^2 - u^2}$$

$$\tan x = \frac{-u}{\sqrt{1^2 - u^2}} = -\frac{u}{\sqrt{1^2 - u^2}}$$

By (1) and (2),

$$\tan x = \pm\frac{u}{\sqrt{1^2 - u^2}}$$

11. In right triangle ABC, if $AB = 15$, $BC = 12$, and $AC = 9$, then the sine of $\angle B$ is

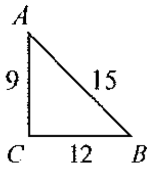
(A) $\dfrac{1}{5}$

(B) $\dfrac{4}{5}$

(C) $\dfrac{3}{5}$

(D) $\dfrac{5}{3}$

Solution: Answer: (C)



$$\sin B = \frac{9}{15} = \frac{3}{5}$$

12. In a triangle ABC, if $\sin B = \frac{1}{6}$ and $\sin C = \frac{3}{5}$, around what is $\sin A$ equal to?

(A) 0.371
(B) 0.573
(C) 0.725
(D) 0.270

Solution: Answer: (C)

$$\begin{cases} x_B = \sin^{-1}\frac{1}{6} \approx 9.5941^\circ \\ x_C = \sin^{-1}\frac{3}{5} \approx 36.8699^\circ \end{cases}$$

$$\downarrow$$

$$x_A = 180 - x_B - x_C \approx 133.536^\circ$$

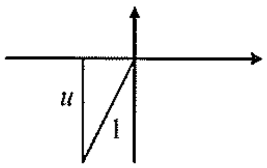
$$\Downarrow$$

$$\sin A = \sin 133.536^\circ \approx 0.725$$

13. If $\sin x = -u$, $u > 0$, in $\frac{\pi}{2} < x < \frac{3}{2}\pi$, which of the following must be true for $\tan x$?

(A) $\frac{u}{\sqrt{1^2 - u^2}}$
(B) $\frac{\sqrt{1^2 - u^2}}{u}$
(C) $\sqrt{1^2 - u^2}$
(D) $\pm\frac{u}{\sqrt{1^2 - u^2}}$

Solution: Answer: (A)



$$\uparrow$$

$$\sin x = -u = -\frac{u}{1}$$

$$\downarrow$$

$\sin x$ occurs in quadrant III.

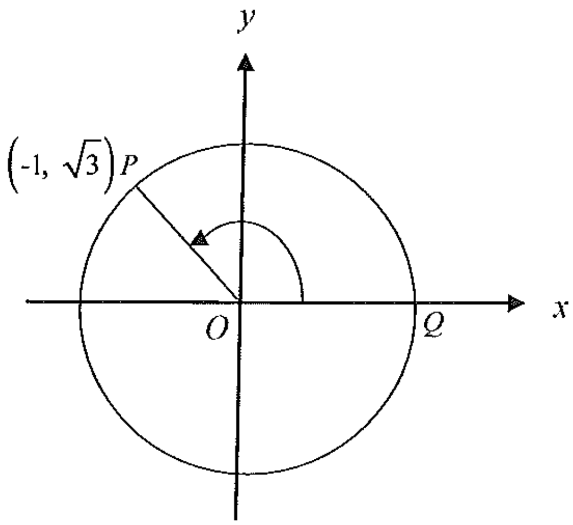
$$\downarrow$$

$$\cos x = -\frac{\sqrt{1^2 - u^2}}{1} = -\sqrt{1^2 - u^2}$$

$$\tan x = \frac{\sin x}{\cos x}$$

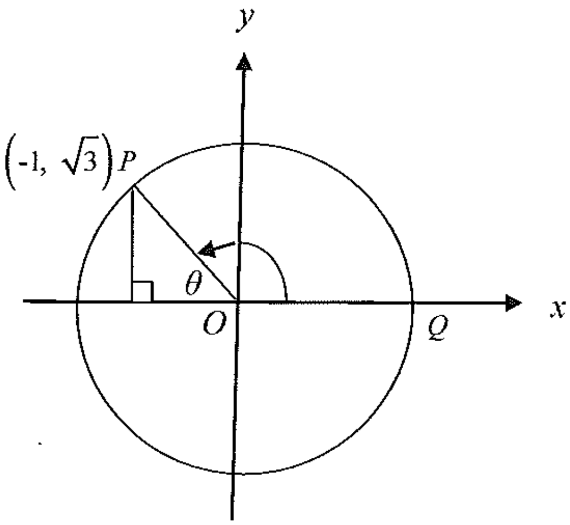
$$\Downarrow$$

$$\tan x = \frac{-u}{-\sqrt{1^2 - u^2}} = +\frac{u}{\sqrt{1^2 - u^2}}$$



14. In the figure above, if the measure of $\angle QOP$ is $\frac{2\pi}{x}$ radians, what is the value of x?

Solution: Answer: 3



$$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{-1}$$

$$\downarrow$$

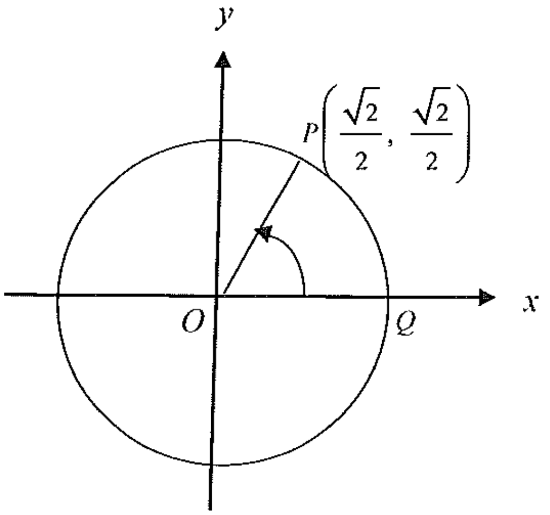
$$\text{Reference angle} = \theta = 60^\circ \cdot \frac{\pi}{180} = \frac{\pi}{3}$$

$$\downarrow$$

$$\angle QOP = \pi - \frac{\pi}{3} = \frac{2\pi}{3}, \quad \frac{2\pi}{3} = \frac{2\pi}{x}$$

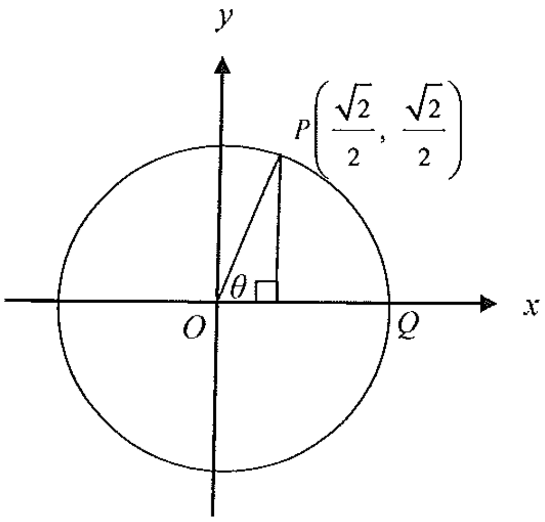
$$\Downarrow$$

$$x = 3$$



15. In the figure above, if the measure of $\angle QOP$ is $\frac{2\pi}{x}$ radians, what is the value of x?

Solution: Answer: 8



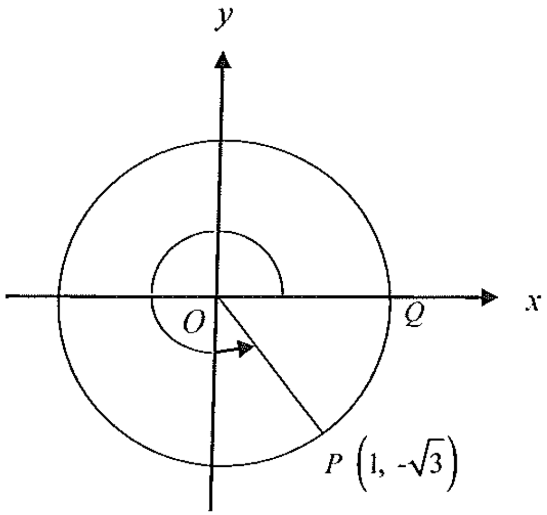
$$\tan\theta = \frac{y}{x} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1, \quad \theta = 45^\circ \cdot \frac{\pi}{180} = \frac{\pi}{4}$$

$$\downarrow$$

$$\angle QOP = \theta, \quad \frac{\pi}{4} = \frac{2\pi}{x}, \quad \frac{4}{1} = \frac{x}{2}$$

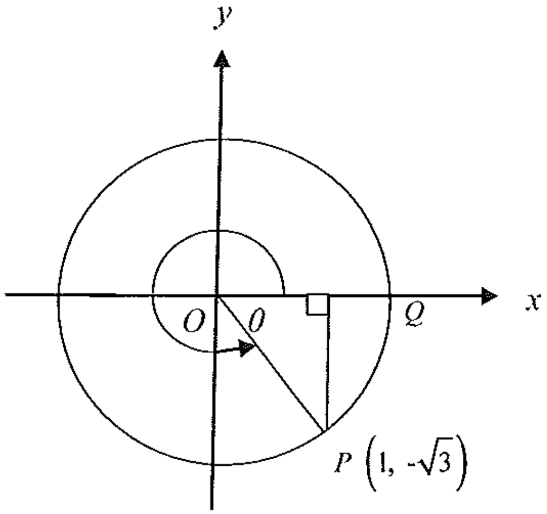
$$\Downarrow$$

$$x = 8$$



16. In the figure above, if the measure of $\angle QOP$ is $\frac{10\pi}{x}$ radians, what is the value of x?

Solution: Answer: 6



$$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{1}$$

$$\downarrow$$

$$\text{Reference angle} = \theta = 60^\circ \cdot \frac{\pi}{180} = \frac{\pi}{3}$$

$$\downarrow$$

$$\angle QOP = 2\pi - \frac{\pi}{3} = \frac{6\pi - \pi}{3} = \frac{5\pi}{3}$$

$$\Downarrow$$

$$\frac{5\pi}{3} = \frac{10\pi}{x}, \quad \frac{3}{5} = \frac{x}{10}, \quad \boxed{x = 6}$$

1. In right triangle ABC, if $AB = 15$, $BC = 12$, and $AC = 9$, then the sine of $\angle A$ is

(A) $\frac{1}{5}$

(B) $\frac{4}{5}$

(C) $\frac{3}{5}$

(D) $\frac{5}{3}$

2. In a triangle ABC, if $\sin B = \frac{1}{4}$ and $\sin C = \frac{2}{5}$, around what is $\sin A$ equal to?

(A) 0.32

(B) 0.57

(C) 0.62

(D) 0.21

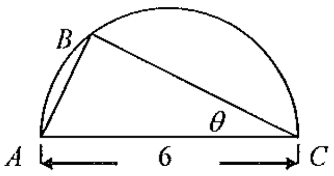


Figure 2

3. In Figure 2, triangle ABC is inscribed in a semicircle. What is the sum of AB and BC in terms of θ ?

(A) 3

(B) $6\sin\theta\cos\theta$

(C) $2\sin(2\theta)$

(D) $6(\sin\theta + \cos\theta)$

Right Triangles and a Triangle in a Circle or Semicircle

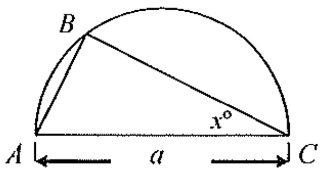


Figure 3

4. In Figure 3, a triangle is inscribed in a semicircle. what is the perimeter of the triangle in terms of x ?

(A) $3a\sin x\cos x$

(B) $a(\sin x+\cos x+1)$

(C) $a\sin x+2a\cos x$

(D) $a(\sin x+\cos x+a)$

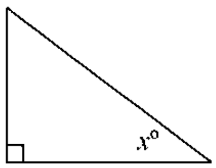


Figure 4

5. In Figure 4, if $\sin x=0.85$, $\cot x=$

(A) 0.33

(B) 2.31

(C) 1.16

(D) 0.62

6. In right triangle ABC, if $AB=13$ and $BC=12$, what is sine of $\angle B$?

(A) $\frac{14}{5}$

(B) $\frac{4}{5}$

(C) $\frac{5}{13}$

(D) $\frac{5}{3}$

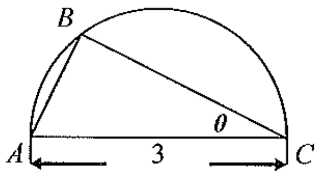


Figure 5

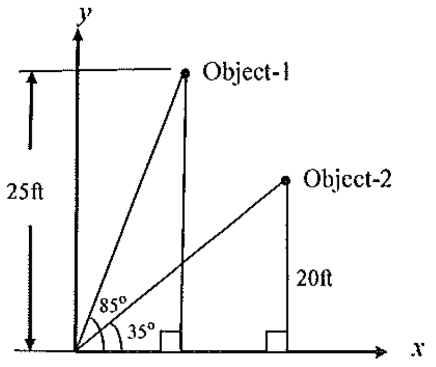
7. In Figure 5, a triangle is inscribed in a semicircle with diameter 3. What is the perimeter of the triangle in terms of θ?

(A) $\frac{9\sin\theta\cdot\cos\theta}{2}$

(B) $\frac{6\sin\theta\cdot\cos\theta}{2}$

(C) $3\sin\theta+\cos\theta+3$

(D) $3(\sin\theta+\cos\theta+1)$



8. In the figure above, what is the distance d, in feet, between Object-1 and Object-2 ?

(A) 26.84

(B) 28.64

(C) 24.64

(D) 26.48

Right Triangles and a Triangle in a Circle or Semicircle

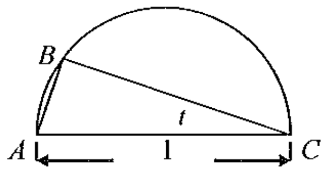


Figure 7

9. In Figure 7, a triangle is inscribed in a semicircle. what is the area of the triangle in terms of t ?

(A) $\sin t \cos t$

(B) $\dfrac{\sin t \cos t}{2}$

(C) $2 \sin t \cos t$

(D) $\dfrac{\sin t \cos t}{4}$

10. If $\sin x = -u$, $u > 0$, in $\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$ and $\dfrac{3\pi}{2} < x < 2\pi$, what is $\tan x$?

(A) $\dfrac{u}{\sqrt{1^2 - u^2}}$

(B) $\dfrac{\sqrt{1^2 - u^2}}{u}$

(C) $\sqrt{1^2 - u^2}$

(D) $\pm \dfrac{u}{\sqrt{1^2 - u^2}}$

11. In right triangle ABC, if $AB = 15$, $BC = 12$, and $AC = 9$, then the sine of $\angle B$ is

(A) $\dfrac{1}{5}$

(B) $\dfrac{4}{5}$

(C) $\dfrac{3}{5}$

(D) $\dfrac{5}{3}$

12. In a triangle ABC, if $\sin B = \frac{1}{6}$ and $\sin C = \frac{3}{5}$, around what is $\sin A$ equal to?

(A) 0.371

(B) 0.573

(C) 0.725

(D) 0.270

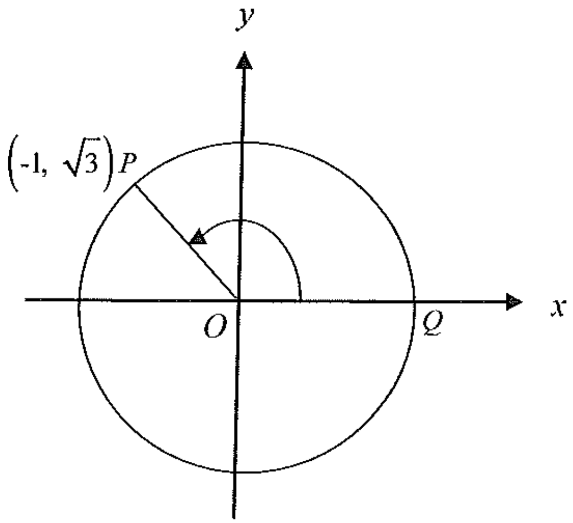
13. If $\sin x = -u$, $u > 0$, in $\frac{\pi}{2} < x < \frac{3}{2}\pi$, which of the following must be true for $\tan x$?

(A) $\frac{u}{\sqrt{1^2 - u^2}}$

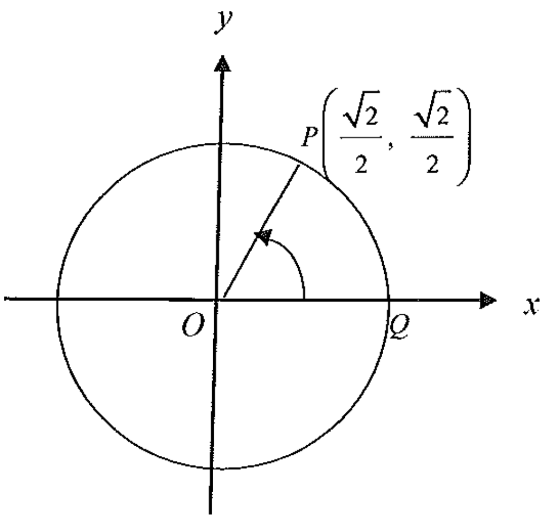
(B) $\frac{\sqrt{1^2 - u^2}}{u}$

(C) $\sqrt{1^2 - u^2}$

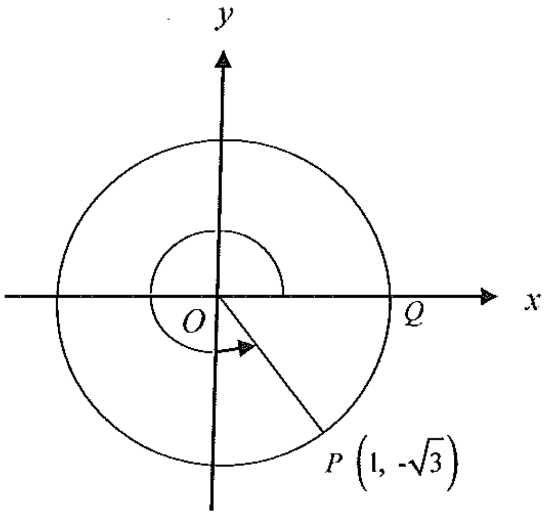
(D) $\pm\frac{u}{\sqrt{1^2 - u^2}}$



14. In the figure above, if the measure of $\angle QOP$ is $\frac{2\pi}{x}$ radians, what is the value of x ?



15. In the figure above, if the measure of $\angle QOP$ is $\frac{2\pi}{x}$ radians, what is the value of x?



16. In the figure above, if the measure of $\angle QOP$ is $\frac{10\pi}{x}$ radians, what is the value of x?

1. Which of the following equations must be true if $x + y = 90^\circ$?

(A) $\dfrac{\sin x}{\cos x} = \dfrac{\sin y}{\cos y}$

(B) $\sin x = \sin y$

(C) $\sin x = -\sin y$

(D) $\dfrac{\sin x}{\cos x} = \dfrac{\cos y}{\sin y}$

Solution: Answer: (D)

They are complementary.

$\downarrow$

$\tan x = \cot y$

$\tan x = \dfrac{\sin x}{\cos x}$, $\cot y = \dfrac{\cos y}{\sin y}$

$\Downarrow$

$\dfrac{\sin x}{\cos x} = \dfrac{\cos y}{\sin y}$

2. What is $\sin(90^\circ - A)$ equal to if $\cos A = 0.21$ and A is an acute angle in degree measure?

(A) 0.72

(B) 0.53

(C) 0.21

(D) 0.44

Solution: Answer: (C)

$\cos A = \sin(90^\circ - A)$

Let $B = 90^\circ - A$.

$\downarrow$

$A + B = 90^\circ$

$\downarrow$

A and B are complementary.

$\Downarrow$

$\sin(90^\circ - A) = 0.21$

3. What is $\sin(90^\circ - A)$ equal to if $\cos A = 0.3$ and A is an acute angle in degree measure?

(A) 0.7

(B) 0.5

(C) 0.3

(D) 0.4

Solution: Answer: (C)

$\cos A = \sin(90^\circ - A)$

$\Downarrow$

$\sin(90^\circ - A) = 0.3$

4. In triangle ABC, $m\angle C$ is 90°. Let $\begin{cases} A = \sin^{-1} x \\ B = \cos^{-1} x \end{cases}$.

If $\dfrac{1}{2}\sin^{-1} x = \cos^{-1} x$, what is the value of x?

(A) $\dfrac{1}{\sqrt{3}}$

(B) $\dfrac{\sqrt{3}}{2}$

(C) $\dfrac{1}{2}$

(D) $\dfrac{1}{3}$

Solution: Answer: (B)

$\begin{cases} A = \sin^{-1} x \\ B = \cos^{-1} x \end{cases} \rightarrow \begin{cases} \sin A = x \\ \cos B = x \end{cases}$

$\dfrac{1}{2}\sin^{-1} x = \cos^{-1} x$

$\downarrow$

$\sin^{-1} x = 2\cos^{-1} x$

$\downarrow$

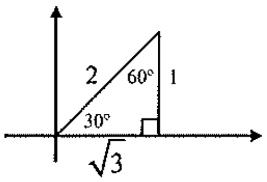
$\begin{cases} A + B = 90 \\ A = 2B \end{cases}$

$\downarrow$

$A = 60^\circ$ and $B = 30^\circ$

$\Downarrow$

$x = \sin 60^\circ = \cos 30^\circ = \dfrac{\sqrt{3}}{2}$



5. Which of the following must be true if $x + y = 90^\circ$?

(A) $\sin x = -\cos y$

(B) $\sin x = \cos y$

(C) $\sin^2 x = \cos y$

(D) $\sin x = \cos^2 y$

Solution: Answer: (B)

x and y are complementary.

sine and cosine are cofunctions.

Cofunctions of complementary angles are equal.

$\Downarrow$

$\sin x = \cos y$

6. In triangle ABC, $m\angle C$ is 90°. Let $\begin{cases} A = \sin^{-1} x \\ B = \cos^{-1} x \end{cases}$.

If $2\sin^{-1} x = \cos^{-1} x$, what is the value of x?

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\frac{1}{3}$

Solution: Answer: (A)

$$\begin{cases} A = \sin^{-1} x \\ B = \cos^{-1} x \end{cases} \rightarrow \begin{cases} \sin A = x \\ \cos B = x \end{cases}$$

$$2\sin^{-1} x = \cos^{-1} x$$

$$\downarrow$$

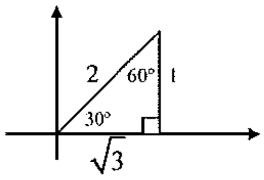
$$\begin{cases} A + B = 90^\circ \\ 2A = B \end{cases}$$

$$\downarrow$$

$$A = 30^\circ \text{ and } B = 60^\circ$$

$$\Downarrow$$

$$x = \sin 30^\circ = \cos 60^\circ = \frac{1}{2}$$



7. In triangle ABC, $m\angle C$ is 90°. Which of the following must be true?

I. $\cos^2 B + \sin^2 B = 1$

II. $\sin B = \cos A$

III. $\cot A = \tan B$

IV. $\cos^2 A + \sin^2 B = 1$

(A) I only (B) II only (C) III only

(D) I, II and III only

Solution: Answer: (D)

$$\begin{cases} \text{I. } \underbrace{\cos^2 B + \sin^2 B = 1}_{\text{formula}} \\ \text{IV. } \underbrace{\cos^2 A + \sin^2 B = 1}_{\text{not "must"}} \end{cases}$$

$$\downarrow$$

I is true.

IV is false.

$$m\angle C = 90^\circ$$

$$\downarrow$$

$\angle A$ and $\angle B$ are complementary angles

$$\downarrow$$

$$\begin{cases} \cos A = \sin B \\ \cot A = \tan B \end{cases}$$

$$\downarrow$$

II and III are true.

$$\Downarrow$$

Answer is (D).

1. Which of the following equations must be true if $x + y = 90^\circ$?

(A) $\dfrac{\sin x}{\cos x} = \dfrac{\sin y}{\cos y}$

(B) $\sin x = \sin y$

(C) $\sin x = -\sin y$

(D) $\dfrac{\sin x}{\cos x} = \dfrac{\cos y}{\sin y}$

2. What is $\sin(90^\circ - A)$ equal to if $\cos A = 0.21$ and A is an acute angle in degree measure?

(A) 0.72

(B) 0.53

(C) 0.21

(D) 0.44

3. What is $\sin(90^\circ - A)$ equal to if $\cos A = 0.3$ and A is an acute angle in degree measure?

(A) 0.7

(B) 0.5

(C) 0.3

(D) 0.4

4. In triangle ABC, $m\angle C$ is 90°. Let $\begin{cases} A = \sin^{-1} x \\ B = \cos^{-1} x \end{cases}$.

If $\dfrac{1}{2}\sin^{-1} x = \cos^{-1} x$, what is the value of x?

(A) $\dfrac{1}{\sqrt{3}}$

(B) $\dfrac{\sqrt{3}}{2}$

(C) $\dfrac{1}{2}$

(D) $\dfrac{1}{3}$

5. Which of the following must be true if $x + y = 90^\circ$?

(A) $\sin x = -\cos y$

(B) $\sin x = \cos y$

(C) $\sin^2 x = \cos y$

(D) $\sin x = \cos^2 y$

6. In triangle ABC, $m\angle C$ is 90°. Let $\begin{cases} A = \sin^{-1} x \\ B = \cos^{-1} x \end{cases}$.

If $2\sin^{-1} x = \cos^{-1} x$, what is the value of x?

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\frac{1}{3}$

7. In triangle ABC, $m\angle C$ is 90°. Which of the following must be true?

I. $\cos^2 B + \sin^2 B = 1$

II. $\sin B = \cos A$

III. $\cot A = \tan B$

IV. $\cos^2 A + \sin^2 B = 1$

(A) I only (B) II only (C) III only

(D) I, II and III only

Others

1. Patricia designed a computer program. It at random chooses a positive one-digit integer. If the integer chosen is even, half that integer is printed. If the integer chosen is odd, the integer itself is printed. If the integer printed is 3, which of the following could have been the integer chosen?

I. 3
II. 6
III. 0

(A) III only
(B) I and II only
(C) II and III only
(D) I and III only

Solution: Answer: (B)

The number 3 printed can be from even number 6 divided by 2 or odd number 3.

⇓

Answer is (B).

2. Bill designed a computer program. It at random chooses a positive 2-digit integer. If the integer chosen is prime, the integer multiplied by 2 is printed. If the integer chosen is even, the integer divided by 11 is printed. If the integer printed is 22, which of the following could have been the integer chosen?

I. 11
II. 2
III. 22

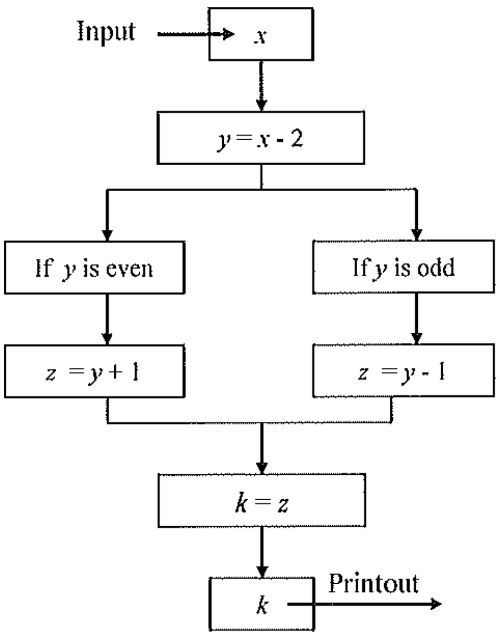
(A) III only
(B) II only
(C) II and III only
(D) I only

Solution: Answer: (D)

The number 22 is not chosen.
The number 22 printed can be from prime number 11 multiplied by 2.

⇓

Answer is (D).



3. Jeff inputs an integer x greater than 1 to a computer program. The program follows the steps in the diagram above to get k. Which of the following statements must be true?

I. $k \geq 0$
II. y is odd
III. k is odd or even

(A) I only
(B) III only
(C) III and I only
(D) I only

Solution: Answer: (C)

Let $x = 2$ (even) and 3 (odd).

Then $y = 0$ and 1.

↓

z = 1 and 0
k = 1 and 0

↓

$k \geq 0$

y may be odd and may be even.
k is odd or even.

↓

III and I = True.

⇓

Answer is (C).

4. Brent designed a computer program. It at random chooses a one-digit integer. If the integer chosen is even, the integer added by 2 is printed. If the integer chosen is prime, the integer itself is printed. If the integer printed is 2, which of the following could have been the integer chosen?

I. 0
II. 2
III. 4

(A) I only
(B) II only
(C) II and III only
(D) I and II only

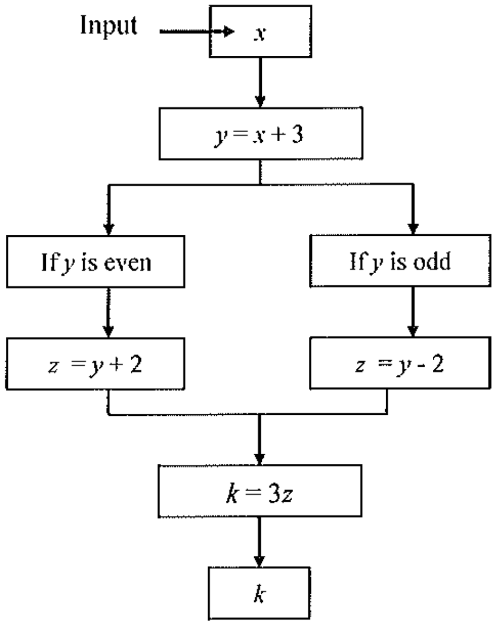
Solution: Answer: (D)

Number 0 is even. Number 2 is prime number.

The number 2 printed can be from even number 2 added by 2 or prime number 2.

$\Downarrow$

Answer is (D).



5. Emily inputs a positive integer x to a computer program. The program follows the steps in the diagram above to get k. Which of the following statements must be true?

I. $k < 0$
II. z is even
III. k is odd

(A) I only
(B) III only
(C) I and III only
(D) None

Solution: Answer: (D)

I. Let x = 1 (odd) and 2 (even), then y = 4 and 5.

$\downarrow$

$\begin{cases} z = 4 \text{ and } 3 \\ k = 12 \text{ and } 9 \end{cases}$

$\downarrow$

$k > 0$

$\begin{cases} z \text{ may be even or not.} \\ k \text{ may be odd or not.} \end{cases}$

$\downarrow$

I, II and III = False.

$\Downarrow$

Answer is (D).

1. Patricia designed a computer program. It at random chooses a positive one-digit integer. If the integer chosen is even, half that integer is printed. If the integer chosen is odd, the integer itself is printed. If the integer printed is 3, which of the following could have been the integer chosen?

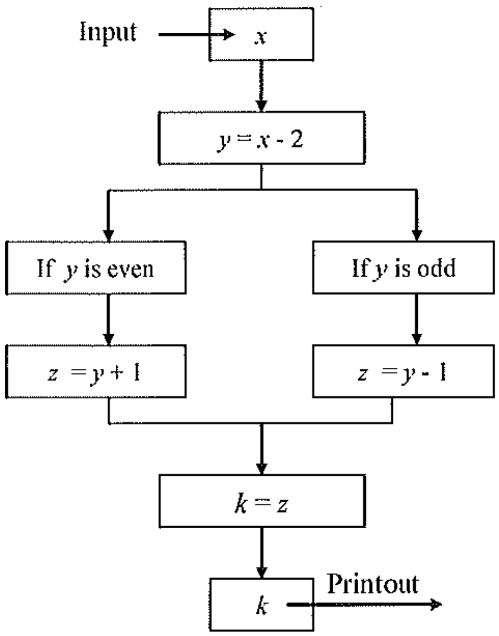
I. 3
II. 6
III. 0

(A) III only
(B) I and II only
(C) II and III only
(D) I and III only

2. Bill designed a computer program. It at random chooses a positive 2-digit integer. If the integer chosen is prime, the integer multiplied by 2 is printed. If the integer chosen is even, the integer divided by 11 is printed. If the integer printed is 22, which of the following could have been the integer chosen?

I. 11
II. 2
III. 22

(A) III only
(B) II only
(C) II and III only
(D) I only



3. Jeff inputs an integer x greater than 1 to a computer program. The program follows the steps in the diagram above to get k. Which of the following statements must be true?

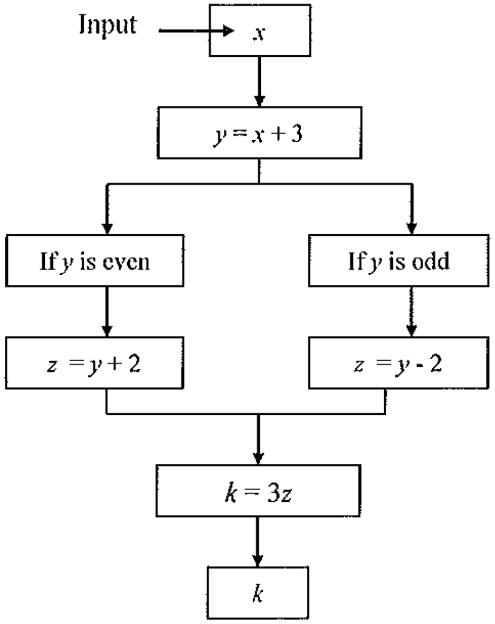
I. $k \geq 0$
II. y is odd
III. k is odd or even

(A) I only
(B) III only
(C) III and I only
(D) I only

4. Brent designed a computer program. It at random chooses a one-digit integer. If the integer chosen is even, the integer added by 2 is printed. If the integer chosen is prime, the integer itself is printed. If the integer printed is 2, which of the following could have been the integer chosen?

I. 0
II. 2
III. 4

(A) I only
(B) II only
(C) II and III only
(D) I and II only



5. Emily inputs a positive integer x to a computer program. The program follows the steps in the diagram above to get k. Which of the following statements must be true?

I. $k < 0$
II. z is even
III. k is odd

(A) I only
(B) III only
(C) I and III only
(D) None

1. If $(a-b)^2 = a^2 + b^2$, which of the following must also be true?

I. $ab = 0$

II. $(a+b)^2 = a^2 + b^2$

III. $b = 0$

(A) I only

(B) II only

(C) III only

(D) I and II only

Solution: Answer: (D)

$$\underline{(a-b)^2} = a^2 + b^2$$

$$\downarrow$$

$$a^2 - 2ab + b^2$$

$$\downarrow$$

$$a^2 - 2ab + b^2 = a^2 + b^2$$

$$\downarrow$$

$$-ab = 0 \rightarrow ab = 0$$

$$\Downarrow$$

$a = 0$ and/or $b = 0$

2. If $p^2 + 2pq + q^2 = p^2 + q^2$, which of the following must also be true?

I. $pq = 0$

II. $(p-q)^2 = p^2 + q^2$

III. $p = 0$

(A) I only

(B) II only

(C) III only

(D) I and II only

Solution: Answer: (D)

$$p^2 + 2pq + q^2 = p^2 + q^2$$

$$\downarrow$$

$$2pq = 0 \rightarrow pq = 0$$

$$\downarrow$$

$p = 0$ and/or $q = 0$

$$\Downarrow$$

I is true.

II. $(p-q)^2 = p^2 - 2pq + q^2$

$$\downarrow$$

$$p^2 - 0 + q^2 = p^2 + q^2$$

$$\Downarrow$$

II is true.

III. p may or may not equal 0,

$$\Downarrow$$

III is false.

3. A list of 10 positive integers has the property that the average, A, of the integers is greater than the median, M, of the integers. Which of the following statements must be true?

I. More of these integers are greater than A than are less than A.

II. More of these integers are greater than M than are equal to M.

III. More of these integers are less than M than are equal to M.

A) I, II, and III

(B) I and II only

(C) II and III only

(D) None

Solution: Answer: (D)

I. The 10 integers can be $\overbrace{1, 1, \cdots, 1}^{9 \text{ of the 10 integers}}, 2$

$$A = \frac{\overbrace{1+1+\cdots+1}^{9 \text{ of the 10 integers}} + 2}{10} = \frac{11}{10}$$

$$\downarrow$$

$$1 < A = \frac{11}{10}$$

$$\downarrow$$

A counter-example exists.

$$\Downarrow$$

I is false.

II. The 10 integers can be $\overbrace{1, 1, \cdots, 1}^{9 \text{ of the 10 integers}}, 2$

$$M = 1$$

$$\downarrow$$

More of the integers $= 1 = M$, not $> M$.

$$\downarrow$$

A counter-example exists.

$$\Downarrow$$

II is false.

III. The 10 integers can be $\overbrace{1, 1, \cdots, 1}^{9 \text{ of the 10 integers}}, 2$

$$M = 1$$

$$\downarrow$$

More of the integers $= 1 = M$, not $< M$.

$$\downarrow$$

A counter-example exists.

$$\Downarrow$$

III is false.

RESULTS OF GAME

Number of Shots	Number of People
1	8
2	7
3	7
4	4
5	3
7	2

4. The chart above shows the results for 31 people who played a certain game. In the game, each person shot a ball at a target till the person missed the target. According to the information in the chart, which of the following must be true?

I. For all of the shots, more hit the target than missed the target.

II. No people hit the target 6 times.

III. Less than half of the people hit the target on their first shot.

(A) I only
(B) II only
(C) I and II only
(D) II and III only

Solution: Answer: (A)

I. (1). Number of targets missed:

The people each missed one time.

$N_{\text{missed}} = 31$

(2). Number of targets hit:

$7 \cdot 1 + 7 \cdot 2 + 4 \cdot 3 + 3 \cdot 4 + 2 \cdot 6 = 57$

$57 > 31$

$\Downarrow$

I is true.

II. 2 people hit the target 6 times.

They missed the targets on their 7th shot.

$\Downarrow$

II is false.

III. $31 - 8 = 23$ people (more than half) hit the targets on their first shot. $23 > \frac{31}{2}$.

$\Downarrow$

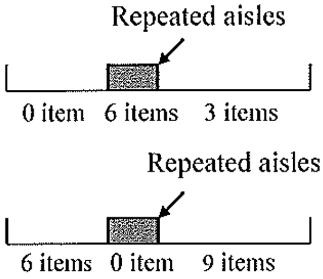
III is false.

5. In a department store, Monica bought 6 items from aisles 1 through 6 and 9 items from aisles 4 through 11. Which of the following could be the total number of items that Monica bought?

I. 8
II. 9
III. 12

(A) None
(B) I and II only
(C) II and III only
(D) I and III only

Solution: Answer: (C)



Consider 2 situations (1) and (2) below.

(1). The 6 items of 9 items came from repeated aisles.

$\downarrow$

The least number of items = 9

(2). All the 9 items from aisles 4 through 11were bought from not repeated aisles.

$\downarrow$

The greatest number of items = 6 + 9 =15.

$\downarrow$

$9 \le x \le 15$

$\Downarrow$

Answer is (C).

6. If x is an integer greater than 1 and $y = x - \frac{1}{x}$, which of the following must be true?

I. $y = (x-1) + \frac{x-1}{x}$

II. y is not an integer.

III. $xy < x^2$

(A) I only

(B) II only

(C) III only

(D) I and II only

Solution: Answer: (D)

I. $y = (x-1) + \frac{x-1}{x} = \frac{x(x-1)+x-1}{x}$

$\downarrow$

$\frac{x^2 \cancel{-x} \cancel{+x} - 1}{x} = \frac{x^2-1}{x}$

$\downarrow$

$\frac{x^2}{x} - \frac{1}{x} = x - \frac{1}{x}$

$\Downarrow$

I is true.

II. $x > 1$

$\downarrow$

$x - \frac{1}{x} \neq$ integer

$\Downarrow$

II is true.

III. $xy = x\left(x - \frac{1}{x}\right)$

$\downarrow$

$x^2 - 1, \quad x^2 - 1 < x^2$

$\Downarrow$

III is true.

7. If x is an integer greater than 1 and $y = \frac{1}{x} - x$, which of the following must be true?

I. $-x > \frac{1}{x} - x$

II. $y < 0$

III. $xy < x^2$

(A) I only (B) II only (C) III only (D) II, III only

Solution: Answer: (D)

I. If $-x > \frac{1}{x} - x$, $-x - \left(\frac{1}{x} - x\right) > 0$.

$-x - \left(\frac{1}{x} - x\right) = \cancel{-x} - \frac{1}{x} + \cancel{x} = -\frac{1}{x}$

$x > 1, \quad -\frac{1}{x} < 0$

$\downarrow$

$-x - \left(\frac{1}{x} - x\right)$ not > 0

$\Downarrow$

I is false.

II. $x > 1$

$\downarrow$

$y = \frac{1}{x} - x < 0$

$\Downarrow$

II is true.

III. $\begin{cases} xy < x^2 \\ \underbrace{x > 1 > 0} \\ \text{When cancel } x, \text{ don't change sign.} \end{cases} \rightarrow \cancel{x}\, y < x^{\cancel{2}}$

$\downarrow$

$y < x$

$\begin{cases} y = \frac{1}{x} - x, \\ y < 0 \text{ (It is proved in II.)} \\ x > 0 \end{cases}$

$\Downarrow$

III is true.

I II III

8. If M is the midpoint of segment AB, then which of the following must be true?

I. $AM = 2BM$

II. $BM > \frac{AB}{3}$

III. $AB = BM + \frac{AB}{2}$

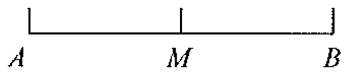
(A) I only

(B) II only

(C) III only

(D) II and III only

Solution: Answer: (D)



I. In the figure, one $AM \neq$ two BM.

$\Downarrow$

I is false.

II. In the figure, $BM = \frac{AB}{2} > \frac{AB}{3}$.

$\Downarrow$

II is true.

III. In the figure, $\begin{cases} BM = \frac{AB}{2} \\ AB = \frac{AB}{2} + \frac{AB}{2} \end{cases}$

$\downarrow$

$AB = BM + \frac{AB}{2}$

$\Downarrow$

III is true.

9. If $0 < x < 1$, which of the following must be true?

I. $x^3 > x^2$

II. $\frac{x}{2} > x$

III. $x > x^2$

(A) I only

(B) II only

(C) III only

(D) II and III only

Solution: Answer: (C)

Method 1

$\begin{cases} x > 1 \\ \downarrow \\ x^n > x^{n-1} \end{cases}$ $\begin{cases} 0 < x < 1 \\ \downarrow \\ x^{n-1} > x^n \end{cases}$

$\Downarrow$

$\begin{cases} \text{I is false.} \\ \text{III is true.} \end{cases}$

II. $\frac{x}{2} > x \rightarrow \frac{1}{2}x > x$

$\Downarrow$

II is false.

Method 2

Let $x = \frac{1}{2}$ to test.

I. $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$, $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\frac{1}{8} < \frac{1}{4}$, $x^3 < x^2$

$\Downarrow$

I is false.

II. $\frac{x}{2} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$, $x = \frac{1}{2}$

$\downarrow$

$\frac{1}{4} < \frac{1}{2}$, $\frac{x}{2} < x$

$\Downarrow$

II is false.

III. $x = \frac{1}{2}$, $x^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\downarrow$

$\frac{1}{2} > \frac{1}{4}$, $x > x^2$

$\Downarrow$

III is true.

10. For all real numbers x and y, let the operation ♣ be defined by $x \clubsuit y = y - xy$. If m and n are integers greater than 0, then which of the following statements could be equal to 0?

I. $m \clubsuit (m - n)$

II. $(m - n) \clubsuit n$

III. $n \clubsuit (m - n)$

(A) I and III only
(B) I only
(C) III only
(D) I, II, and III

Solution: Answer: (D)

The question doesn't point out that m and n are different. So you should do not forget to check the special situation: $m = n$.

$x \clubsuit y = y - xy$ and "could be"

$\downarrow$

I. $m \clubsuit (m-n) = (m-n) - m(m-n)$

$\downarrow$

$(1-1) - 1(1-1) = 0$

$\Downarrow$

I is true.

II. $(m-n) \clubsuit n = n - (m-n)n$

$\downarrow$

$1 - (1-1)1 = 1 \neq 0$

But, when $m = 2$ and $n = 1$,

$(m-n) \clubsuit n = n - (m-n)n = 1 - (2-1)1 = 0$

$\Downarrow$

II is true.

III. $n \clubsuit (m-n) = (m-n) - n(m-n)$

$\downarrow$

$(1-1) - 1(1-1) = 0$

$\Downarrow$

III is true.

11. If x is divisible by 5 and y is divisible by 7, which of the following must be divisible by 35?

I. $5x + 7y$

II. xy

III. $7x + 5y$

(A) I only (B) II only (C) I and II only
(D) II and III only

Solution: Answer: (D)

Let m and n be integers, $x = \underbrace{5m}_{\text{Multiple of 5}}$, and $y = \underbrace{7n}_{\text{Multiple of 7}}$.

I. $5x = 5 \cdot 5m = 25m, \quad 7y = 7 \cdot 7n = 49n$

$\downarrow$

$$\frac{5x+7y}{35} = \frac{25m+49n}{35}$$

When $m = n = 1$, $\dfrac{25m+49n}{35} = \dfrac{25+49}{35} = \dfrac{74}{35}$.

$\downarrow$

$\dfrac{74}{35} \neq$ integer (not divisible)

$\Downarrow$

I is false.

II. $$\frac{5m \cdot 7n}{35} = \frac{\cancel{35}mn}{\cancel{35}} = \underbrace{mn}_{\text{integer}}$$

$\downarrow$

xy is divisible by 35.

$\Downarrow$

II is true.

III. $7x = 7 \cdot 5m = 35m, \quad 5y = 5 \cdot 7n = 35n$

$\downarrow$

$$\frac{7x+5y}{35} = \frac{35m+35n}{35}$$

$\downarrow$

$$\frac{\cancel{35}(m+n)}{\cancel{35}} = \underbrace{m+n}_{\text{integer}}$$

$\downarrow$

$(7x+5y)$ is divisible by divisible.

$\Downarrow$

III is true.

12. A computer program at random chooses a positive 3-digit integer. If the integer chosen is even, twice that integer is printed. If the integer printed is 204, which of the following could have been the integer chosen?

I. 306
II. 102
III. 204

(A) II and III only
(B) II only
(C) III only
(D) I only

Solution: Answer: (B)

The integer printed is 204.

I. If a number chosen is 306, number 612 is printed.
⇓
I is false.

III. If a number chosen is 102, number 204 is printed.
⇓
II is true.

III. If a number chosen is 204, number 408 is printed.
⇓
III is false.

13. If 2 sides of a right triangle are 17 and 8, which of the following numbers could be the length of the 3rd side?

I. $\sqrt{353}$

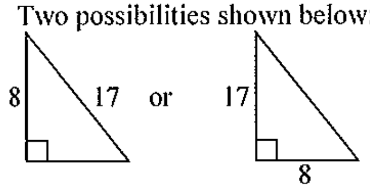
II. 15

III. 14

(A) I only
(B) III only
(C) I and II only
(D) I and III only

Solution: Answer: (C)

Two possibilities shown below:



(1) $\sqrt{17^2-8^2}=\sqrt{289-64}=\sqrt{225}=15$

(2) $\sqrt{17^2+8^2}=\sqrt{289+64}=\sqrt{353}$

⇓

I and II

14. A computer program at random chooses a positive 3-digit integer. If the integer chosen is odd, twice that integer is printed. If the integer chosen is even, the integer itself is printed. If the integer printed is 202, which of the following could have been the integer chosen?

I. 101
II. 202
III. 404

(A) I only
(B) II only
(C) III only
(D) I and II only

Solution: Answer: (D)

The integer printed is 202.

I. If the number chosen is 101 that is odd, $2 \cdot 102 = 204$ is printed.
⇓
I is true.

II. If the number chosen is 202 that is even, 202 is printed.
⇓
II is true.

III. If the number chosen is 404 that is even, 404 is printed.
⇓
III is false.

15. If x is divisible by 4 and y is divisible by 6, which of the following must be divisible by 24?

I. $4x+6y$

II. xy

III. $6x+4y$

(A) I only
(B) II only
(C) III only
(D) II and III only

Solution: Answer: (D)

Let m and n be integers, $x = 4m$ (Multiple of 4), and $y = 6n$ (Multiple of 6).

I. $4x = 4\cdot 4m = 16m, \quad 6y = 6\cdot 6n = 36n$

$\downarrow$

$$\frac{4x+6y}{24} = \frac{16m+36n}{24}$$

When $m = n = 1$, $\frac{16m+36n}{24} = \frac{16+36}{24} = \frac{52}{24} \neq$ integer

$\downarrow$

$4x+6y$ is not divisible by 24.

$\Downarrow$

I is false.

II. $\frac{xy}{24} = \frac{4m\cdot 6n}{24} = \frac{\cancel{24}\,mn}{\cancel{24}} = \underbrace{mn}_{\text{integer}}$

$\downarrow$

xy is divisible by 24.

$\Downarrow$

II. is true.

III. $6x = 6\cdot 4m = 24m, \quad 4y = 4\cdot 6n = 24n$

$\downarrow$

$$\frac{6x+4y}{24} = \frac{24m+24n}{24} = \frac{\cancel{24}\,(m+n)}{\cancel{24}} = \text{integer}$$

$\downarrow$

$6x+4y$ is divisible by 24.

$\Downarrow$

III is true.

16. Which of the following has or have the graphs that are composed of two perpendicular lines?

I. $xy = 1$

II. $\frac{|y|}{|x|} = 1$

III. $2xy = 0$

(A) I only
(B) II only
(C) III only
(D) II and III only

Solution: Answer: (D)

I. $xy = 1, \quad y = \frac{1}{x}$

Its graph is a hyperbola, not a line.

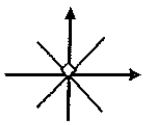
$\Downarrow$

I is false.

II. $\frac{|y|}{|x|} = 1$, $|y| = |x|$

A hole exists but it does not affect that perpendicular lines exist.

$$\begin{cases} +y = |x| \\ -y = |x| \end{cases} \rightarrow \begin{cases} y = |x| \\ y = -|x| \end{cases}$$

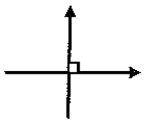


$\Downarrow$

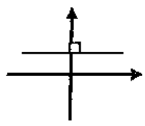
II is true.

III. $2xy = 0, \quad xy = 0$

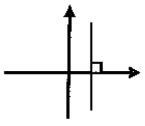
(1). $x = 0$ and $y = 0$



(2). $x = 0$ and $y \neq 0$



(3). $y = 0$ and $x \neq 0$



$\Downarrow$

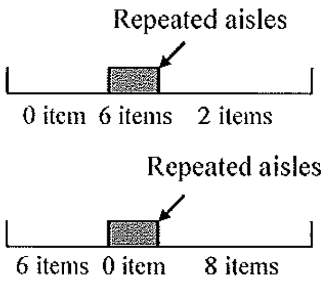
III is true.

17. In a department store, Doris bought 6 items from aisles 1 through 6 and 8 items from aisles 4 through 11. Which of the following could be the total number of items that Doris bought?

I. 6
II. 7
III. 12

(A) I and III only
(B) I and II only
(C) II and III only
(D) III only

Solution: Answer: (D)



Consider 2 situations (1) and (2) below

(1). The 6 items of 8 items came from repeated aisles.

↓

The least number of items = 8

(2). All the 8 items from aisles 3 through 10 were bought from not repeated aisles.

↓

The greatest number of items = 6 + 8 =14.

↓

$8 \le x \le 14$

⇓

Answer is (D).

18. Which of the following could be true if $(k-1)(1-b)=0$?

I. $k=b$

II. $b=1$

III. $k=1$

(A) I only
(B) II only
(C) I and II only
(D) I, II, and III

Solution: Answer: (D)

"could be" and $(k-1)(1-b)=0$

I. If $k=b=1$, then $k=b$.

⇓

I is true.

II. If $1-b=0$, then $b=1$.

⇓

II is true.

III. If $k-1=0$, then $k=1$.

⇓

III is true.

19. A list of 11 positive integers has the property that the average, A, of the integers is greater than the median, M, of the integers. Which of the following statements must be true?

I. More of these integers are greater than A than are less than A.
II. More of these integers are greater than M than are equal to M.
III. More of these integers are less than M than are equal to M.

(A) I, II, and III
(B) I and II only
(C) II and III only
(D) None

Solution: Answer: (D)

I. The 11 integers can be $\overbrace{1,\ 1,\ \cdots,\ 1}^{10 \text{ of the } 11 \text{ integers}},\ 2$

$$A=\frac{\overbrace{1+1+\cdots+1}^{10 \text{ of the } 11 \text{ integers}}+2}{11}=\frac{12}{11}$$

↓

$$1<A=\frac{12}{11}$$

↓

A counter-example exists.

⇓

I is false.

II. The 11 integers can be $\overbrace{1,\ 1,\ \cdots,\ 1}^{10 \text{ of the } 11 \text{ integers}},\ 2$

$M=1$

↓

More of the integers = 1 = M, not > M.

↓

A counter-example exists.

⇓

II is false.

III. The 11 integers can be $\overbrace{1,\ 1,\ \cdots,\ 1}^{10 \text{ of the } 11 \text{ integers}},\ 2$

$M=1$

↓

More of the integers = 1 = M, not < M.

↓

A counter-example exists.

⇓

III is false.

1. If $(a-b)^2 = a^2 + b^2$, which of the following must also be true?

I. $ab = 0$

II. $(a+b)^2 = a^2 + b^2$

III. $b = 0$

(A) I only

(B) II only

(C) III only

(D) I and II only

2. If $p^2 + 2pq + q^2 = p^2 + q^2$, which of the following must also be true?

I. $pq = 0$

II. $(p-q)^2 = p^2 + q^2$

III. $p = 0$

(A) I only

(B) II only

(C) III only

(D) I and II only

3. A list of 10 positive integers has the property that the average, A, of the integers is greater than the median, M, of the integers. Which of the following statements must be true?

I. More of these integers are greater than A than are less than A.

II. More of these integers are greater than M than are equal to M.

III. More of these integers are less than M than are equal to M.

A) I, II, and III

(B) I and II only

(C) II and III only

(D) None

I II III

RESULTS OF GAME

Number of Shots	Number of People
1	8
2	7
3	7
4	4
5	3
7	2

4. The chart above shows the results for 31 people who played a certain game. In the game, each person shot a ball at a target till the person missed the target. According to the information in the chart, which of the following must be true?

I. For all of the shots, more hit the target than missed the target.

II. No people hit the target 6 times.

III. Less than half of the people hit the target on their first shot.

(A) I only

(B) II only

(C) I and II only

(D) II and III only

5. In a department store, Monica bought 6 items from aisles 1 through 6 and 9 items from aisles 4 through 11. Which of the following could be the total number of items that Monica bought?

I. 8
II. 9
III. 12

(A) None
(B) I and II only
(C) II and III only
(D) I and III only

6. If x is an integer greater than 1 and $y = x - \frac{1}{x}$, which of the following must be true?

I. $y = (x-1) + \frac{x-1}{x}$

II. y is not an integer.

III. $xy < x^2$

(A) I only

(B) II only

(C) III only

(D) I and II only

7. If x is an integer greater than 1 and $y = \frac{1}{x} - x$, which of the following must be true?

I. $-x > \frac{1}{x} - x$

II. $y < 0$

III. $xy < x^2$

(A) I only (B) II only (C) III only (D) II, III only

I II III

8. If M is the midpoint of segment AB, then which of the following must be true?

I. $AM = 2BM$

II. $BM > \frac{AB}{3}$

III. $AB = BM + \frac{AB}{2}$

(A) I only

(B) II only

(C) III only

(D) II and III only

9. If $0 < x < 1$, which of the following must be true?

I. $x^3 > x^2$

II. $\frac{x}{2} > x$

III. $x > x^2$

(A) I only

(B) II only

(C) III only

(D) II and III only

10. For all real numbers x and y, let the operation ♣ be defined by x ♣ $y = y - xy$. If m and n are integers greater than 0, then which of the following statements could be equal to 0?

I. m ♣ $(m - n)$

II. $(m - n)$ ♣ n

III. n ♣ $(m - n)$

(A) I and III only
(B) I only
(C) III only
(D) I, II, and III

11. If x is divisible by 5 and y is divisible by 7, which of the following must be divisible by 35?

I. $5x + 7y$

II. xy

III. $7x + 5y$

(A) I only (B) II only (C) I and II only
(D) II and III only

I II III

12. A computer program at random chooses a positive 3-digit integer. If the integer chosen is even, twice that integer is printed. If the integer printed is 204, which of the following could have been the integer chosen?

I. 306
II. 102
III. 204

(A) II and III only
(B) II only
(C) III only
(D) I only

13. If 2 sides of a right triangle are 17 and 8, which of the following numbers could be the length of the 3rd side?

I. $\sqrt{353}$

II. 15

III. 14

(A) I only
(B) III only
(C) I and II only
(D) I and III only

14. A computer program at random chooses a positive 3-digit integer. If the integer chosen is odd, twice that integer is printed. If the integer chosen is even, the integer itself is printed. If the integer printed is 202, which of the following could have been the integer chosen?

I. 101
II. 202
III. 404

(A) I only
(B) II only
(C) III only
(D) I and II only

15. If x is divisible by 4 and y is divisible by 6, which of the following must be divisible by 24?

I. $4x + 6y$

II. xy

III. $6x + 4y$

(A) I only
(B) II only
(C) III only
(D) II and III only

16. Which of the following has or have the graphs that are composed of two perpendicular lines?

I. $xy = 1$

II. $\frac{|y|}{|x|} = 1$

III. $2xy = 0$

(A) I only

(B) II only

(C) III only

(D) II and III only

I II III

17. In a department store, Doris bought 6 items from aisles 1 through 6 and 8 items from aisles 4 through 11. Which of the following could be the total number of items that Doris bought?

I. 6
II. 7
III. 12

(A) I and III only
(B) I and II only
(C) II and III only
(D) III only

18. Which of the following could be true if $(k-1)(1-b)=0$?

I. $k=b$

II. $b=1$

III. $k=1$

(A) I only

(B) II only

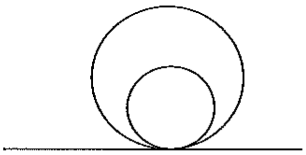
(C) I and II only

(D) I, II, and III

19. A list of 11 positive integers has the property that the average, A, of the integers is greater than the median, M, of the integers. Which of the following statements must be true?

I. More of these integers are greater than A than are less than A.
II. More of these integers are greater than M than are equal to M.
III. More of these integers are less than M than are equal to M.

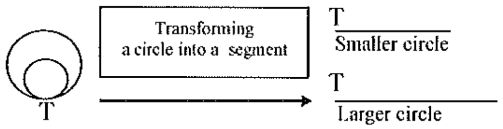
(A) I, II, and III
(B) I and II only
(C) II and III only
(D) None



Note: Figure above not drawn to scale.

1. Two wheels are tangent at one point as shown above. Their radii measure 3 and 5 inches, respectively. They start moving to the same direction simultaneously. What is the least number of revolutions of the smaller wheel so that both wheels are at the tangent point of each wheel again?

Solution: Answer: 5



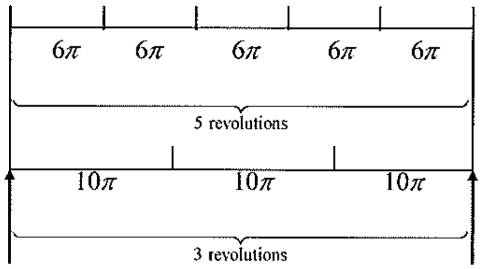
One revolution = One Circumference

$r_S = 3,\ r_L = 5$

$C_{\text{smaller}} = 2r_S\pi = 2\times 3\times\pi = 6\pi$

$C_{\text{larger}} = 2r_L\pi = 2\times 5\times\pi = 10\pi$

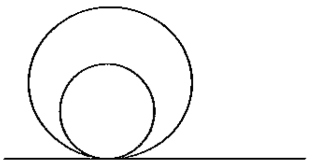
Least Common Multiple = 30π



For smaller wheel:

$\Downarrow$

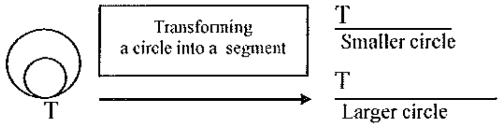
The least number = 5



Note: Figure above not drawn to scale.

2. Two wheels are tangent at one point d as shown above. Their radii measure 12 and 20 inches, respectively. They start moving to the same direction simultaneously. What is the second least number of revolutions of the larger wheel so that both wheels are at the tangent point of each wheel again?

Solution: Answer: 6



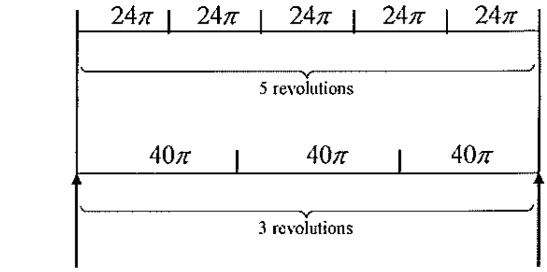
One revolution = One Circumference

$r_S = 12,\ r_L = 20$

$C_{\text{smaller}} = 2r_S\pi = 2\times 12\times\pi = 24\pi$

$C_{\text{larger}} = 2r_L\pi = 2\times 20\times\pi = 40\pi$

Least Common Multiple = 120π



For larger wheel:

The least number = 3

$\Downarrow$

The second least number= $3\cdot 2 = 6$

Meeting Again

3. One toy bell chimes per 4 seconds and another per 6 seconds. If they chime simultaneously, in how many seconds will they first chime together again?

Solution: Answer: 12

Number 12 is the least common multiple of 4 and 6.

$\Downarrow$

They first chime together again = 12.

4. One toy bell chimes per 6 seconds and another per 7 seconds. If they chime simultaneously, in how many seconds will they first chime together again?

Solution: Answer: 42

Number 42 is the least common multiple of 6 and 7.

$\Downarrow$

They first chime together again = 42

5. One toy bell chimes per 6 seconds and another per 7 seconds. If they chime simultaneously, in how many seconds will they second chime together again?

Solution: Answer: 84

Number 42 is the least common multiple of 6 and 7.

$\downarrow$

They first chime together again = 42

$\Downarrow$

They second chime together again = $2 \times 42 = 84$

6. One toy bell chimes per 4 seconds and another per 6 seconds. If they chime simultaneously, in how many seconds will they second chime together again?

Solution: Answer: 24

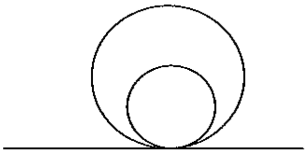
Number 12 is the least common multiple of 4 and 6.

$\downarrow$

They first chime together again = 12

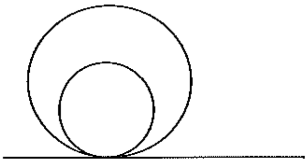
$\Downarrow$

They second chime together again = $2 \times 12 = 24$



Note: Figure above not drawn to scale.

1. Two wheels are tangent at one point as shown above. Their radii measure 3 and 5 inches, respectively. They start moving to the same direction simultaneously. What is the least number of revolutions of the smaller wheel so that both wheels are at the tangent point of each wheel again?



Note: Figure above not drawn to scale.

2. Two wheels are tangent at one point d as shown above. Their radii measure 12 and 20 inches, respectively. They start moving to the same direction simultaneously. What is the second least number of revolutions of the larger wheel so that both wheels are at the tangent point of each wheel again?

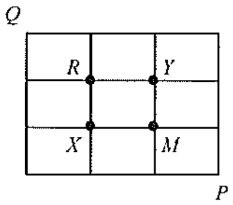
Meeting Again

3. One toy bell chimes per 4 seconds and another per 6 seconds. If they chime simultaneously, in how many seconds will they first chime together again?

4. One toy bell chimes per 6 seconds and another per 7 seconds. If they chime simultaneously, in how many seconds will they first chime together again?

5. One toy bell chimes per 6 seconds and another per 7 seconds. If they chime simultaneously, in how many seconds will they second chime together again?

6. One toy bell chimes per 4 seconds and another per 6 seconds. If they chime simultaneously, in how many seconds will they second chime together again?



1. In the figure above, a path from the corner point P to the corner point Q is determined by moving along the grid lines.
No overlap of any portion is allowed for each path. How many different paths that do not contain X, Y, or M can be drawn from P to Q?

(A) 2
(B) 8
(C) 4
(D) 6

Solution: Answer: (C)

Point $P \rightarrow$ Point Q, ~~X~~, ~~Y~~, and ~~M~~

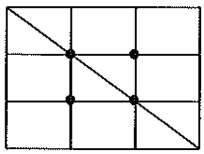
$\downarrow$

1. P – upward – Q
2. P – upward – R – Q

} 2 paths

and

Remaining set of paths is symmetric about the diagonal.

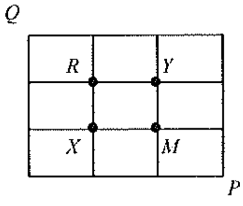


$\downarrow$

Total paths = $2 \times 2 = 4$

$\Downarrow$

Answer is (C).



2. In the figure above, a path from the corner point Q to the corner point P is determined by moving along the grid lines. No overlap of any portion is allowed for each path. How many different paths that do not contain either X or Y can be drawn from Q to P?

Solution: Answer: 8

From point Q to point P, ~~X~~ or ~~Y~~

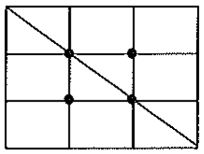
$\downarrow$

1. Q – right – P
2. Q – right – M – P
3. Q – downward – R – P
4. Q – downward – R – M – P

} 4 paths

and

Remaining set of paths is symmetric about the diagonal.



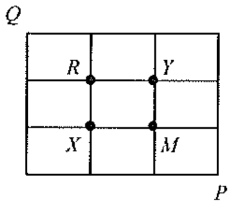
$\downarrow$

Total paths = $2 \cdot 4 = 8$

$\Downarrow$

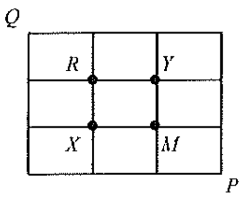
Answer is 8.

Paths



1. In the figure above, a path from the corner point P to the corner point Q is determined by moving along the grid lines. No overlap of any portion is allowed for each path. How many different paths that do not contain X, Y, or M can be drawn from P to Q?

(A) 2
(B) 8
(C) 4
(D) 6



2. In the figure above, a path from the corner point Q to the corner point P is determined by moving along the grid lines. No overlap of any portion is allowed for each path. How many different paths that do not contain either X or Y can be drawn from Q to P?

4 1 3 2

1. The number sequence above is allowed to change in either of two methods. Either two adjacent numbers may be exchanged or entire sequence may be reversed. What is the least number of such changes needed to put the numbers into the order from small to big?

(A) 2
(B) 5
(C) 4
(D) 3

Solution: Answer: (D)

(1) Reverse the entire sequence.
2 3 1 4
(2) Exchange 2 adjacent numbers.
2 1 3 4
(3) Exchange 2 adjacent numbers.
1 2 3 4

⇓

Answer is (D).

D A C B

2. The letter sequence above is allowed to change in either of two methods. Either two adjacent letters may be exchanged or entire sequence may be reversed. What is the least number of such changes needed to put the letters into the alphabetical order from A to Z?

(A) 5
(B) 2
(C) 3
(D) 4

Solution: Answer: (C)

(1) Reverse the entire sequence.
B C A D
(2) Exchange 2 adjacent numbers.
B A C D
(3) Exchange 2 adjacent numbers.
A B C D

⇓

Answer is (C).

R O Q P

3. The letter sequence above is allowed to change in either of two methods. Either two adjacent letters may be exchanged or entire sequence may be reversed. What is the least number of such changes needed to put the letters into the alphabetical order from A to Z?

(A) 5
(B) 3
(C) 2
(D) 4

Solution: Answer: (B)

(1) Reverse the entire sequence.
P Q O R
(2) Exchange 2 adjacent numbers.
P O Q R
(3) Exchange 2 adjacent numbers.
O P Q R

⇓

Answer is (B).

6 2 4 3

4. The number sequence above is allowed to change in either of two methods. Either two adjacent numbers may be exchanged or entire sequence may be reversed. What is the least number of such changes needed to put the numbers into the order from small to big?

(A) 2
(B) 5
(C) 4
(D) 3

Solution: Answer: (D)

(1) Reverse the entire sequence.
3 4 2 6
(2) Exchange 2 adjacent numbers.
3 2 4 6
(3) Exchange 2 adjacent numbers.
2 3 4 6

⇓

Answer is (D).

Reordering

4 1 3 2

1. The number sequence above is allowed to change in either of two methods. Either two adjacent numbers may be exchanged or entire sequence may be reversed. What is the least number of such changes needed to put the numbers into the order from small to big?

(A) 2
(B) 5
(C) 4
(D) 3

D A C B

2. The letter sequence above is allowed to change in either of two methods. Either two adjacent letters may be exchanged or entire sequence may be reversed. What is the least number of such changes needed to put the letters into the alphabetical order from A to Z?

(A) 5
(B) 2
(C) 3
(D) 4

R O Q P

3. The letter sequence above is allowed to change in either of two methods. Either two adjacent letters may be exchanged or entire sequence may be reversed. What is the least number of such changes needed to put the letters into the alphabetical order from A to Z?

(A) 5
(B) 3
(C) 2
(D) 4

6 2 4 3

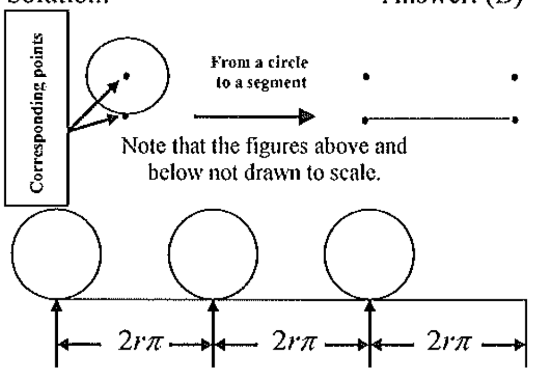
4. The number sequence above is allowed to change in either of two methods. Either two adjacent numbers may be exchanged or entire sequence may be reversed. What is the least number of such changes needed to put the numbers into the order from small to big?

(A) 2
(B) 5
(C) 4
(D) 3

1. If the radius of a motorcycle wheel is 0.2 meter, how many meters has the center of the wheel traveled when the wheel has made 3 complete revolutions along a straightaway?

(A) 0.6π
(B) 1.2π
(C) 0.9π
(D) 0.8π

Solution: Answer: (B)



$$3\times(2r\pi)=3\times(2\times0.2\pi)=1.2\pi$$

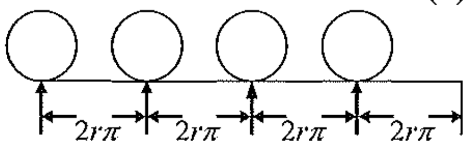
$$\Downarrow$$

Answer is (B).

2. If the radius of a motorcycle wheel is 0.25 meter, how many meters has the center of the wheel traveled when the wheel has made 4 complete revolutions along a straightaway?

(A) 1.0π
(B) 1.2π
(C) 2.0π
(D) 0.8π

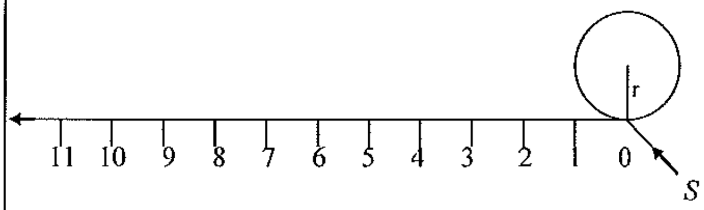
Solution: Answer: (C)



$$4\times(2r\pi)=4\times(2\times0.25\pi)=2\pi$$

$$\Downarrow$$

Answer is (C).



Note: Figure above not drawn to scale.

3. In the figure above, a circle of radius $r = 3/2\pi$ rolls to left along the line without slipping. In the starting position, point S on the circle touches the line for the first touch at point 0 on the line. At what point on the line will point S touch the line for the fourth touch?

(A) 9
(B) 3
(C) 6
(D) 4

Solution: Answer: (A)

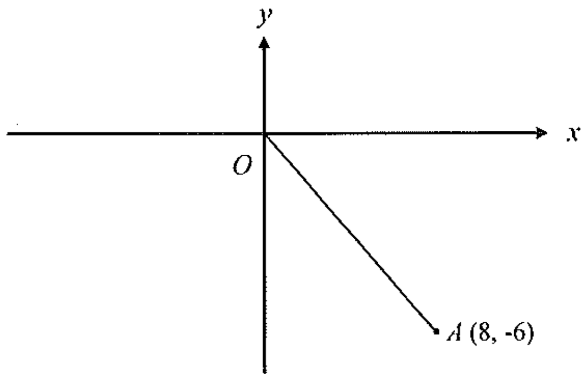
Remember: 1 R (revolution) = 1 C (circumference)

$$C=2r\pi=2\times(3/2\pi)\pi=3,\ R=3,\ 3R=3\times3=9$$

The first touch is at point 0.

$$\Downarrow$$

The fourth touch should be at point 9.

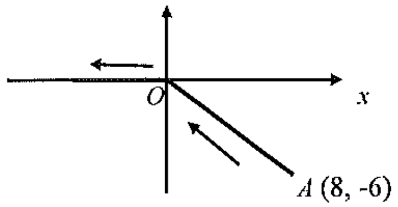


Note: Figure above not drawn to scale.

4. In the figure above, a ring with area π is placed on an incline. point A with coordinates (8, -6) is on the ring. The ring is rolled up the incline, and once point A of the ring touches the origin, it is then rolled horizontally along the x-axis to the left. What is the x-coordinate of point A, where point A on the ring touches the x-axis or the incline for the 7^{th} time?

(A) 27.7
(B) -31.2
(C) -27.7
(D) 12π

Solution: Answer: (C)



$$A=r^2\pi,\ \pi=r^2\pi,\ r^2=1,\ r=1$$

$$\downarrow$$

1 revolution = 1circumference

1circumference c = 1 segment s

$$\downarrow$$

$$s=c=2r\pi=2\cdot1\cdot\pi=2\pi$$

$6-8-10$ right triangle $\rightarrow AO=10$

$$\downarrow$$

7 touches $\rightarrow$ 6 segments

$$\downarrow$$

$$|x|+10=6s,\ x=6s-10$$

$$|x|=6\cdot2\pi-10\approx27.7$$

$$\Downarrow$$

$$x\approx-27.7$$

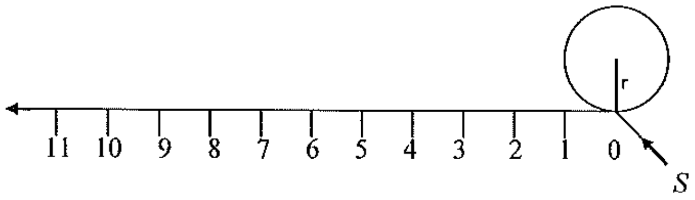
Revolutions

1. If the radius of a motorcycle wheel is 0.2 meter, how many meters has the center of the wheel traveled when the wheel has made 3 complete revolutions along a straightaway?

(A) 0.6π
(B) 1.2π
(C) 0.9π
(D) 0.8π

2. If the radius of a motorcycle wheel is 0.25 meter, how many meters has the center of the wheel traveled when the wheel has made 4 complete revolutions along a straightaway?

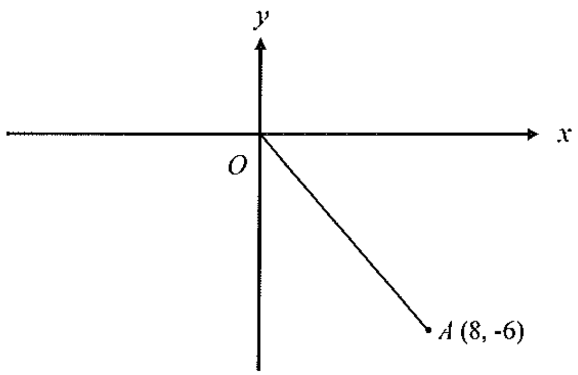
(A) 1.0π
(B) 1.2π
(C) 2.0π
(D) 0.8π



Note: Figure above not drawn to scale.

3. In the figure above, a circle of radius $r = 3/2\pi$ rolls to left along the line without slipping. In the starting position, point S on the circle touches the line for the first touch at point 0 on the line. At what point on the line will point S touch the line for the fourth touch?

(A) 9
(B) 3
(C) 6
(D) 4



Note: Figure above not drawn to scale.

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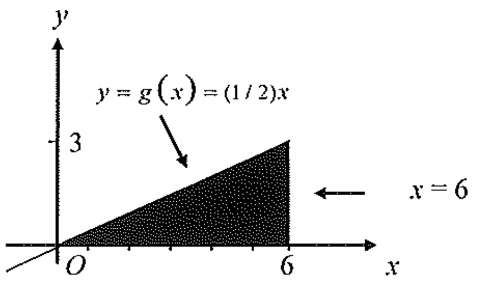
(A) 27.7
(B)-31.2
(C)-27.7
(D) 12π



1. In the figure above, the shaded portion is bounded by the x-axis, the line $x = 6$, and the graph of $y = g(x)$. The boundary is part of the shaded portion. Which of the following must be true if the point (m, n) lies in the shaded portion?

I. $0 \le m \le 6$
II. $n \le m$
III. $n \le g(m)$

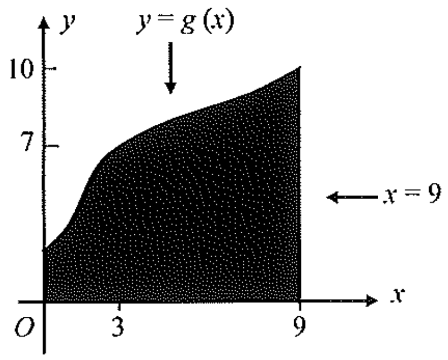
(A) I only
(B) II only
(C) I and III only
(D) I, II, and III

Solution: Answer: (D)

I. The boundary is part of the shaded portion.
m is the values of x-coordinate.
$0 < m \le 6$
$\Downarrow$
I = True

II. At point (0, 0), $n = m$.
At any remaining point (m, n), $n < m$.
The slope $= \frac{n}{m} = \frac{1}{2}$ of the function g is the greatest one of the slopes of the segments formed by any point inside the shaded area to point (0, 0).
The graph of the function is a line passing through point (0, 0).The value of y of any point on the line is one-half of the value of x.
$\Downarrow$
II = True

III. $g(x)$ is the top boundary of the shaded portion and it is all y-coordinates of the boundary. This boundary is a segment. On the boundary segment, each point is (m, n) also. That means that each value of n corresponding to each value of m is greatest value and that each value of the remaining n corresponding to each value of m is smaller than the values of n that lie on the boundary.
$\Downarrow$
III = True



2. In the figure above, the shaded portion is bounded by the y-axis, the x-axis, the line $x = 9$, and the graph of $y = g(x)$. The boundary is part of the shaded portion. Which of the following must be true if the point (m, n) lies in the shaded portion?

I. $n \le m$
II. $0 \le m \le 9$
III. $n \le g(m)$

(A) I only
(B) II only
(C) I and III only
(D) II and III only

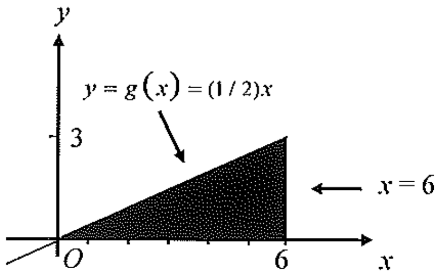
Solution: Answer: (D)

I. A counter-example
When $m = 3$, $n \approx 7$.
$\downarrow$
$n > m$
$\Downarrow$
I is false.

II. The boundary is part of the shaded portion.
$\downarrow$
$0 \le m \le 9$
$\Downarrow$
II is true.

III. $g(x)$ is the top boundary of the shaded portion and it is all y-coordinates of the boundary. This boundary is a segment. On the boundary segment, each point is (m, n) also. That means that each value of n corresponding to each value of m is greatest value and that each value of the remaining n corresponding to each value of m is smaller than the values of n that lie on the boundary.
$\Downarrow$
III = True

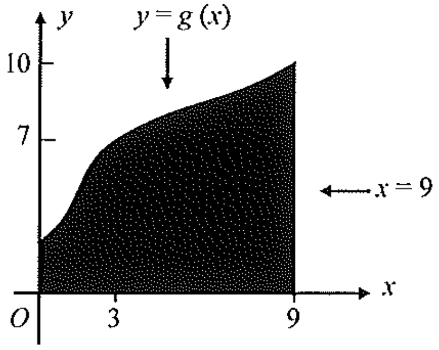
Shaded Regions Surrounded by Function Graphs



1. In the figure above, the shaded portion is bounded by the x-axis, the line $x = 6$, and the graph of $y = g(x)$. The boundary is part of the shaded portion. Which of the following must be true if the point (m, n) lies in the shaded portion?

I. $0 \le m \le 6$
II. $n \le m$
III. $n \le g(m)$

(A) I only
(B) II only
(C) I and III only
(D) (D) I, II, and III



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I. $n \le m$
II. $0 \le m \le 9$
III. $n \le g(m)$

(A) I only
(B) II only
(C) I and III only
(D) II and III only

Formula for simple interest: $i = art$	where
Formula for simple interest: $b = a(1+rt)$	i is year interest,
Formula for compound interest: $b = a(1+r)^t$	b is the balance,
	a is starting value,
	r is the annual interest rate,
	t is the time in years.

1. If a \$3,000 deposit earns 6% simple annual interest, which of the following is the interest earned in 6 years?

(A) 108
(B) 416
(C) 1,080
(D) 800

Solution: Answer: (C)

$$i = art = 30\cancel{00} \times \frac{6}{1\cancel{00}} \times 6$$

$$\Downarrow$$

$$i = 1,080$$

2. If you save \$2,800 of your pay and deposit it into your an account that earns simple annual interest, the balance is \$2,850, which of the following is the annual interest rate in 10 months?

(A) None of the below
(B) 0.2
(C) 0.02
(D) 0.03

Solution: Answer: (C)

$$b = a(1+rt)$$

$$\downarrow$$

$$285\cancel{0} = 280\cancel{0}\left(1 + r\frac{10}{12}\right)$$

$$\downarrow$$

$$\frac{12}{10}\left(\frac{285}{280} - 1\right) = r$$

$$\Downarrow$$

$$r = \frac{6}{5} \times 0.0179 \approx 0.02$$

3. If a \$2,000 bond earns 7% simple annual interest, which of the following is the interest earned in 5 years?

(A) 700
(B) 7
(C) 35
(D) 140

Solution: Answer: (A)

$$i = art = 20\cancel{00} \times \frac{7}{1\cancel{00}} \times 5$$

$$\Downarrow$$

$$i = 700$$

4. If you save \$2,800 of your pay and deposit it into your an account that earns 2.6 percent interest compounded annually, which of the following is the balance in 5 years?

(A) 8892.23
(B) 8982.23
(C) 3138.43
(D) 3183.43

Solution: Answer: (D)

$$b = a(1+r)^t$$

$$\downarrow$$

$$b = 2800(1+0.026)^5$$

$$b = 2800(1.026)^5$$

$$\Downarrow$$

$$b \approx 3183.43$$

5. If you save \$800 of your pay and deposit it into your saving account that earns 4 percent simple annual interest, which of the following is the balance in 8 years?

(A) 1,056
(B) 1,065
(C) 8,010
(D) 5,681

Solution: Answer: (A)

$$b = a(1+rt)$$

$$\downarrow$$

$$b = 800(1+0.04\times 8)$$

$$\Downarrow$$

$$b = 1056$$

Simple and Compound Interest

6. If you save \$600 of your pay and deposit it into your an account that earns 5.5 % interest compounded annually, which of the following is the balance in seven years?

(A) 12896.54
(B) 12896.45
(C) 872.91
(D) 872.81

Solution: Answer: (D)

$$b = a(1+r)^t$$
$$\downarrow$$
$$b = 600(1+0.055)^7$$
$$\downarrow$$
$$b = 600(1.055)^7$$
$$\Downarrow$$
$$b \approx 872.81$$

7. If you deposit \$5,000 into your an saving account that earns five percent simple annual interest, which of the following is the balance of the saving account in half year?

(A) 5,125
(B) 2,650
(C) 6,280
(D) 5,215

Solution: Answer: (A)

$$b = a(1+rt)$$
$$\downarrow$$
$$b = 5000\left(1+0.05\times\frac{6}{12}\right)$$
$$\downarrow$$
$$5000(1+0.025)$$
$$\Downarrow$$
$$b = 5000\times 1.025 = 5125$$

8. If you deposit \$300 into your an account that earns 5.5 % interest compounded annually, which of the following is the balance B in eight years?

(A) $B = 300(1.55)^8$
(B) $B = 300(1.0055)^8$
(C) $B = 300(1.055)^8$
(D) $B = 300^8(1.055)^8$

Solution: Answer: (C)

$$b = a(1+r)^t$$
$$\downarrow$$
$$b = 300(1+0.055)^8$$
$$\Downarrow$$
$$b = 300(1.055)^8$$

9. If you deposit \$$D$ into your an saving account that earns p percent simple annual interest, which of the following is the balance B of the saving account in m months?

(A) $D = B\left(1+\frac{p}{100}\times\frac{m}{12}\right)$
(B) $B = D\left(1+\frac{p}{100}\times m\right)$
(C) $B = D(1+p\times m)$
(D) $B = D\left(1+\frac{p}{100}\times\frac{m}{12}\right)$

Solution: Answer: (D)

Formula: $b = a(1+rt)$

$$\Downarrow$$
$$B = D\left(1+\frac{p}{100}\times\frac{m}{12}\right)$$

Formula for simple interest: $i = art$

Formula for simple interest: $b = a(1+rt)$

Formula for compound interest: $b = a(1+r)^t$

where

i is year interest,

b is the balance,

a is starting value,

r is the annual interest rate,

t is the time in years.

1. If a \$3,000 deposit earns 6% simple annual interest, which of the following is the interest earned in 6 years?

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(B) 416

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2. If you save \$2,800 of your pay and deposit it into your an account that earns simple annual interest, the balance is \$2,850, which of the following is the annual interest rate in 10 months?

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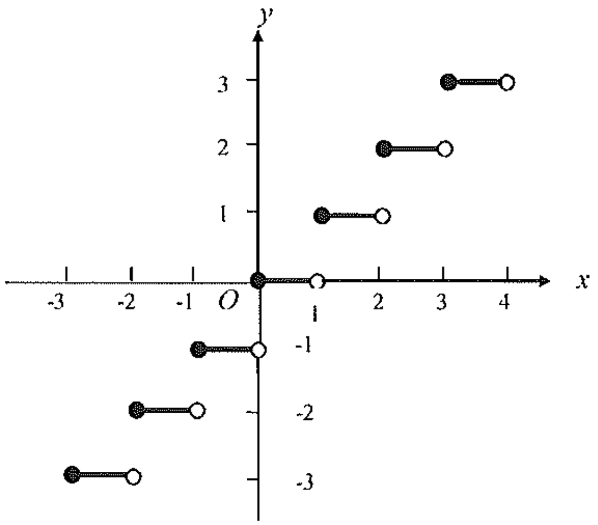
(A) $D = B\left(1+\frac{p}{100}\times\frac{m}{12}\right)$
(B) $B = D\left(1+\frac{p}{100}\times m\right)$
(C) $B = D(1+p\times m)$
(D) $B = D\left(1+\frac{p}{100}\times\frac{m}{12}\right)$



1. The figure above shows the graph of the greatest integer function g represented by $g(x)=[x]$. What is the value of the step function $f(x)=[x]-1$ if $x=\frac{5}{3}$?

(A) $\frac{2}{3}$

(B) $-\frac{2}{3}$

(C) 0

(D) 1

Solution: Answer: (C)

$$g\left(\frac{5}{3}\right)=\left[\frac{5}{3}\right]=\left[1\frac{2}{3}\right]=1$$

$$\Downarrow$$

$$f\left(\frac{5}{3}\right)=\left[\frac{5}{3}\right]-1=1-1=0$$

2. The figure above shows the graph of the greatest integer function g represented by $g(x)=[x]$. What is the value of the step function $f(x)=[x]+2$ if $x=-\frac{7}{3}$?

(A) -1

(B) $-\frac{1}{3}$

(C) 0

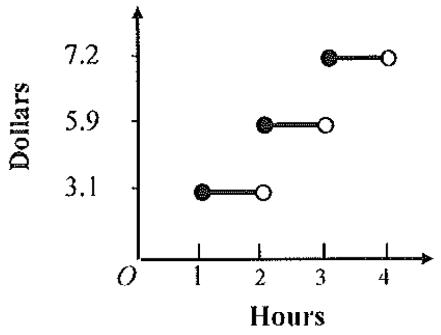
(D) 1

Solution: Answer: (A)

$$g\left(-\frac{7}{3}\right)=\left[-\frac{7}{3}\right]=\left[-2\frac{1}{3}\right]=-3$$

$$\Downarrow$$

$$f\left(-\frac{7}{3}\right)=\left[-\frac{7}{3}\right]+2=-3+2=-1$$



3. The figure above shows the part of graph of the greatest integer function used for charges of a computer rental company. What is the sum, in doallars, of the charges of the 1.1 hours, 1.9 hours and 3.1 hours?

(A) 16.2

(B) 13.4

(C) 14.9

(D) 13.2

Solution: Answer: (B)

$$[1.1]=3.1,\quad [1.9]=3.1,\quad [3.1]=7.2$$

$$\Downarrow$$

$$\text{Sum}=3.1+3.1+7.2=13.4$$

4. The figure above shows the part of graph of the greatest integer function used for charges of a computer rental company. What is the sum, in doallars, of the charges of the 1.9 hours, 2.1 hours and 3.9 hours?

(A) Not enough information to determine

(B) 13.4

(C) 14.9

(D) 16.2

Solution: Answer: (D)

$$[1.9]=3.1,\quad [2.1]=5.9,\quad [3.9]=7.2$$

$$\Downarrow$$

$$\text{Sum}=3.1+5.9+7.2=16.2$$

5. The figure above shows the part of graph of the greatest integer function used for charges of a computer rental company. What is the sum, in doallars, of the charges of the 1.99 hours, 2.11 hours and 2.99 hours?

(A) Not enough information

(B) 13.4

(C) 14.9

(D) 13.2

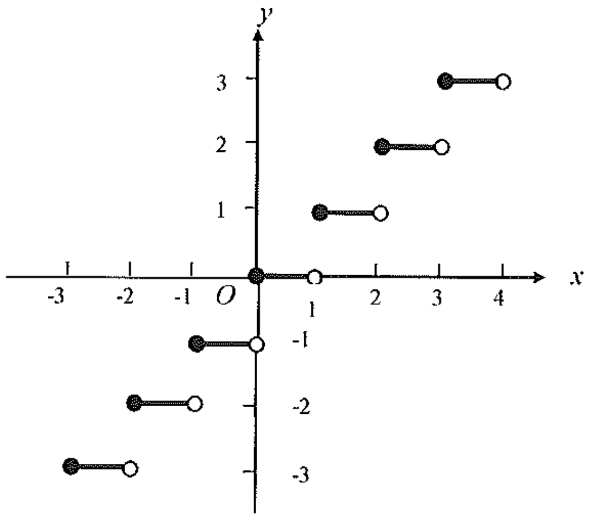
Solution: Answer: (C)

$$[1.99]=3.1,\quad [2.11]=5.9,\quad [2.99]=5.9$$

$$\Downarrow$$

$$\text{Sum}=3.1+5.9+5.9=14.9$$

Greatest Integer Functions



1. The figure above shows the graph of the greatest integer function g represented by $g(x) = [x]$. What is the value of the step function $f(x) = [x] - 1$ if $x = \frac{5}{3}$?

(A) $\frac{2}{3}$

(B) $-\frac{2}{3}$

(C) 0

(D) 1

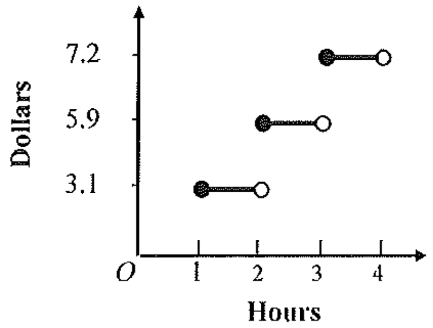
2. The figure above shows the graph of the greatest integer function g represented by $g(x) = [x]$. What is the value of the step function $f(x) = [x] + 2$ if $x = -\frac{7}{3}$?

(A) -1

(B) $-\frac{1}{3}$

(C) 0

(D) 1



3. The figure above shows the part of graph of the greatest integer function used for charges of a computer rental company. What is the sum, in doallars, of the charges of the 1.1 hours, 1.9 hours and 3.1 hours?

(A) 16.2

(B) 13.4

(C) 14.9

(D) 13.2

4. The figure above shows the part of graph of the greatest integer function used for charges of a computer rental company. What is the sum, in doallars, of the charges of the 1.9 hours, 2.1 hours and 3.9 hours?

(A) Not enough information to determine

(B) 13.4

(C) 14.9

(D) 16.2

5. The figure above shows the part of graph of the greatest integer function used for charges of a computer rental company. What is the sum, in doallars, of the charges of the 1.99 hours, 2.11 hours and 2.99 hours?

(A) Not enough information

(B) 13.4

(C) 14.9

(D) 13.2

1. If $f(x)=\sqrt{x^2-x-6}$, which of the following indicates the set of all values of x at which the function is not defined?

(A) $-2 < x < -3$

(B) $x < -3$

(C) $x < -4$

(D) $x < -2$

Solution: Answer: (A)

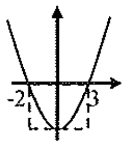
$$\begin{cases} f(x)=\sqrt{x^2-x-6} \\ \text{undefined} \end{cases}$$

$$\downarrow$$

$$y = x^2 - x - 6 < 0$$

$$\downarrow$$

$$y = (x-3)(x+2) < 0$$



By the figure above, it's clear that when $-2 < x < 3$, all values of y corresponding the x are less than 0.

$$\Downarrow$$

Answer is (A).

2. If x and $f(x)$ are real numbers and $f(x) = 2\sqrt{4-x^2}$, what is the maximum value of $f(x)$?

(A) 4

(B) 6

(C) 8

(D) 10

Solution Answer: (A)

When $x = 0$,

$f(x) = y$ has the maximum value.

$$\Downarrow$$

$$y = 2\sqrt{4-0^2} = 2\sqrt{4} = 2\times 2 = 4$$

3. Which of the following numbers is the number that is not in the domain of g if $g(x) = \dfrac{x^4-16}{x^4+16}$?

(A) No any number is not in the domain.

(B) $\dfrac{1}{2}$

(C) $-\dfrac{1}{2}$

(D) 1

Solution: Answer: (A)

$x^4 + 16$ has no chance to equal zero.

$$\downarrow$$

The domain of g is all the real numbers.

$$\Downarrow$$

Answer is (A).

4. If $f(x)=\sqrt{x^2-9}$, which of the following indicates the set of all values of x at which the function is defined?

(A) $-3 < x < 3$

(B) $x < -3$

(C) $x \le -3,\ x \ge 3$

(D) $x > -3$

Solution: Answer: (C)

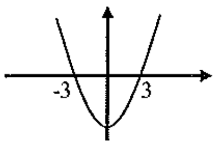
$$\begin{cases} f(x)=\sqrt{x^2-9} \\ \text{"defined" = domain} \end{cases}$$

$$\downarrow$$

$$y = x^2 - 9 \ge 0$$

$$\downarrow$$

$$y = (x-3)(x+3) \ge 0$$



According to the figure above, it is clear that when $-3 \le x$ and $x \ge 3$, all the values of y corresponding the x are equal or greater than 0.

$$\Downarrow$$

Answer is (C).

5. What is the domain of $f(x) = \dfrac{\sqrt{x-2}}{x+3}$?

(A) $x \ne -3$

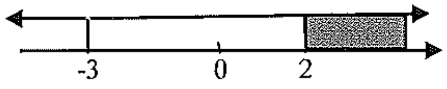
(B) $x \ge 2$

(C) $-3 \le x \le 2$

(D) $x \ge 3$

Solution: Answer: (B)

$$\begin{cases} \text{The domain of } \sqrt{x-2} \\ \quad x-2 \ge 0,\ x \ge 2 \\ \text{The domain of } (x+3) \\ \quad x+3 \ne 0,\ x \ne -3 \end{cases}$$



$$\Downarrow$$

The common domain : $x \ge 2$

or

intersection domain

Domain, Range, and "Undefined"

1. If $f(x)=\sqrt{x^2-x-6}$, which of the following indicates the set of all values of x at which the function is not defined?

(A) $-2 < x < -3$

(B) $x < -3$

(C) $x < -4$

(D) $x < -2$

2. If x and $f(x)$ are real numbers and $f(x)=2\sqrt{4-x^2}$, what is the maximum value of $f(x)$?

(A) 4

(B) 6

(C) 8

(D) 10

3. Which of the following numbers is the number that is <u>not</u> in the domain of g if $g(x)=\dfrac{x^4-16}{x^4+16}$?

(A) No any number is not in the domain.

(B) $\dfrac{1}{2}$

(C) $-\dfrac{1}{2}$

(D) 1

4. If $f(x)=\sqrt{x^2-9}$, which of the following indicates the set of all values of x at which the function is defined?

(A) $-3 < x < 3$

(B) $x < -3$

(C) $x \le -3,\ x \ge 3$

(D) $x > -3$

5. What is the domain of $f(x)=\dfrac{\sqrt{x-2}}{x+3}$?

(A) $x \ne -3$

(B) $x \ge 2$

(C) $-3 \le x \le 2$

(D) $x \ge 3$

1. If for all real numbers x, a function f is defined by $f(x)=\begin{cases}10, & x=10\\ 20, & x\neq 10\end{cases}$, then $f(1)-f(-1)=$

(A) 10

(B) 0

(C) 1

(D) 20

Solution: Answer: (B)

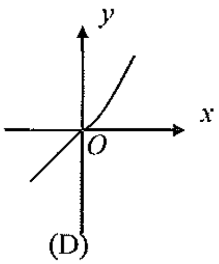
$f(x)=20$ except $x=10$

$\Downarrow$

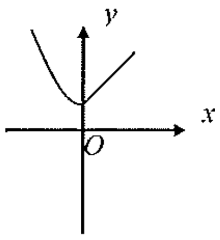
$f(0)-f(-1)=20-20=0$

2. Which of the following graphs could be the function $f(x)=\begin{cases}x^2+2, & x<0\\ x+2, & x\geq 0\end{cases}$

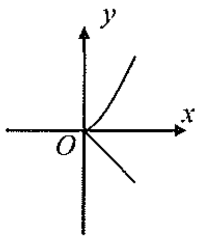
(A)



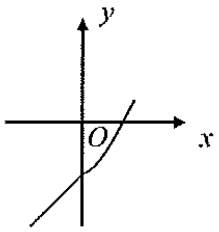
(B)



(C)



(D)



Solution: Answer: (B)

$f(x)=\begin{cases}x^2+2, & x<0 \rightarrow \text{the graph is a Parabola.}\\ x+2, & x\geq 0 \rightarrow \text{the graph is a line.}\end{cases}$

$\downarrow$

For the parabola, when $x=0$, $f(0)=0^2+2=2$.

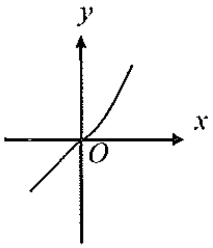
For the line, when $x=0$, $f(0)=0+2=2$.

$\Downarrow$

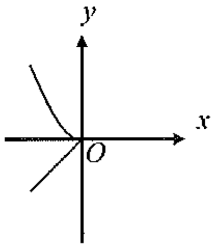
The graph of (B)

3. Which of the following graphs could be the function $f(x)=\begin{cases}x^2+2, & x>0\\ x+2, & x\leq 0\end{cases}$

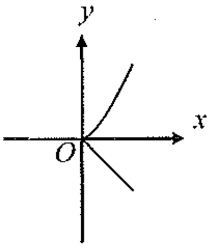
(A)



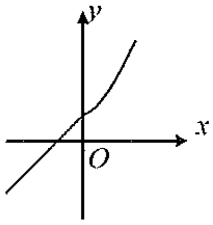
(B)



(C)



(D)



Solution: Answer: (D)

$f(x)=\begin{cases}x^2+2, & x>0 \rightarrow \text{the graph is a Parabola.}\\ x+2, & x\leq 0 \rightarrow \text{the graph is a line.}\end{cases}$

$\downarrow$

For the parabola, when $x=0$, $f(0)=0^2+2=2$.

For the line, when $x=0$, $f(0)=0+2=2$;

when $y=0$, $0=x+2$, $x=-2$.

$\Downarrow$

The graph of (D)

Piecewise Functions

4. If for all real numbers x, a function f is defined by $f(x)=\begin{cases}10, & x=10\\ 20, & x\neq 10\end{cases}$, then $f(10)-f(-10)=$

(A) 10

(B) -10

(C) 1

(D) 20

Solution: Answer: (B)

$$f(10)-f(-10)$$
$$\Downarrow$$
$$10-20=-10$$

5. If for all real numbers x, a function f is defined by $f(x)=\begin{cases}7, & x=10\\ 6, & x\neq 10\end{cases}$, then $f(10)-f(11)=$

(A) 3

(B) 1

(C) 2

(D) -2

Solution: Answer: (B)

$$f(10)-f(11)$$
$$\Downarrow$$
$$7-6=1$$

6. If $g(x)=\begin{cases}\dfrac{x^2-16}{x-4}, & \text{when } x\neq 4\\ b, & \text{when } x=4\end{cases}$, what must the value(s) of b be in order for g to be a continuous function?

(A) 8

(B) 7

(C) 9

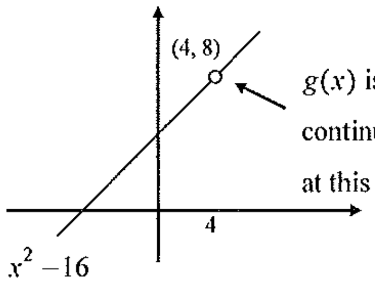
(D) 6

Solution: Answer: (A)

$$y=\frac{x^2-16}{x-4}=\frac{\cancel{(x-4)}(x+4)}{\cancel{x-4}}$$
$$\downarrow$$
$$y=x+4$$

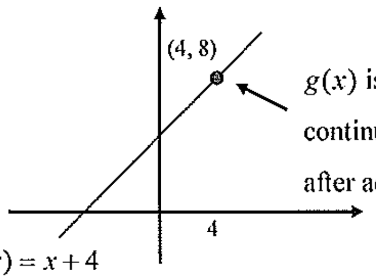
When $x=4$, $y=8$.

$$\downarrow$$



$g(x)$ is not continuous at this point.

$$y=\frac{x^2-16}{x-4}$$



$g(x)$ is continuous after adding.

$$g(x)=x+4$$

$$\begin{cases}g(x)=b, & \text{when } x=4\\ g(x)=x+4\end{cases}$$
$$\Downarrow$$
$$b=g(4)=4+4=8$$

1. If for all real numbers x, a function f is defined by

$$f(x)=\begin{cases}10, & x=10\\ 20, & x\neq 10\end{cases}, \text{ then } f(1)-f(-1)=$$

(A) 10

(B) 0

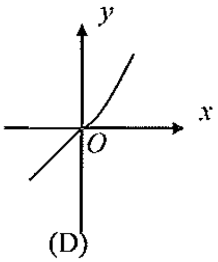
(C) 1

(D) 20

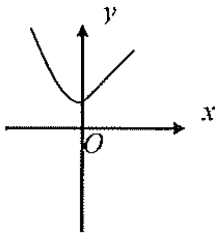
2. Which of the following graphs could be the function

$$f(x)=\begin{cases}x^2+2, & x<0\\ x+2, & x\geq 0\end{cases}$$

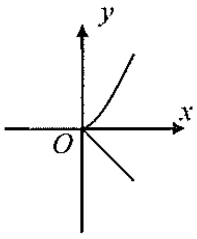
(A)



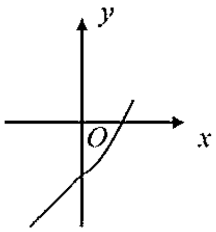
(B)



(C)



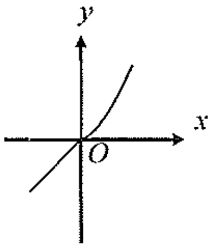
(D)



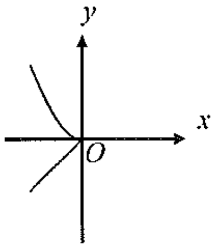
3. Which of the following graphs could be the function

$$f(x)=\begin{cases}x^2+2, & x>0\\ x+2, & x\leq 0\end{cases}$$

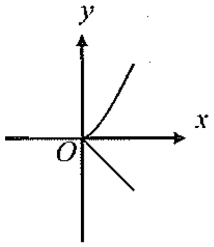
(A)



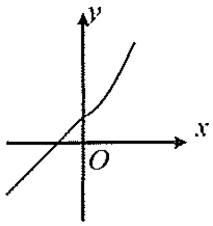
(B)



(C)



(D)



Piecewise Functions

4. If for all real numbers x, a function f is defined by $f(x)=\begin{cases}10, & x=10\\ 20, & x\neq 10\end{cases}$, then $f(10)-f(-10)=$

(A) 10

(B) -10

(C) 1

(D) 20

5. If for all real numbers x, a function f is defined by $f(x)=\begin{cases}7, & x=10\\ 6, & x\neq 10\end{cases}$, then $f(10)-f(11)=$

(A) 3

(B) 1

(C) 2

(D) -2

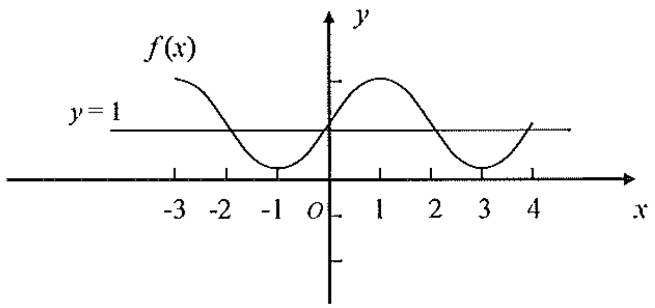
6. If $g(x)=\begin{cases}\dfrac{x^2-16}{x-4}, & \text{when } x\neq 4\\ b, & \text{when } x=4\end{cases}$, what must the value(s) of b be in order for g to be a continuous function?

(A) 8

(B) 7

(C) 9

(D) 6



1. In the figure above, the graphs of $f(x)$ and $y = 1$ are intersected. For how many values of x does $f(x) = 1$?

(A) 1
(B) 2
(C) 3
(D) 4

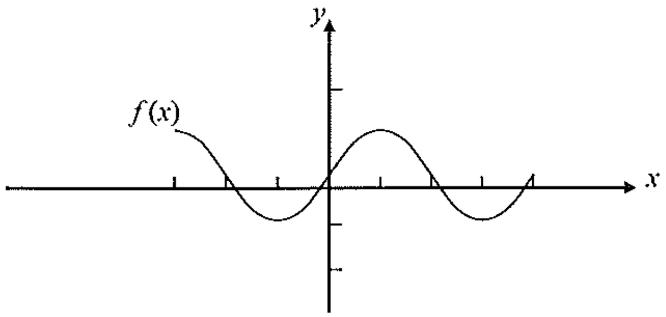
Solution: Answer: (D)

The figure shows that there are 4 points of intersections.

Each of them has a pair of coordinate values x and y. So $f(x) = 1$ for 4 values of x.

⇓

The 4 points are: (-2, 1), (0, 1), (2, 1), (4, 1).



2. In the figure above, the graphs of $f(x)$ and $y = 0$ are intersected. For how many values of x does $f(x) = 0$?

(A) 5
(B) 4
(C) 3
(D) 2

Solution: Answer: (B)

$y = f(x) = 0$

↓

$f(x)$ is the *x-axis*.

↓

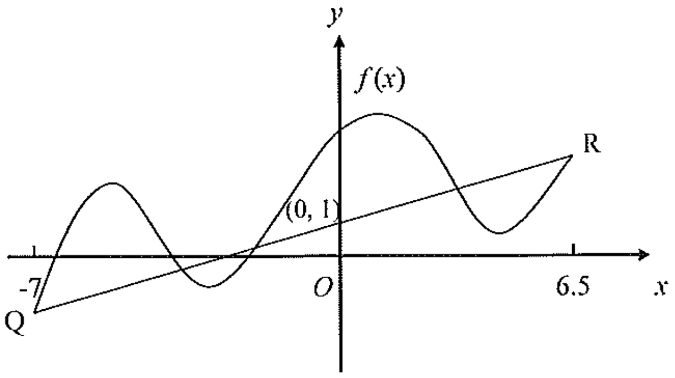
There are 4 intersection points.

↓

There are 4 values of x when $f(x) = 0$.

⇓

Answer is (B).



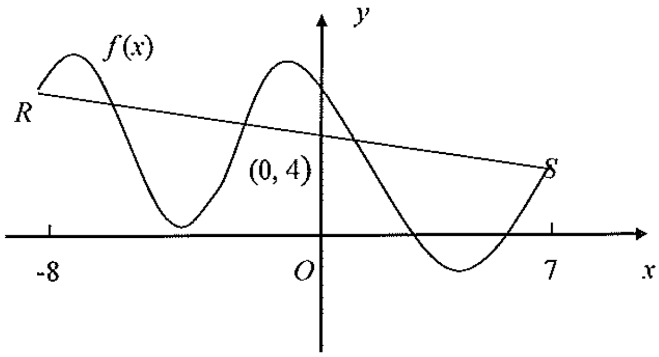
3. The figure above shows the graph of the function f and segment $\overline{QR}$ that has a y-intercept of 1. For how many values of x between –7 and 6.5 does $f(x) = 1$?

Solution: Answer: 5

Draw a horizontal line, which passes through the point (0, 1). The function of the line is $y = 1$. The line intersects the graph of f at 5 points between –7 and 6.5.

⇓

Answer = 5



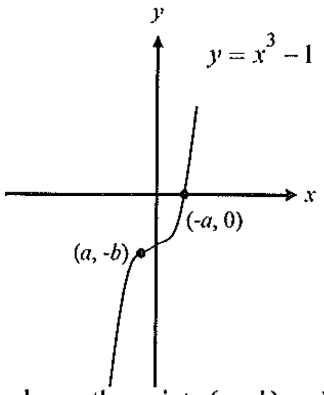
4. The figure above shows the graph of the function g and segment $\overline{RS}$ that has a y-intercept of 4. For how many values of x between –8 and 7 does $f(x) = 4$?

Solution: Answer: 3

Draw a horizontal line, which passes through the point (0, *4*). The function of the line is $y = 4$. The line intersects the graph of f at 3 points between –8 and 7.

⇓

Answer = 3



5. In the figure above, the points $(a, -b)$ and $(-a, 0)$ lie on the graph of $y = x^3 - 1$. What is the value of b?

(A) 5
(B) 4
(C) 3
(D) 2

Solution: Answer: (D)

$$y = x^3 - 1$$

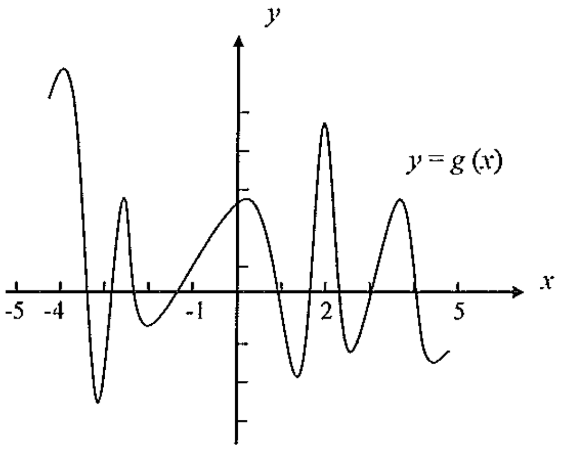
$$\downarrow$$

$$\begin{cases} 0 = (-a)^3 - 1 \\ -b = a^3 - 1 \end{cases} \rightarrow \begin{cases} a^3 = -1 \\ a^3 = 1 - b \end{cases}$$

$$a^3 = a^3,\ -1 = 1 - b$$

$$\Downarrow$$

$$b = 2$$



6. The figure above shows the graph of the function g. Which of the following values is greater than $g(2)$?

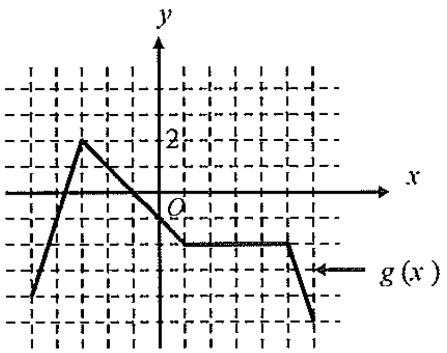
(A) $g(-1)$
(B) $g(0)$
(C) $g(-4)$
(D) $g(-3)$

Solution: Answer: (C)

$g(b)$ = a certain value of y,
where b is a certain value of x.
The value of y of point $(-4, g(-4))$ is
$g(-4)$ that is the greatest value of y
that is only value greater than $f(2)$.

$$\Downarrow$$

Answer is (C).



7. The figure above shows the graph of $y = g(x)$. g is a function. If $g(k) = g(3k)$, which of the following could be the value of k?

(A) -3
(B) 2
(C) -1
(D) 1

Solution: Answer: (D)

$$\begin{cases} \text{When } k = 1,\ g(1) = -2 \\ \text{When } 3k = 3,\ g(3) = -2 \end{cases}$$

$$g(1) = g(3)$$

$$\Downarrow$$

$$k = 1$$

8. If the equation of a parabola is $y = x^2 - 97x - 77$, where does the graph of the parabola cross the y-axis?

(A) 77
(B) -77
(C) -97
(D) 97

Solution: Answer: (B)

$$y = x^2 - 97x - 77$$

When $x = 0$, the parabola crosses the y-axis.

$$\Downarrow$$

$$y = 0^2 - 97 \cdot 0 - 77 = -77.$$

9. If $f(x) = 20$ for all real numbers x, what does $f(x + 20)$ equal?

(A) 40
(B) 20
(C) 18
(D) 30

Solution: Answer: (B)

$$f(x \pm a) = f(x)$$

$$\downarrow$$

$$f(x + 20) = f(x)$$

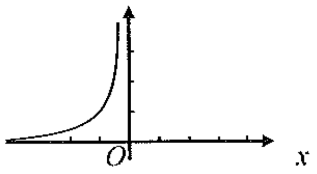
$$f(x) = 20$$

$$\Downarrow$$

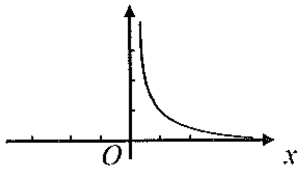
$$f(x + 20) = 20$$

10. Which of the following graphs has the feature that the x-coordinate is greater than the y-coordinate for which point on the graph?

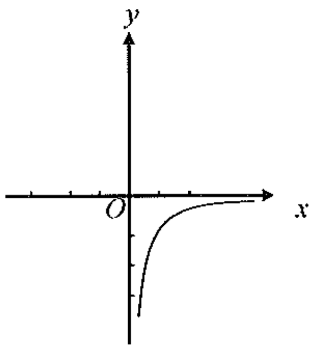
(A)



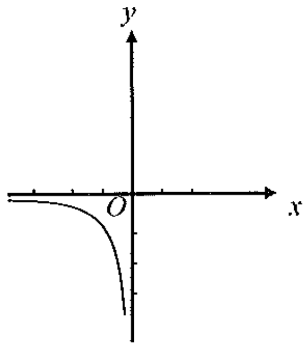
(B)



(C)



(D)

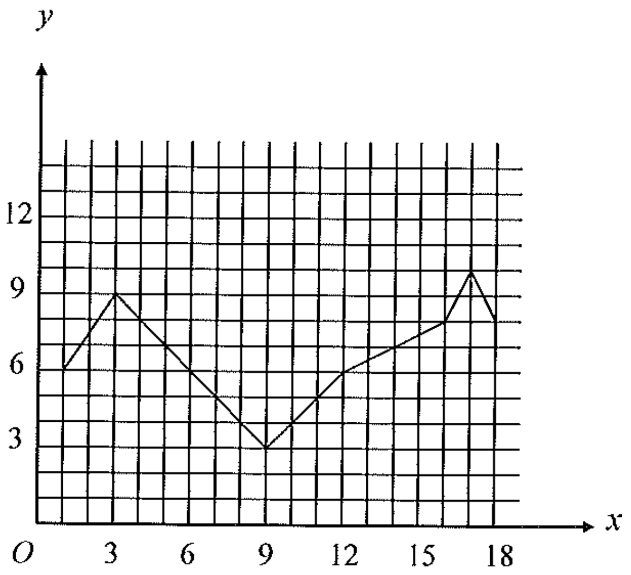


Solution: Answer: (C)

On the graph of (C), the x-coordinate is more than y-coordinate for each point.

$$\Downarrow$$

Answer is (C).



11. The graph of $y = f(x)$ is shown above. If $f(5) = a$, which of the following could be the value of $f(a)$?

(A) 2
(B) 4
(C) 3
(D) 5

Solution: Answer: (D)

In the xy-plane, $f(k)$ = value of y when $x = k$.

$\Downarrow$

$f(5) = 7,\ a = 7,\ f(7) = 5$

12. The graph of $y = g(x)$ is shown above. If $g(9) = p$, which of the following could be the value of $g(p)$?

(A) 9
(B) 4
(C) 3
(D) 5

Solution: Answer: (A)

In the xy-plane, $f(k)$ = value of y when $x = k$.

$\Downarrow$

$g(9) = 3,\ p = 3,\ g(3) = 9$

13. The graph of $y = h(x)$ is shown above. If $h(18) = k$, which of the following could be the value of $h(k)$?

(A) 2
(B) 4
(C) 3
(D) 5

Solution: Answer: (B)

In the xy-plane, $f(b)$ = value of y when $x = b$.

$\Downarrow$

$h(18) = 8,\ k = 8,\ h(8) = 4$

14. If the graph of the function g in the xy-coordinate plane contains the points (0, -8), (2, -3), and (5, 0), which of the following cannot be true?

(A) The graph of g has a minimum value.
(B) The graph of g has a maximum value.
(C) The function g is a quadratic function.
(D) The function g is a linear function.

Solution: Answer: (D)

If the function is a linear one, the slopes obtained from any pair of points on the line should be the same.

$\downarrow$

$$S_{\text{Slope-1}} = \frac{-8-(-3)}{0-2} = \frac{5}{2},\ S_{\text{Slope-2}} = \frac{0-(-3)}{5-2} = 1$$

$\downarrow$

$$S_{\text{Slope-1}} \neq S_{\text{Slope-2}}$$

$\Downarrow$

Answer is (D).

15. If the graph of the function g in the xy-coordinate plane contains the points (0, -7), (1, -3), and (5, 2), which of the following cannot be true?

(A) The graph of function g is a straight line.
(B) The graph of g has a maximum value.
(C) The function g is a quadratic function.
(D) The graph of g has a minimum value.

Solution: Answer: (A)

If the function is a linear one, the slopes obtained from any pair of points on the line must be the same.

$\downarrow$

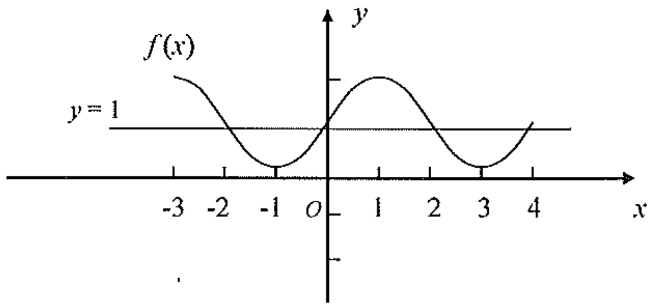
$$S_{\text{Slope-1}} = \frac{-7-(-3)}{0-1} = 4,\ S_{\text{Slope-2}} = \frac{2-(-3)}{5-1} = \frac{5}{4}$$

$\downarrow$

$$S_{\text{Slope-1}} \neq S_{\text{Slope-2}}$$

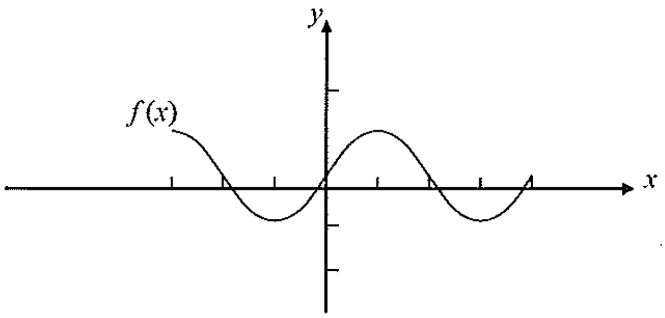
$\Downarrow$

Answer is (A).



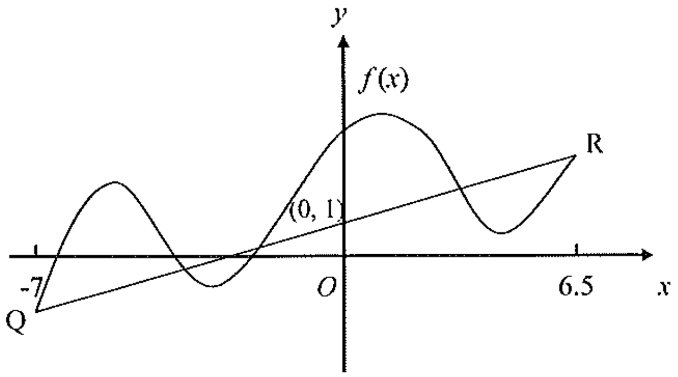
1. In the figure above, the graphs of $f(x)$ and $y = 1$ are intersected. For how many values of x does $f(x) = 1$?

(A) 1
(B) 2
(C) 3
(D) 4

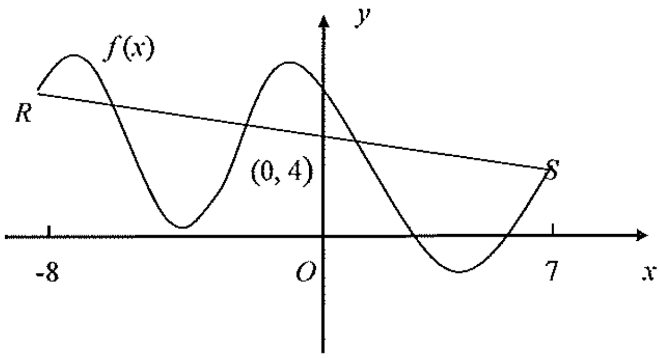


2. In the figure above, the graphs of $f(x)$ and $y = 0$ are intersected. For how many values of x does $f(x) = 0$?

(A) 5
(B) 4
(C) 3
(D) 2

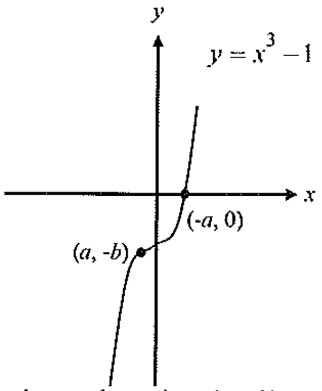


3. The figure above shows the graph of the function f and segment $\overline{QR}$ that has a y-intercept of 1. For how many values of x between –7 and 6.5 does $f(x) = 1$?



4. The figure above shows the graph of the function g and segment $\overline{RS}$ that has a y-intercept of 4. For how many values of x between –8 and 7 does $f(x) = 4$?

Qualitative Behaviors of Graphs of Functions



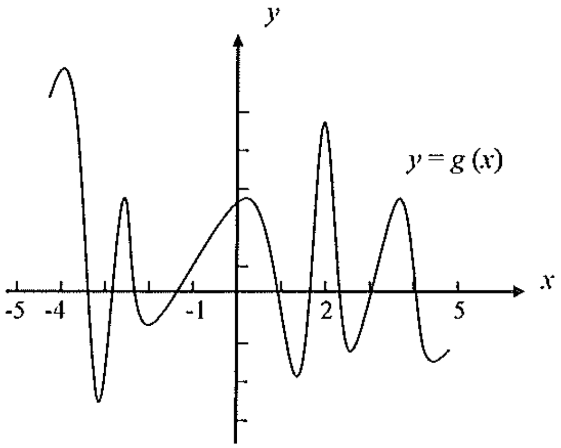
5. In the figure above, the points $(a, -b)$ and $(-a, 0)$ lie on the graph of $y = x^3 - 1$. What is the value of b?

(A) 5

(B) 4

(C) 3

(D) 2



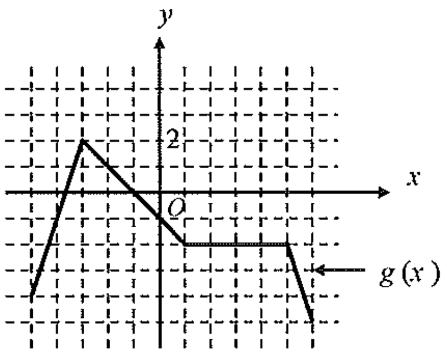
6. The figure above shows the graph of the function g. Which of the following values is greater than $g(2)$?

(A) $g(-1)$

(B) $g(0)$

(C) $g(-4)$

(D) $g(-3)$



7. The figure above shows the graph of $y = g(x)$. g is a function. If $g(k) = g(3k)$, which of the following could be the value of k?

(A) -3
(B) 2
(C) -1
(D) 1

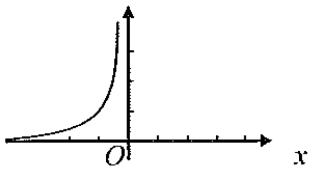
8. If the equation of a parabola is $y = x^2 - 97x - 77$, where does the graph of the parabola cross the y-axis?

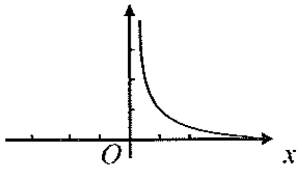
(A) 77
(B) -77
(C) -97
(D) 97

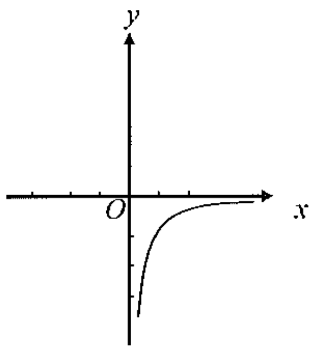
9. If $f(x) = 20$ for all real numbers x, what does $f(x + 20)$ equal?

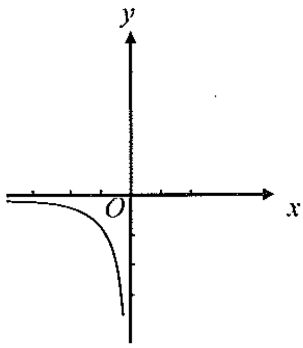
(A) 40
(B) 20
(C) 18
(D) 30

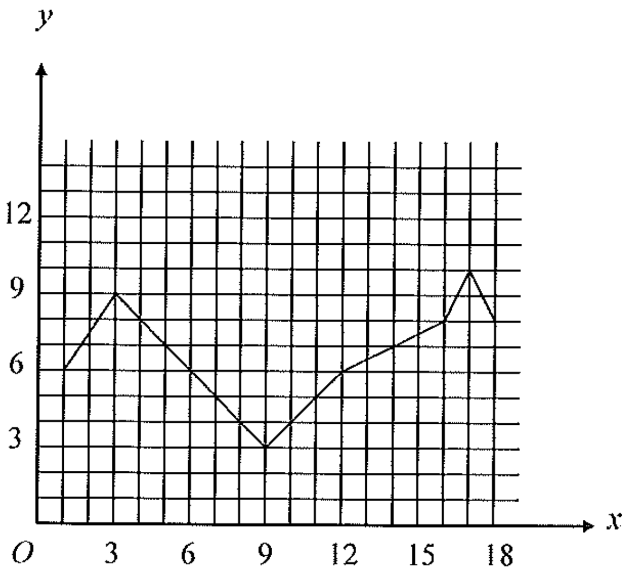
10. Which of the following graphs has the feature that the x-coordinate is greater than the y-coordinate for which point on the graph?

(A)


(B)


(C)


(D)




11. The graph of $y = f(x)$ is shown above. If $f(5) = a$, which of the following could be the value of $f(a)$?

(A) 2
(B) 4
(C) 3
(D) 5

12. The graph of $y = g(x)$ is shown above. If $g(9) = p$, which of the following could be the value of $g(p)$?

(A) 9
(B) 4
(C) 3
(D) 5

13. The graph of $y = h(x)$ is shown above. If $h(18) = k$, which of the following could be the value of $h(k)$?

(A) 2
(B) 4
(C) 3
(D) 5

14. If the graph of the function g in the xy-coordinate plane contains the points (0, -8), (2, -3), and (5, 0), which of the following cannot be true?

(A) The graph of g has a minimum value.
(B) The graph of g has a maximum value.
(C) The function g is a quadratic function.
(D) The function g is a linear function.

15. If the graph of the function g in the xy-coordinate plane contains the points (0, -7), (1, -3), and (5, 2), which of the following cannot be true?

(A) The graph of function g is a straight line.
(B) The graph of g has a maximum value.
(C) The function g is a quadratic function.
(D) The graph of g has a minimum value.

	Type X	Type Y	Type Z
Number of bells	8	6	5
Rings n times on the n^{th} hour	√	√	
Rings once on the hour			√
Rings once on the half hour	√		√

1. In the table of the bell information above, what is the total number of rings of bells in the 120-minute period from $6:25_{am}$ to $8:25_{am}$?

Solution: Answer: 246

X: At the 7^{th} and 8^{th} hours, one bell rings $7+8=15$ times.

At the 6:30 and 7:30, one bell rings 2 times.

Subtotal $= 8\times(15+2)=136$

Y: At the 7^{th} and 8^{th} hours, one bell rings $7+8=15$ times.

Subtotal $= 6\times 15=90$

Z: At the 7:00 and 8:00 o'clocks, one bell rings 2 times.

At the 6:30 and 7:30, one bell rings 2 times.

Subtotal $= 5\times(2+2)=20$

$\Downarrow$

Total: $136+90+20=\boxed{246}$

	Seniors	Juniors	Period-1	Period-2
Math	20	25	16	n
English	20	35	k	19

2. The chart above shows the number of seniors and juniors taking math and English classes at a high school and gives the enrollment for these classes in period-1 and period-2, which are the only periods math and English are taught. Only seniors and juniors take these classes. What is the total number of students who take math and English in period-2?

(A) 52
(B) 48
(C) 36
(D) 24

Solution: Answer: (B)

$\text{Total}_{\text{Math}} = 20+25=45$

$\downarrow$

$n = 45-16=29$

$\Downarrow$

$\text{Total}_{\text{Period-2}} = n+19=29+19=48$

Type	Number of bells	Rings n times on the n^{th} hour	Rings once on the hour	Rings once on the half hour
X	9		√	√
Y	10	√		√

3. In the table of the bell information above, what is the total number of rings of bells in the 150-minute period from $6:25_{pm}$ to $8:55_{pm}$?

Solution: Answer: 225

X: At the 7:00 and 8:00 o'clocks, one bell rings 2 times.

At the 6:30, 7:30, and 8:30, one bell rings 3 times.

Subtotal $= 9\times(2+3)=45$

Y: At the 7^{th} and 8^{th} hours, one bell rings $7+8=15$ times.

At the 6:30, 7:30, and 8:30, one bell rings 3 times.

Subtotal $= 10\times(15+3)=180$

$\Downarrow$

Total: $45+180=\boxed{225}$

	Seniors	Juniors	Period-1	Period-2
Math	20	25	16	n
English	20	35	k	19

4. The chart above shows the number of seniors and juniors taking math and English classes at a high school and gives the enrollment for these classes in period-1 and period-2, which are the only periods math and English are taught. Only seniors and juniors take these classes. What is the total number of students who take math and English in period-1?

(A) 52
(B) 48
(C) 36
(D) 24

Solution: Answer: (A)

$\text{Total}_{\text{English}} = 20+35=55$

$\downarrow$

$k = 55-19=36$

$\Downarrow$

$\text{Total}_{\text{Period-1}} = 16+k=16+36=52$

Table Data

	1	0	3	2	
0		0			A
1		0			B
2		0			C
3		0			D

5. Each of 16 squares in the grid above is to be filled with either 0 or 1. Each number to the left of the grid is the sum of the numbers in the row to its right, and each number above the grid is sum of the numbers in the column below it. If the 1s and 0s are all input correctly into the grid, which of the following represents row *C*?

(A)	1	1	0	0
(B)	0	0	1	1
(C)	0	1	1	0
(D)	1	0	0	1

Solution: Answer: (B)

	1	0	3	2	
0	0	0	0	0	A
1	0	0	1	0	B
2	0	0	1	1	C
3	1	0	1	1	D

Step 3 Step 1 Step 2

⇓

Answer is (B).

PRICES OF ITEMS

	Sofa	Tea Table
2005	$ 670	$105
2006	$ 850	$125
2007	$ 980	$135
2008	$ 990	$145

CAPACITY OF INVENTORY

	Warehouse		
	A	*B*	*C*
Sofas	42	93	40
Tea Tables	308	255	200

6. A furniture store sells one type of sofas and one type of tea tables.The chart on the left above shows the prices of the sofas and tea tables in four years. The chart on the right above shows the maximum number of sofas and tea tables that can be stocked in each of three warehouses, *A*, *B*, and *C*. According to the prices shown in the chart above, what was the maximum possible value of the inventory of the sofas and tea tables in warehouse *C* in 2007 ?

(A) 60,400
(B) 58,870
(C) 39,400
(D) 66,200

Solution: Answer: (D)

$$40\times 980+135\times 200$$

⇓

$$39200+27000=66200$$

7. A furniture store sells one type of sofas and one type of tea tables.The chart on the left above shows the prices of the sofas and tea tables in four years. The chart on the right above shows the maximum number of sofas and tea tables that can be stocked in each of three warehouses, *A*, *B*, and *C*. According to the prices shown in the chart above, what was the maximum possible value of the inventory of the sofas and tea tables in warehouse *A* in 2008 ?

(A) 86,240
(B) 88,840
(C) 79,400
(D) 60,450

Solution: Answer: (A)

$$42\times 990+145\times 308$$

⇓

$$41580+44660=86240$$

	Type X	Type Y	Type Z
Number of bells	8	6	5
Rings n times on the n^{th} hour	√	√	
Rings once on the hour			√
Rings once on the half hour	√		√

1. In the table of the bell information above, what is the total number of rings of bells in the 120-minute period from $6:25_{am}$ to $8:25_{am}$?

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(A) 52

(B) 48

(C) 36

(D) 24

Table Data

	1	0	3	2	
0		0			A
1		0			B
2		0			C
3		0			D

5. Each of 16 squares in the grid above is to be filled with either 0 or 1. Each number to the left of the grid is the sum of the numbers in the row to its right, and each number above the grid is sum of the numbers in the column below it. If the 1s and 0s are all input correctly into the grid, which of the following represents row *C*?

(A)	1	1	0	0
(B)	0	0	1	1
(C)	0	1	1	0
(D)	1	0	0	1

PRICES OF ITEMS

	Sofa	Tea Table
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CAPACITY OF INVENTORY

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(A) 86,240

(B) 88,840

(C) 79,400

(D) 60,450

1. If $\frac{18x^2+15x-35}{kx-2}=6x+9-\frac{17}{kx-2}$ is true for all values of $x\neq\frac{2}{k}$, where k is a constant, then what is the value of k ?

(A) 3
(B) -2
(C) -3
(D) 2

Solution: Answer: (A)

$$kx-2\overline{)18x^2+15x-35}\quad \begin{matrix}6x\\ k6x^2\end{matrix}$$

$$\Downarrow$$

$$k\cdot 6=18.\quad k=3$$

2. If $\frac{18x^2+15x-35}{kx-3}=9x+21+\frac{28}{kx-3}$ is true for all values of $x\neq\frac{3}{k}$, where k is a constant, then what is the value of k ?

(A) 3
(B) 2
(C) -3
(D) -2

Solution: Answer: (B)

$$kx-3\overline{)18x^2+15x-35}\quad \begin{matrix}9x\\ k9x^2\end{matrix}$$

$$\Downarrow$$

$$k\cdot 9=18.\quad k=2$$

3. If $\frac{16x^2+14x-35}{kx+3}=8x-5-\frac{20}{kx+3}$ is true for all values of $x\neq-\frac{3}{k}$, where k is a constant, then what is the value of k ?

(A) 4
(B) -2
(C) 2
(D) -5

Solution: Answer: (C)

$$kx+3\overline{)16x^2+14x-35}\quad \begin{matrix}8x\\ k8x^2\end{matrix}$$

$$\Downarrow$$

$$k\cdot 8=16.\quad k=2$$

4. If $9x^2-6x+3$ is divided by $3x+2$, the result is $(3x-4)+\frac{r}{3x+2}$, where r is a constant. Which of the following is the value of the constant r ?

(A) 8
(B) 9
(C) 10
(D) 11

Solution: Answer: (D)

$$3x+2\overline{)9x^2-6x+3}\quad \begin{matrix}3x-4\\ \underline{9x^2+6x}\\ -12x+3\\ \underline{-12x-8}\\ 11\end{matrix}$$

$$\Downarrow$$

$$r=11$$

5. $\frac{6x-5}{x-2}$ is equal to which of the following?

(A) $6-\frac{17}{x-2}$
(B) $6+\frac{7}{x-2}$
(C) $7+\frac{6}{x-2}$
(D) $-17+\frac{6}{x-2}$

Solution: Answer: (B)

Polynomial Division

$$\frac{\text{Dividend}}{\text{Divisor}}=\text{Quotient}+\frac{\text{Remainder}}{\text{Divisor}}$$

$$x-2\overline{)6x-5}\quad \left.\begin{matrix}6\\ \underline{6x-12}\\ 7\end{matrix}\right\}$$

$$\Downarrow$$

$$\frac{6x-5}{x-2}=6+\frac{7}{x-2}$$

6. $\frac{6x+5}{x+2}$ is equal to which of the following?

(A) $6-\frac{7}{x+2}$
(B) $6+\frac{7}{x+2}$
(C) $\frac{6}{x+2}-7$
(D) $6+\frac{17}{x+2}$

Solution: Answer: (A)

Polynomial Division

$$\frac{\text{Dividend}}{\text{Divisor}}=\text{Quotient}+\frac{\text{Remainder}}{\text{Divisor}}$$

$$x+2\overline{)6x+5}\quad \left.\begin{matrix}6\\ \underline{6x+12}\\ -7\end{matrix}\right\}$$

$$\Downarrow$$

$$\frac{6x+5}{x+2}=6-\frac{7}{x-2}$$

The Long Division

1. If $\frac{18x^2+15x-35}{kx-2}=6x+9-\frac{17}{kx-2}$ is true for all values of $x\neq\frac{2}{k}$, where k is a constant, then what is the value of k ?

(A) 3

(B) -2

(C) -3

(D) 2

2. If $\frac{18x^2+15x-35}{kx-3}=9x+21+\frac{28}{kx-3}$ is true for all values of $x\neq\frac{3}{k}$, where k is a constant, then what is the value of k ?

(A) 3

(B) 2

(C) -3

(D) -2

3. If $\frac{16x^2+14x-35}{kx+3}=8x-5-\frac{20}{kx+3}$ is true for all values of $x\neq-\frac{3}{k}$, where k is a constant, then what is the value of k ?

(A) 4

(B) -2

(C) 2

(D) -5

4. If $9x^2-6x+3$ is divided by $3x+2$, the result is $(3x-4)+\frac{r}{3x+2}$, where r is a constant. Which of the following is the value of the constant r ?

(A) 8

(B) 9

(C) 10

(D) 11

5. $\frac{6x-5}{x-2}$ is equal to which of the following?

(A) $6-\frac{17}{x-2}$

(B) $6+\frac{7}{x-2}$

(C) $7+\frac{6}{x-2}$

(D) $-17+\frac{6}{x-2}$

6. $\frac{6x+5}{x+2}$ is equal to which of the following?

(A) $6-\frac{7}{x+2}$

(B) $6+\frac{7}{x+2}$

(C) $\frac{6}{x+2}-7$

(D) $6+\frac{17}{x+2}$

1. What are the solutions to $6x^2 + 24x + 12 = 0$?

(A) $\pm 2 - \sqrt{2}$

(B) $-2 \pm \sqrt{2}$

(C) $-2 - \sqrt{2}$

(D) $\pm 2 + \sqrt{2}$

Solution: Answer: (B)

$$\not{6}\left(x^2 + 4x + 2\right) = 0$$

$$\downarrow$$

$$\text{Use } x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\downarrow$$

$$x_{1,2} = \frac{-4 \pm \sqrt{16 - 8}}{2} = \frac{-4 \pm \sqrt{4 \cdot 2}}{2}$$

$$\Downarrow$$

$$\frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

2. What are the solutions to $6x^2 + 24x - 12 = 0$?

(A) $-2 \pm \sqrt{6}$

(B) $\pm 2 - \sqrt{6}$

(C) $\pm 2 + \sqrt{6}$

(D) $+2 \pm \sqrt{6}$

Solution: Answer: (A)

$$\not{6}\left(x^2 + 4x - 2\right) = 0$$

$$\downarrow$$

$$\text{Use } x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\downarrow$$

$$x_{1,2} = \frac{-4 \pm \sqrt{16 + 8}}{2} = \frac{-4 \pm \sqrt{4 \cdot 6}}{2}$$

$$\Downarrow$$

$$\frac{-4 \pm 2\sqrt{6}}{2} = -2 \pm \sqrt{6}$$

3. What are the solutions to $6x^2 + 24x - 24 = 0$?

(A) $-2 \pm \sqrt{3}$

(B) ± 2

(C) $2 + \sqrt{2}$

(D) -2

Solution: Answer: (D)

$$\not{6}\left(x^2 + 4x + 4\right) = 0$$

$$\downarrow$$

$$\text{Use } x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\downarrow$$

$$x_{1,2} = \frac{-4 \pm \sqrt{16 - 16}}{2} = \frac{-4 \pm 0}{2}$$

$$\Downarrow$$

$$x_{1,2} = -2$$

4. What are the solutions to $6x^2 + 24x + 30 = 0$?

(A) $-2 - \sqrt{2} \cdot i$

(B) $-2 \pm \sqrt{2}$

(C) $-2 \pm \sqrt{2} \cdot i$

(D) $\pm 2 + \sqrt{2} \cdot i$

Solution: Answer: (C)

$$\not{6}\left(x^2 + 4x + 5\right) = 0$$

$$\downarrow$$

$$\text{Use } x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\downarrow$$

$$x_{1,2} = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$$

$$\downarrow$$

$$\frac{-4 \pm \sqrt{4} \cdot \sqrt{-1}}{2} = \frac{-4 \pm 2\sqrt{2} \cdot i}{2}$$

$$\Downarrow$$

$$x_{1,2} = -2 \pm \sqrt{2} \cdot i$$

The Quadratic Formula

1. What are the solutions to $6x^2 + 24x + 12 = 0$?

(A) $\pm 2 - \sqrt{2}$

(B) $-2 \pm \sqrt{2}$

(C) $-2 - \sqrt{2}$

(D) $\pm 2 + \sqrt{2}$

2. What are the solutions to $6x^2 + 24x - 12 = 0$?

(A) $-2 \pm \sqrt{6}$

(B) $\pm 2 - \sqrt{6}$

(C) $\pm 2 + \sqrt{6}$

(D) $+2 \pm \sqrt{6}$

3. What are the solutions to $6x^2 + 24x - 24 = 0$?

(A) $-2 \pm \sqrt{3}$

(B) ± 2

(C) $2 + \sqrt{2}$

(D) -2

4. What are the solutions to $6x^2 + 24x + 30 = 0$?

(A) $-2 - \sqrt{2} \cdot i$

(B) $-2 \pm \sqrt{2}$

(C) $-2 \pm \sqrt{2} \cdot i$

(D) $\pm 2 + \sqrt{2} \cdot i$

Questions 1 and 2 refer to the information below.

FINANCIAL PLAN (millions of dollars)			
	Financial Year		
Program	2014	2015	2016
General Fund	**$61,234**	**$63.524**	**$70.629**
State Fund	**$96.355**	**$98.698**	**$102.551**
Capital Budget	**$7.951**	**$7.995**	**$9.125**
Federal Operating Aid	**$32.492**	**$35.342**	**36.271**
All Funds	**$120.227**	**$138.655**	**$139.642**

The table above provides part of certain Financial Plan information of a state for FY 2014, FY 2015, and FY 2016.

1. Which of the following best approximates the average rate of change in the financial plan for State Fund from 2014 to 2016 ?

(A) $\$3.1\times10^5$

(B) $\$3.1\times10^6$

(C) $\$3.7\times10^6$

(D) $\$3.7\times10^5$

Solution: Answer: (B)

The changes happen in 2, not 3 periods.

from 2014 to 2015 and from 2015 to 2016

$$\Downarrow$$

$$r=\frac{102.551-96.355}{2}=\frac{6.196}{2}\approx 3.1$$

2. Which of the following program's ratio of its 2014 financial plan to its 2016 financial plan most approximats to the Capital Budget program's ratio of its 2014 financial plan to its 2016 financial plan ?

(A) All Funds

(B) Federal Operating Aid

(C) General Fund

(D) State Fund

Solution: Answer: (B)

$$\begin{cases}\text{Capital Budget}: & \frac{7.95}{9.13}\approx\frac{8}{9}\\ \text{Federal Operating Aid}: & \frac{32}{36}\approx\frac{8}{9}\end{cases}$$

$$\Downarrow$$

Federal Operating Aid

Questions 1 and 2 refer to the information below.

FINANCIAL PLAN
(millions of dollars)

	Financial Year		
Program	2014	2015	2016
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