Pearson & MINISTRY OF EDUCATION AND TECHNICAL EDUCATION

est EGYPTIAN SCHOLASTIC TEST

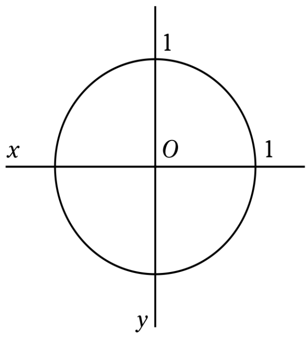
EST I MATH

LEVEL UP

2025 EDITION

STRATEGIES

Take a Number, Any Number: The Mathematics Sections



In this part . . .

- Discover the ins and outs of the EST Math sections.
- Strengthen your knowledge and understanding of algebra and work on algebra-related practice questions.
- Get the scoop on problem solving and data analysis and then test your skills on some practice questions.
- Find out what you need to know about advanced math as it pertains to the EST.
- Become comfortable with other types of math topics found on the EST.

Meeting Numbers Head-On: The EST Math Section

In This Chapter

- Surveying the Mathematics portion of the EST
- Choosing the right calculator and using it efficiently during the exam
- Tackling time constraints
- Getting good at grid-ins
- Adopting the best strategies for EST math questions

If you're one of those people who whined to your ninth-grade math teacher, "No one in the real world ever has to calculate the value of $6x - y$," the EST is about to prove you wrong. You can't get much more real world than a test that helps to determine where you go to college and maybe even what sort of job you get afterward. And on the EST, the value of $6x - y$ is fair game. So are absolute value (and I'm not talking about the great price you got on that orange sweater), exponential growth (the kind your tuition payments will display), and plenty of other stuff. In this chapter, we show you what's where, how to prepare, and most important, how to survive the EST Math section.

Having Fun with Numbers: EST Math 101

The EST that you begin one chilly morning in the near future will contain two sections of math that count toward your score: one 55-minute section and one 25-minute section. You may also encounter an unscored section that allows the EST-makers to try out new questions. In other words, you pay them to try out experimental questions on you. Nice, huh?

The experimental section doesn't count toward your final score, but because you may not know which section is equating, don't blow off anything. You could be ignoring a section that matters.

Each Math section begins with a little gift basket: a set of formulas to help you solve the problems — the area and circumference of a circle, the area of a square, the angles and sides of "special" triangles, and so forth. As you plod through an EST Math section, look back whenever you need this information so you're sure that your nerves haven't changed, say, the area of a rectangle from $A = lw$ to $A = lw^2$.

Outside the possible unscored experimental section, 45 EST Math questions are multiple-choice with four answer choices. Thirteen questions are *grid-ins,* which require you to bubble in the numbers you come up with, thus, giving you no hint whatsoever about the correct answer. Two of these 13 grid-ins are *Extended Thinking,* meaning they're more complicated and worth 2 points each. They are also related and based on the same information. (Check out the section "Knowing When to Grid and Bear It," later in this chapter, for everything you need to know about these questions.) Expect to see problems relating to topics commonly covered in the first three years of high-school math. In other words, you'll see questions on numbers and operations, algebra and functions, geometry, trigonometry, statistics, probability, and data interpretation. For more specifics, read on.

Algebra and functions

These problems account for 35 percent of your math score. They ask you to analyze and solve equations, create expressions to represent quantity relationships, and rearrange and interpret formulas. They include the following:

- **Absolute value:** How far away from a particular point on the number line is another number? That's the absolute value, which may show up in equations or functions.
- **Inequalities:** No, it's not social injustice. It's whether one number is larger than another, or if x is greater than 2, or what happens when you multiply both sides by –1.
- **Exponents:** These little numbers tell you how many times to multiply something by itself, as in x^4. You may see positive, negative, and fractional exponents.
- **Factoring:** Factoring is the math equivalent of extracting the cocoa powder and flour from a brownie after it's baked. Here's a typical factoring problem: If a rectangle has a width of $x+3$ and an area of $x^2+8x+15$, what is its length, in terms of x?
- **Functions:** We're not talking about cousin Thelma's fundraiser for impoverished beekeepers but about problems in which you take a number, do some stuff to it, and end up with a new number. Functions, which are written as $f(x)$ or other letters, including $g(h)$, appear in a number of forms, including the graphs of linear and quadratic functions.
- **Special symbols:** These strange figures have been created just for the EST; in other words, they don't exist in *normal* math. You have to figure out, given the definition, how to manipulate these symbols.

Problem solving and data analysis

These problems, worth 28 percent of your math score, examine whether you can analyze relationships by using ratios and proportions as well as interpret and summarize graphs. They include the following:

- **Arithmetic:** You have to add, subtract, multiply, and divide, plus show understanding of even and odd numbers, positive and negative numbers, consecutive integers, and primes.
- **Arithmetic sequences:** You have to see how numbers fit together to make a sequence, or pattern. For example, if you get 20, 24, 28, 32, and 36 on your five most recent math quizzes, what will you get on the next one, assuming that the sequence stays the same?
- **Coordinate geometry:** The EST asks about slopes of lines, including parallel and perpendicular lines. Also, if point G has the coordinates (x_1, y_1) and point W has coordinates (x_2, y_2), what is the midpoint or the distance between the points? You may

also have to interpret the graph of a function and to answer questions about transformations of a function. An example: If $f(x)$ measures how much time Gloria spends on her cellphone, how will $f(x)$ change the day after her unlimited calling plan starts?

- **Exponential growth sequences, also known as geometric sequences:** These questions require you to multiply by a certain number to get to the next term in the sequence. For example, the number of bent wire hangers at the bottom of your closet increases by a factor of 3 each day: 4, 12, 36, 108, 324 . . . you get the idea. You may be asked to create a mathematical statement expressing the way this wire hanger collection grows.
- **Percents:** You will be asked to find the amount that you pay if your book bill increases by 4,000 percent and books are 10 percent of your budget.
- **Ratios and proportions:** The EST asks about values that are in proportion. If the ratio of tuba players that try out for Prestigious University to those who get in is 200 to 3, how many tuba players are accepted out of the 400 who apply?
- **Sets, including union, intersection, and elements:** The EST may ask you to identify common elements or ask other questions about two or more sets. The set of all the dog treats given and the set of all the dog treats that the dogs will actually eat (instead of strew around the living room) overlap slightly.

Passport to advanced math

Problems here, worth 27 percent of your math score, ask you to solve quadratic equations and rewrite expressions based on the math structure. It's also a useful bucket for questions that don't fall into the other three categories.

- **Averages:** Make friends with the three *m*'s — mean, median, and mode.
- **Geometric probability:** If you're hanging a picture on the kitchen wall, what's the probability that you'll drive the nail right through a hot-water pipe?
- **Logic:** This topic covers those horrible problems you never see in real life, such as *What is the seating plan if Mr. Green can't sit next to Ms. Red but must sit across from Violet and behind Orchid or he throws popcorn. . . .* Wait, this sounds like the seating plan at a wedding. You *do* use this stuff in real life!
- **Equations and inequalities:** Equal signs represent what things are, as in $h = 3w + 4$ represents the number of hours you work while learning stuff like this. Or, if $3w + 4$ isn't enough time, your hours are represented with the inequality $h > 3w + 4$. This category also includes quadratic equations, which have x^2 in them, such as $x^2 + 8x + 15 = 0$.
- **Probability:** If you have 12 pairs of black socks and one pair of white socks, what is the likelihood that you'll match two socks right out of the drawer?
- **Graphs and charts:** These are data represented in a drawing. The test-writers may show you a bunch of dots or some other pattern, where the *x*-axis represents the amount of time students spend reading this book and the *y*-axis shows their EST scores. You may have to answer questions on this, such as exactly how it was the right move to pick up this new edition of *EST For Dummies.*

Anything here sounds like a foreign language? Probably, because math has its own language. If you need to brush up on one or more of these topics, check out the relevant chapters in Part IV for review and practice problems.

Chapter 10

Numb and Numbering: The Ins and Outs of Numbers and Operations

In This Chapter

- Identifying types of numbers and following the order of operations
- Calculating percents and working with ratios
- Figuring out rate/time/distance problems
- Eyeing radicals and absolute value
- Understanding sequences and sets

Once upon a time, you could take care of all the numbers you needed for school purposes with ten fingers and, in a pinch, a couple of toes. Sadly, life has changed. For the EST you need to know what's prime and what's not as well as how to calculate and manipulate percents, ratios, means, and the like. Not to mention sets and sequences! Never fear. Even though you've moved way beyond body-part math, this chapter tells you everything you need to know about numbers and operations, at least as they appear on the EST.

Meeting the Number Families

Mathematics is based on numbers, and different groups of numbers work in different ways. It helps to know about these before taking the exam, so in this section we explore the variations.

You may be wondering why you need a vocabulary lesson to do well on EST math. The fact of the matter is, the EST makers love to tuck these terms into the questions, as in "How many prime numbers are . . ." or "If the sum of three consecutive integers is 99, what is . . ." and the like. If you don't know the vocabulary, you're sunk before you start.

Check out these different groups of numbers:

- **Whole numbers:** *Whole numbers* aren't very well named, because they include 0, which isn't a whole lot of anything. The whole numbers are the ones you (hopefully) remember from grade school: 0, 1, 2, 3, 4, 5, 6 . . . you get the idea. Whole numbers, by definition, don't include fractions or decimals.

  Whole numbers can't be negative, but they can be even or odd. *Even numbers* are divisible by 2, and *odd numbers* aren't.

- **Prime numbers:** *Prime numbers* are whole numbers divisible only by themselves and by 1. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, and 19. Zero and 1 aren't prime numbers. They're considered "special." (The kids in grade school said that about us, too.) Two is the only even prime number. No negative number is ever prime because all negative numbers are divisible by –1.

  One common misconception is that all odd numbers are prime. Don't fall into that trap. Tons of odd numbers (9 and 15, for example) aren't prime, because they're divisible by at least one other number besides 1 and itself.

- **Composite numbers:** Any whole number that's not prime or special is *composite.* If you can divide a number by some smaller whole number (other than 1) without getting a remainder, you have a composite number. A few composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, and so on.

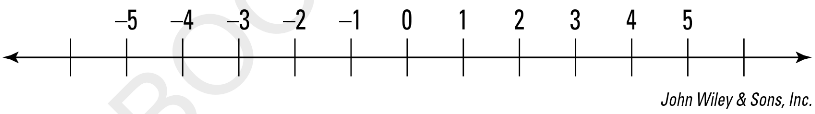
  Speaking of divisibility, remembering these points will win you EST points:

  - All numbers whose digits add up to a multiple of 3 are also divisible by 3. For example, the digits of 789 add up to 24 ($7+8+9=24$); because 24 is divisible by 3, so is 789.
  - Ditto for multiples of 9. If the digits of a number add up to a multiple of 9, you can divide the number itself by 9. For example, the digits of 729 add up to 18; because 18 is divisible by 9, so is 729.
  - All numbers ending in 0 or 5 are divisible by 5.
  - All numbers ending in 0 are also divisible by 10.

  Consider the number 365. It's not even, so it can't be divided by 2. Its digits add up to 14, which isn't divisible by 3 or 9, so it's not divisible by either 3 or 9. Because 365 ends in 5, it's divisible by 5. Because it doesn't end in 0, it's not divisible by 10.

- **Integers:** The whole numbers and all their opposites — also known as *negative numbers* — are *integers.* The whole numbers go all the way up to infinity, but the integers are even more impressive. Integers reach infinity in both directions, as the number line in Figure 10-1 shows.

Figure 10-1: Integers go on forever.



TIP

When you're asked to compare integers, remember that the farther to the right a number is, the greater it is. For example, –3 is greater than –5.

- **Rational numbers:** All integers are *rational numbers.* In addition, any number that can be written as a fraction — proper or improper — is a rational number. (In a *proper fraction,* the number on top is smaller than the number on the bottom, and in an *improper fraction,* the top number is greater than the bottom number.) Plus, any decimal that either ends, such as 1.2, or repeats, such as $0.\overline{33}$ (the decimal for 1/3), is a rational number. The following are also rational: –2, 0.234, 787/23, $3.21\overline{33}$.

- **Irrational numbers:** These are numbers for which a padded room is necessary. Kidding. *Irrational numbers* have decimals that never end or repeat. Practically speaking, you need to worry about only two kinds of irrational numbers:

  - Radicals (such as $\sqrt{2}$ and $\sqrt{3}$)
  - π, which you've seen from working with a circle

Getting Your Priorities Straight: Order of Operations

How many times has your mom told you to turn off the PlayStation and start your homework because you "have to get your priorities straight"? We won't comment on the annoyance that authority figures generate, especially when they're right, but we're going to tell you that in math, priorities matter.

Consider the problem $3 + 4 \times 2$. If you add $3 + 4$, which of course equals 7, and multiply by 2, you get 14. Nice answer, but wrong, because you forgot about Aunt Sally. "Aunt Sally," or more accurately, "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally" — *PEMDAS* — is a mnemonic device you can use to help you remember what mathematicians call *order of operations*. When faced with a multipart problem, just follow the order of operations that "Aunt Sally" calls for. Note the italicized letters in the following step list, which tells you what "Aunt Sally" really means:

1. **Do everything in *p*arentheses.**
2. **Calculate all *e*xponents.**
3. ***M*ultiply and *d*ivide, from left to right.**
4. ***A*dd and *s*ubtract, from left to right.**

Back to the sample problem, $3 + 4 \times 2$. No parentheses or exponents, so the first operations up are multiplication and division. Because there's no division, you're left with 4×2, which equals 8. Onward to addition and subtraction (in this problem, subtraction isn't present, so forget about subtracting). Just add 3 to 8, at which point you arrive at 11, the correct answer.

Many calculators know the "Aunt Sally" rules, but on older ones, sometimes you have to input the numbers according to the "Aunt Sally" rules to ensure the right answer. Be sure to figure out which kind of calculator you have before test day.

Here's another chance to visit Aunt Sally:

The expression $20 - (40 \div 5 \times 2) + 3^2$ equals

(A) –5

(B) 7

(C) 10

(D) 13

The answer is Choice (D). Start with what's in the parentheses: $40 \div 5 \times 2$. Don't fall into the trap of multiplying 5×2 first; proceed from left to right: $40 \div 5 = 8$ and $8 \times 2 = 16$. Next, tackle the exponent: $3^2 = 9$. At this stage, you have $20 - 16 + 9$. Again, resist the temptation to start by adding; just go left to right ($20 - 16 = 4$ and then $4 + 9 = 13$).

Playing Percentage Games

The EST loves percentages, perhaps because math teachers who are sick of the question "Am I ever going to use this stuff in real life?" actually write the math portion of the exam. With percentages, the answer is yes if you're taking out a loan (interest rates) or investing the earnings from your part-time job in mutual funds (still interest, but this time it's a good thing) or buying something on sale (15 percent off the regular price). *Percents* represent how much of each hundred you're talking about.

Taking a percentage of a number is a simple task if you're using a calculator with a "%" button. Just hit the "%" and "×" buttons. For example, to find 60 percent of 35, multiply 60% by 35. The answer is 21. If you're not blessed with such a calculator, you can turn a percent into a decimal by moving the decimal point two spaces to the left, as in $60\% = 0.60$. (Other examples of percents include $12.5\% = 0.125$, $0.4\% = 0.004$, and so on.) Or turn the percent into a fraction. The "cent" in *percent* means hundred, (as in a *century* is 100 years,) so 60 percent literally means, "60 per 100," or 60/100.

For more complicated problems, fall back on the formula you mastered in grade school:

$$\frac{\text{is}}{\text{of}} = \frac{\%}{100}$$

Suppose you're asked "40% of what number is 80?" The number you're looking for is the number you're taking the percent *of,* so x will go in the *of* space in the formula:

$$\frac{80}{x} = \frac{40}{100}$$

Now cross-multiply: $40x = 8{,}000$. Dividing by 40 gives you $x = 200$.

You can also consider that the percent of the whole equals the part. In other words, 40% of x is 80. Set up the equation and solve for x:

$$\begin{aligned} 40\%x &= 80 \\ 0.4x &= 80 \\ x &= \frac{80}{0.4} \\ x &= 200 \end{aligned}$$

One subtopic of percentages is a problem that involves a percent increase or decrease. A slight variation of the percentage formula helps you out with this type of problem. Here's the formula and an example problem to help you master it:

$$\frac{\text{amount of change}}{\text{starting amount}} = \frac{x}{100}$$

EXAMPLE

The value of your investment in the winning team of the National Softball League increased from \$1,500 to \$1,800 over several years. What was the percentage increase of the investment?

(A) 300

(B) 120

(C) 50

(D) 20

The correct answer is Choice (D). The key here is that the number 1,800 shouldn't be used in your formula. Before you can find the *percent* of increase, you need to find the *amount* of increase, which is $1,800 - 1,500 = 300$. To find the percentage of increase, set up this equation:

$$\frac{300}{1,500} = \frac{x}{100}$$

Cross-multiply to get $1,500x = 30,000$. Dividing tells you that $x = 20$ percent.

The EST-makers often try to confuse you by asking about something that doesn't appear in the original question, as in this example:

At one point in the season, the New York Yankees had won 60 percent of their games. The Yanks had lost 30 times and never tied. (As you know, there are no ties in the world's noblest sport, baseball.) How many games had the team played?

(A) 12

(B) 18

(C) 50

(D) 75

The answer is Choice (D). Did you find the catch? The winning percentage was 60 percent, but the question specified the number of losses. What to do? Well, because ties don't exist, the wins and losses must have represented all the games played, or 100 percent. Thus the percentage of losses must be $100\% - 60\%$, which is 40%. Putting the formula to work:

$$\frac{30}{x} = \frac{40}{100}$$

As always, cross-multiply: $40x = 3,000$, and $x = 75$

Keeping It in Proportion: Ratios

After you know the tricks, ratios are some of the easiest problems to answer quickly. Here are the points to remember:

- A ratio is written as $\frac{\text{of}}{\text{to}}$ or of:to.
  - The ratio *of* sunflowers *to* roses = $\frac{\text{sunflowers}}{\text{roses}}$.
  - The ratio *of* umbrellas *to* heads = umbrellas : heads.
- A possible total is a multiple of the *sum* of the numbers in the ratio.

You may have to confront a proportion problem like this on the test:

> At a party, the ratio of blondes to redheads is 4:5. What could be the total number of blondes and redheads at the party?

This one's easy. Just add the numbers in the ratio: $4 + 5 = 9$. The total must be a multiple of 9, such as 9, 18, 27, 36, and so on. If this "multiple of" stuff is confusing, think of it another way: The sum must divide evenly into the total. That is, the total must be divisible by 9. Can the total, for example, be 54? Yes, 9 goes evenly into 54. Can it be 64? No, 9 doesn't go evenly into 64.

Check out another example.

While creating his special dish, Thomas uses 7 teaspoons of whipped topping for every 5 teaspoons of chocolate mousse. Which of the following could be the total number of teaspoons of whipped topping and chocolate mousse in his special dish?

(A) 75

(B) 57

(C) 48

(D) 35

The correct answer is Choice (C). Add the numbers in the ratio: $7+5=12$. The total must be a multiple of 12. (It must be evenly divisible by 12.) Here, only *48,* Choice (C), is evenly divisible by 12. Of course, 75 and 57 try to trick you by using the numbers 7 and 5 from the ratio.

Notice how the question has been what *can be* the *possible* total. The total can be *any* multiple of the sum. If a question asks you which of the following *is* the total, you have to answer, "It cannot be determined." You know only which *can be* true.

Another ratio headache strikes when you're given a ratio and a total and asked to find a specific term. To find a specific term, do the following, in order:

1. **Add the numbers in the ratio.**
2. **Divide that sum into the total.**
3. **Multiply that quotient by each term in the ratio. (The *quotient* is the answer you get when you divide.)**
4. **Add the answers to double-check that they sum up to the total.**

Pretty confusing stuff. Take it one step at a time. Look at this example problem:

> To congratulate his team, which had just won the last game for an undefeated 21-in-0 season, the ecstatic coach took his team to the local pizza joint, where each player ordered either a deep dish pizza or a calzone. If there were 3 deep dishes for every 4 calzones, and if every member of the 28-man squad ordered either one or the other, how many deep dishes were there?

Here's how to solve it:

1. **Let $3x$ be the number of players who ordered a pizza.**
2. **Let $4x$ be the number of players who ordered a calzone.**
3. **Together they ordered 28 meals: $3x+4x=28$.**
4. **Solve for x: $x=4$.**
5. **Plug in 4 for x to get your numbers: $3(4)=12$ pizzas and $4(4)=16$ calzones.**

Getting DIRTy: Time, Rate, and Distance

Time to dish the dirt, as in D.I.R.T. **D**istance **Is** **R**ate × **T**ime or D = *RT.* When the EST throws a time, rate, and distance problem at you, use this formula. Make a chart with the formula across the top and fill in the spaces on the chart. Here's an example to help you master this formula:

> Jennifer drives 40 miles an hour for $2\frac{1}{2}$ hours. Her friend Ashley goes the same distance but drives at $1\frac{1}{2}$ times Jennifer's speed. How many *minutes* longer does Jennifer drive than Ashley?

Don't start making mad formulas with *x*'s and *y*'s. Make the DIRT chart using the distance formula: Distance = Rate × Time.

When you fill in the 40 miles per hour and $2\frac{1}{2}$ hours for Jennifer, you can calculate that she went 100 miles. Think of it this way: If she goes 40 miles per hour for 1 hour, that's 40 miles. For a second hour, she goes another 40 miles. In a half hour, she goes $\frac{1}{2}$ of 40, or 20 miles. (See? You don't have to write down $40 \times 2\frac{1}{2}$ and do all that pencil-pushing; use your brain, not your yellow No. 2 pencil or your calculator.) Add them together: $40 + 40 + 20 = 100$. Jennifer drives 100 miles.

	Distance	=	***Rate***	×	***Time***
Jennifer	100		40 mph		$2\frac{1}{2}$ hours

Because Ashley drives the same distance, fill in 100 under distance for her. She goes $1\frac{1}{2}$ times as fast. Uh-uh, put down that calculator. Use your brain: 1×40 is 40; $\frac{1}{2} \times 40$ is 20. Add $40 + 20 = 60$. Ashley drives 60 miles per hour. Now this gets really easy. If Ashley drives at 60 miles per hour, she drives one mile a minute (60 minutes in an hour, 60 miles in an hour). Therefore, to go 100 miles takes her 100 minutes. Because your final answer is asked for in minutes, don't bother converting this to hours; leave it the way it is.

	Distance	=	***Rate***	×	***Time***
Ashley	100		60 mph		100 minutes

Last step. Jennifer drives $2\frac{1}{2}$ hours. How many minutes is that? Do it the easy way, in your brain. One hour is 60 minutes. A second hour is another 60 minutes. A half hour is 30 minutes. Add them together: $60 + 60 + 30 = 150$ minutes. If Jennifer drives for 150 minutes and Ashley drives for 100 minutes, Jennifer drives 50 minutes more than Ashley.

	Distance	=	***Rate***	×	***Time***
Jennifer	100		40 mph		150 minutes
Ashley	100		60 mph		100 minutes

Be careful to note whether the people are traveling in the same direction or opposite directions. Suppose you're asked how far apart drivers are at the end of their trip. If you're told that Jordan travels 40 miles per hour east for 2 hours and Connor travels 60 miles per hour west for 3 hours, they're going in opposite directions. If they start from the same point at the same time, Jordan has gone 80 miles one way, and Connor has gone 180 miles the opposite way. They're 260 miles apart. The trap answer is 100, because careless people (not *you*) simply subtract 80 from 180.

Demonstrating the Value of Radicals

In math-speak, a *radical* is a square root as well as the symbol indicating square root, $\sqrt{\ }$. The *square root* of number *x*, written as $\sqrt{x}$, is the positive number, which, multiplied by itself, gives you *x*. As a classic example, $\sqrt{9} = 3$, because $3 \times 3 = 9$. Although most numbers have square roots that are decidedly not pretty, ($\sqrt{7}$, for example, equals approximately 2.646), most of the radicals you encounter on the EST will either simplify nicely (such as $\sqrt{25} = 5$) or can be left in the radical form (such as $\sqrt{2}$ or $3\sqrt{5}$).

The rules for multiplication and division of radicals are simple. Just multiply and divide the numbers normally: $\sqrt{5} \times \sqrt{6} = \sqrt{30}$ and $\sqrt{21} \div \sqrt{7} = \sqrt{3}$. However, you can't add or subtract radicals. For example, $\sqrt{3} + \sqrt{5}$ doesn't equal $\sqrt{8}$. You can break down any radical by factoring out a perfect square and simplifying it, so $\sqrt{27} = \sqrt{9}\sqrt{3} = 3\sqrt{3}$, and $\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$. Also, you can add and subtract radicals that have the same number under the square root symbol, so $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$. In a sense, radicals work like variables. When the variables or radicals are the same, you can add or subtract their coefficients: $5x + 3x = 8x$ and $5\sqrt{2} + 3\sqrt{2} = 8\sqrt{2}$ work the same way.

A few squares show up all the time on the EST. Scan Table 10-1 so you're familiar with these numbers when you see them.

Table 10-1 Simple Square Roots

Numbers	$-x$	–1	0	1	2	3	4	5	6	7	8	9	x
Squares	x^2	1	0	1	4	9	16	25	36	49	64	81	x^2

Notice how the square of both x and $-x$ is x^2? Conveniently, when you multiply two negative numbers, the result is positive, as it is when you multiply two positive numbers. So the square of the same negative and positive number is always the same: $(-8)^2 = 64$ and $(+8)^2 = 64$.

A couple of notes on this. First of all, the exponent affects only what it's touching. For example, $5x^2$ is equivalent to $5 \cdot x \cdot x$. You may wonder why the 5 isn't also squared: It's because the exponent is only touching the *x*. If you want to square the 5 also, put the expression in parentheses, and then put the exponent on it: $(5x)^2 = 5 \cdot x \cdot 5 \cdot x = 25x^2$. This is also true with the negative sign: Put -5^2 in the calculator, and it returns –25. This is because the calculator reads -5^2 as $-5 \cdot 5 = -25$. In other words, it squares the 5 and places the negative on it: –25. To square the entire –5, place it in parentheses. The calculator reads $(-5)^2$ as $(-5) \times (-5)$ and returns the answer you were expecting: 25.

Secondly, if $x^2 = 64$, x could equal either 8 or –8. However, the square root of a number always has the positive value. For example, $\sqrt{64}$ equals 8 but never –8. This is because the square root can only be on a positive number.

Computing Absolute Value

Absolute value is a simple concept that's annoyingly easy to mess up. *Absolute value* is the number, shorn of its positive or negative value. The number has pipes on either side, like this: $|-15|$. The absolute value of 3 is written $|3|$, which equals 3; the absolute value of –3 is written $|-3|$, which also equals 3.

1. **Simplify whatever is inside the absolute value symbol, if possible.**
2. **If the answer is negative, switch it to positive.**

Some people have the (incorrect) idea that absolute value changes subtraction to addition. Nope. If you're working with $|3-4|$, don't change the quantity to $3+4$. Calculate whatever is inside the absolute value symbols first, $|3-4|=|-1|$, and only *then* change the result to a positive number, in this case 1.

Finding the Pattern

Check out this arithmetic sequence: 2, 5, 8, 11, 14. . . . Notice how each number is 3 more than the previous number? In an arithmetic sequence, you always add or subtract the same number to the previous term to get the next term. Another example of an arithmetic sequence is 80, 73, 66, 59. . . . In this one, you're subtracting 7 from the previous term.

A geometric sequence is similar to an arithmetic sequence, but it works by multiplication or division. In the sequence 2, 6, 18, 54, . . . every term is multiplied by 3 to get the next term. In 88, 44, 22, 11, . . . each term is divided by 2 to get the next term.

Often, the best way to solve these problems is just to make a list and follow the pattern. However, if the test writers ask you for something like the 20th term of the sequence, this process can take forever. Each type of sequence has a useful formula, which is worth memorizing if you have the time and the room in your head:

- For an arithmetic sequence, the *n*th term = a (the first term) + $(n-1)d$, where d is the difference between consecutive terms in the sequence. In the sequence 2, 5, 8, 11, 14, the difference between consecutive terms is +3, because you add 3 each time. What would be the 20th term? Take 2, the first term, and add 3 nineteen times, so it's $2+19(3)=2+57=59$.
- For a geometric sequence, the *n*th term = a (the first term) $\times r^{(n-1)}$, where r is the ratio of one term to the next. Huh? Well, you probably remember that taking something to a power (that's what the exponent stands for) means multiplying it by itself a bunch of times. For example, 4 to the 3rd power = $4\times4\times4$, which equals 64. You can do powers on most calculators by using either the "y^x" or the "^" button.
- Check out this sequence: 2, 6, 18, 54. The ratio is 3, because you multiply by 3 each time. To find the 10th term (the 20th would be way too big to handle), take 2×3^9 (that's 3 to the 9th power): $3^9=19{,}683$, and $2\times19{,}683=39{,}366$, so that's the answer.

To find the *n*th term, you always use $n-1$, no matter what kind of sequence it is.

The bacteria population in my day-old wad of chewing gum doubles every 3 hours. If there are 100 bacteria at 12:00 noon on Friday, how many bacteria will be present at midnight of the same day?

(A) 200

(B) 300

(C) 800

(D) 1,600

The right answer is Choice (D). To solve this problem, make a chart. Because the population doubles every 3 hours, count off 3-hour intervals, doubling as you go:

12:00 (noon) = 100 bacteria

3:00 p.m. = 200 bacteria

6:00 p.m. = 400 bacteria

9:00 p.m. = 800 bacteria

12:00 (midnight) = 1,600 bacteria

Or use the formula for $100 \times 2^4 = 100 \times 16 = 1{,}600$.

And here's another example in which the formulas come in handy:

Author A, an extraordinarily fast writer who zips through a chapter a day, gets paid $100 for her first chapter, $200 for her second, $300 for her third, and so on. Author B, also a member of the chapter-a-day club, gets paid $1 for his first chapter, $2 for his second, $4 for his third, $8 for his fourth, and so on. On the 12th day,

(A) Author A is paid $76 more.

(B) Author B is paid $24 more.

(C) Author A is paid $1,178 more.

(D) Author B is paid $848 more.

The correct answer is Choice (D). Author A's plan is an arithmetic sequence, increasing by $100 each time, so on the 12th day she's paid $100 + 11(100) = 100 + 1{,}100 = \$1{,}200$. Author B's plan is a geometric sequence, multiplied by 2 each time, so on the 12th day, he's paid $1 \times 2^{11} = 1 \times 2{,}048 = \$2{,}048$. Because $\$2{,}048 - \$1{,}200 = \$848$, author B is paid $848 more.

Setting a Spell

A *set* is just a collection of things — marbles, hockey pucks, Legos, whatever. In math, a set is a collection of *elements,* usually numbers, which you find inside brackets: $\{\ldots\}$. For example, the set of positive integers less than 6 is a set with five elements: $\{1, 2, 3, 4, 5\}$. Some sets go on forever, and three dots at the end tell you so. The set of positive odd numbers is $\{1, 3, 5, 7, \ldots\}$

because it reaches infinity. A set may have nothing inside of it; this is the "empty set" or "null set," and it's written either $\{\ \}$ or, more commonly, $\varnothing$.

you need to know about two specific things when it comes to sets — the union and the intersection of sets. The *union* of two sets is just the two sets put together; thus, the union of $\{1, 2, 3\}$ and $\{5, 7, 8\}$ is $\{1, 2, 3, 5, 7, 8\}$.

Even if something shows up in both sets, it shows up only once in the union. Thus, the union of $\{2, 3, 4\}$ and $\{3, 4, 5\}$ is $\{2, 3, 4, 5\}$, *not* $\{2, 3, 3, 4, 4, 5\}$. The following steps help you find the number of elements in the union of two sets:

1. **Add up the number of elements in each set.**
2. **Subtract the number of elements that show up in both.**

To count the elements in the preceding example, $3+3=6$; but because 3 and 4 show up in both sets, you have to subtract 2. The union has 4 elements. The *intersection* of two sets, on the other hand, contains only those elements that show up in both sets. The intersection of $\{1, 2, 4\}$ and $\{4, 6, 7\}$ is $\{4\}$; the intersection of $\{3, 5, 7\}$ and $\{2, 4, 6\}$ is $\varnothing$.

Chapter 11

Practicing Problems in Numbers and Operations

That old saying, "Practice makes perfect," is annoying yet true. In this chapter, we hit you with two sets of numbers and operations questions along with explanations of the answers.

After you practice each question in the first set, check your answers and read the explanations for any questions you answered incorrectly. (The answers immediately follow each question. Use a piece of paper to cover the answers as you work.) If you're confused about any point, check Chapter 10 for more details on the kind of problem that's stumping you.

The second set is set up like the real test: You do all the problems and then check your work with the answer key that follows the last question.

Set One: Trying Out Some Guided Questions

1. If you invest \$2,000 for one year at 5% annual interest, the total amount you would have at the end of the year would be

(A) \$100

(B) \$2,005

(C) \$2,100

(D) \$2,500

Solve the question like this: 5% = 0.05, so 5% of $\$2,000 = 0.05 \times \$2,000 = \$100$. But wait! Before you choose \$100 as your answer, remember that you still have the \$2,000 that you originally invested, so you now have $\$2,000 + \$100 = \$2,100$. Choice (C) is correct.

2. Which number is an element of the set of prime numbers but not of the set of odd numbers?

(A) 0

(B) 1

(C) 2

(D) 3

Because 2 is the only prime number that isn't odd, Choice (C) is correct.

3. 100 percent of 99 subtracted from 99 percent of 100 equals

(A) –1

(B) 0

(C) 0.99

(D) 1

Keep in mind that 100 percent of anything is itself, so 100 percent of 99 is 99. And 99 percent of 100 equals $0.99 \times 100 = 99$ (not a big surprise because percent means "out of one hundred"). And $99 - 99 = 0$, so Choice (B) is the correct answer.

4. The tenth number of the sequence 50, 44.5, 39, 33.5, . . . is

(A) –4

(B) 0.5

(C) 1

(D) 1.5

The numbers decrease by 5.5 every time. The simplest way to do this problem is to continue the pattern: 50, 44.5, 39, 33.5, 28, 22.5, 17, 11.5, 6, 0.5. You can also use the following formula to find the tenth term: the *n*th term = the first term $+(n-1)d$, where *d* is the difference between terms in the sequence. Therefore, $50+9(-5.5) = 50 - 49.5 = 0.5$ $50+(10-1)(-5.5) = 50-49.5 = 0.5$. Choice (B) is the correct answer.

5. If *E* represents the set of even numbers and *N* represents the set of numbers divisible by 9, which number is in the intersection of *E* and *N*?

(A) 99

(B) 92

(C) 66

(D) 54

An element is in the intersection of two sets only if it's in both of them. You can go through the choices until you find the right one: 99 isn't even; 92 isn't divisible by 9; 66 isn't divisible by 9; 54 is even *and* divisible by 9. Thus, 54 is the only one that works, and Choice (D) is the right answer.

6. The first three elements of a geometric sequence are 1, 2, and 4. What is the eighth element of the sequence?

(A) 14

(B) 16

(C) 29

(D) 128

The formula for geometric sequences tells you that the answer is $1 \times 2^7 = 1 \times 128 = 128$. (Remember that in this formula, the exponent is one less than the number of the term you're being asked for.) Go with Choice (D).

7. The expression $3^2 - 4 + 5\left(\frac{8}{2}\right)$ equals

(A) –27

(B) –15

(C) 5

(D) 25

"Aunt Sally" can help with this problem. (See Chapter 10 for the lowdown on our favorite relative.) First, do the operation in parentheses, $\frac{8}{2} = 4$, and then calculate 3^2, which equals 9. That leaves you with $9 - 4 + 5(4)$. Next, multiply $5 \times 4 = 20$. Now the expression is $9 - 4 + 20$. You have a trap to avoid: Did you see it? Don't do addition before subtraction; just go left to right: $9 - 4 = 5$, and $5 + 20 = 25$. Aunt Sally says Choice (D) is correct.

8. Which of the following numbers is rational?

(A) π

(B) 0.12112111211112 . . .

(C) $\sqrt{8}$

(D) $\sqrt{9}$

To do this problem, you need to remember the definitions of rational and irrational numbers. π is irrational by definition. (Yes, it's worth memorizing this fact.) The number 0.12112111211112 . . . is irrational because the decimal never ends or repeats. (For those of you who are still awake, it doesn't repeat because the number of 1s keeps increasing.) All radicals are irrational if the number underneath the radical symbol isn't a perfect square, so $\sqrt{8}$ is irrational. However, because $\sqrt{9} = 3$, it's rational. Choice (D) is correct.

9. Given that there are 30 days in April, the ratio of rainy days to sunny days during the month of April could *not* be

(A) 5:3

(B) 3:2

(C) 5:1

(D) 4:1

The rule for ratios states that the total must be divisible by the sum of the numbers in the ratio. Because $5 + 3 = 8$, and 30 isn't divisible by 8, Choice (A) is correct. Just to be sure, check that all the other possible sums do go into 30.

10. At a sale, a shirt normally priced at \$60 was sold for \$48. What was the percentage of the discount?

(A) 12%

(B) 20%

(C) 25%

(D) 48%

Use the percentage formula, $\frac{\text{is}}{\text{of}} = \frac{x}{100}$, but, as always, be extra careful. The problem asks for the percentage of the discount, so don't just plug in 48. Instead, first figure out the amount of the discount, which was $60 - 48 = 12$. Using 12, write $\frac{12}{60} = \frac{p}{100}$, where p is the percentage of the discount. Cross-multiplying, you get $1{,}200 = 60p$, and $p = 20$. You can still get the right answer using 48. If you use 48 in the formula, you get 80%. Because the shirt now costs 80% of what it used to, the discount is 100% – 80% = 20%. Choice (B) is correct.

Set Two: Practicing Some Questions on Your Own

Note: Two questions (2 and 6) are grid-ins. On the blank grids in this section, write and bubble in your answers. (See Chapter 9 for the proper way to bubble in your answers for grid-in questions.)

1. The total number of even three-digit numbers is

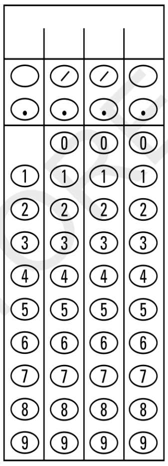
   (A) 49

   (B) 100

   (C) 449

   (D) 450

2. Evaluate $\left|10-\left(42\div\left|1-4\right|\right)\right|$.



3. A shark is eating the fish in a certain lake. Every eight days, exactly half of the fish in the lake are eaten. If there are 1,000 fish in the lake on March 3, how many remain on March 27?

   (A) 0

   (B) 100

   (C) 125

   (D) 250

4. If a number n is the product of two distinct primes, x and y, how many factors does n have, including 1 and itself?

   (A) 2

   (B) 3

   (C) 4

   (D) 5

5. Which number is 30% greater than 30?

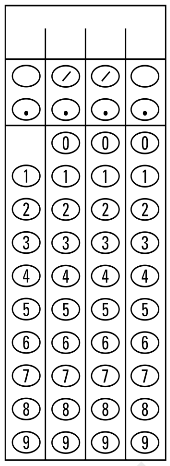
(A) 27

(B) 30.9

(C) 33

(D) 39

6. A recipe for French toast batter calls for 1/2 teaspoon of cinnamon for every 5 eggs. How many teaspoons of cinnamon would be needed if a restaurant made a batch of batter using 45 eggs?



7. Which of the following is *not* equivalent to $\sqrt{40}$?

(A) $2\sqrt{10}$

(B) $\sqrt{30} + \sqrt{10}$

(C) $\sqrt{5} \times \sqrt{8}$

(D) $\sqrt{90} - \sqrt{10}$

8. Janice wrote down all the numbers from 11 to 20. Darren wrote down all the positive numbers less than 30 that are divisible by 6. How many numbers are in the union of their two lists?

(A) 2

(B) 12

(C) 14

(D) 15

9. Elena drove for one hour at 60 miles per hour and for half an hour at 30 miles per hour. Returning home along the same route, she maintained a constant speed. If the journey home took the same total amount of time as the original drive, what was her speed on the journey home?

(A) 40 mph

(B) 42 mph

(C) 45 mph

(D) 50 mph

$$\begin{array}{r} ABC \\ \text{10.}\quad \underline{+6BC} \\ 1{,}B5A \end{array}$$

In the correctly solved addition problem above, *A, B,* and *C* all stand for different numbers from 1 to 9. The value of *C* must be

(A) 8

(B) 7

(C) 6

(D) 5

Answers to Set Two

1. **D.** Counting all the even three-digit numbers would take a really long time, so try to figure out this question logically. The three-digit numbers start with 100 and end with 999. How many numbers do you have? It's 900, not 899. (Yes, there is a formula you can use here: Subtract the numbers and add 1. Works every time.) How many of these numbers are even? Well, because even and odd numbers alternate on this list, half of them are even, and half are odd. So you have 450 of each type. Choice (D) is right.

2. **4.** When doing an absolute value problem, treat the absolute value symbols as parentheses when trying to figure out the order of operations. Because this problem has a bunch of parentheses and absolute values, work from the inside out:

$$\begin{gathered} \left|10-\left(42\div\left|1-4\right|\right)\right| \\ \left|10-\left(42\div\left|-3\right|\right)\right| \\ \left|10-\left(42\div3\right)\right| \\ \left|10-\left(14\right)\right| \\ \left|-4\right| \\ 4 \end{gathered}$$

3. **C.** On March 3, 1,000 fish exist. On March 11, 500 fish are alive. On March 19, 250 fish are swimming. And on March 27, 125 fish remain. Choice (C) is correct.

4. **C.** Prime numbers have only two factors: 1 and themselves. Pretend in your problem that $x = 5$ and $y = 7$. Then $n = 5 \times 7 = 35$. The factors of 35 are 1, 5, 7, and 35. Because you can't break down 5 or 7, there are no other factors. As long as you pick prime numbers for *x* and *y,* you'll always get four factors for *n*. Choice (C) is correct.

5. **D.** Solve it like this: 30% of $30 = 0.30 \times 30 = 9$. Because the answer is 30% greater than 30, add $30 + 9 = 39$. Go with Choice (D).

6. **4.5 or 9/2.** If 1/2 teaspoon is needed for 5 eggs, then 9 times that (because $9 \times 5 = 45$) is needed to make 45 eggs: $9 \times 1/2 = 4.5$ or $9/2$. (Don't grid in $4\frac{1}{2}$. The machine will read it as $41/2$, which is wrong.)

7. **B.** You could use a calculator to figure out what each choice equals, but it's better to work with the radicals, because you'll catch mistakes easier. Start with $2\sqrt{10}$. To multiply these,

turn 2 into $\sqrt{4}$; now, $\sqrt{4}\times\sqrt{10}=\sqrt{40}$. On to Choice (B): You can't add radicals, and there's no way to break down $\sqrt{30}$ or $\sqrt{10}$, because no perfect square goes into either one. So you're stuck on this one. In Choice (C), $\sqrt{5}\times\sqrt{8}=\sqrt{40}$. In Choice (D), which is $\sqrt{90}-\sqrt{10}$, $\sqrt{90}$ becomes $\sqrt{9}\times\sqrt{10}$ and then $3\sqrt{10}$. Now $3\sqrt{10}-\sqrt{10}=2\sqrt{10}$, which matches Choice (A), which you already know is equivalent to $\sqrt{40}$. Bottom line: They all equal $\sqrt{40}$, except for Choice (B). If you check it out on a calculator, $\sqrt{40}\approx 6.32$, but $\sqrt{30}+\sqrt{10}\approx 8.64$. Thus, Choice (B) is correct.

8. **B.** Janice's list has 10 numbers: $\{11,\ 12,\ 13,\ 14,\ 15,\ 16,\ 17,\ 18,\ 19,\ 20\}$. Darren's list has 4 numbers: $\{6,\ 12,\ 18,\ 24\}$. Now, don't fall into the trap of thinking that there are 14 numbers in the union; even though 12 and 18 show up in both sets, you're not allowed to count them twice in the union. The total number of elements in the union is $14-2=12$. Choice (B) is correct.

9. **D.** The original trip took $1\frac{1}{2}$ hours. Elena traveled 60 miles plus half of 30, which is $60+15$ or 75 miles. The return trip took the same amount of time: 1 hour + $\frac{1}{2}$ hour = $1\frac{1}{2}$ hours. And 75 miles divided by $1\frac{1}{2}$ hours equals 50 miles per hour. Choice (D) is correct.

10. **A.** You know that

$$\begin{array}{r} ABC \\ \underline{+6BC} \\ 1{,}B5A \end{array}$$

This problem takes some inductive reasoning. A must be an even number because you get it by adding $C+C$, but A can't be 2, because then the two numbers wouldn't add up to something bigger than 1,000. So A = 4, 6, or 8.

Now look at the tens column. The sum of $B+B$ can't be 5 unless you carried a "1" from the ones column. That means that $C + C$ = 14, 16, or 18, so C = 7, 8, or 9.

What about B? The sum of $B+B+1$ (you carried, remember) gives you 5. So B could be either 2 or 7, because $7+7+1=15$. But if B = 7, then the hundreds column makes no sense. (Try it and you'll see why.) So B must be 2. Because B = 2, A must be 6 to make the hundreds column work, and that makes C = 8. Check the original problem:

$$\begin{array}{r} 628 \\ \underline{+628} \\ 1{,}256 \end{array}$$

It works: Choice (A) is correct.

Hitting a vocabulary homer

Had enough math for the moment? Take a TV break. Have you ever seen anyone less ***hirsute*** (hairy) than Homer Simpson? This famous ***gourmand*** (someone who loves to eat and drink a large quantity) isn't exactly a fan of ***gourmet*** cooking (featuring excellent or high-quality food and drink). Homer is ***gullible*** (he'll believe anything) and ***indolent*** (don't wake him up when he's "working"). Three words you'll never use to describe Homer are ***adroit*** (clever), ***lithe*** (graceful), and ***emaciated*** (starving). Yet he is lovable.

Chapter 12

X Marks the Spot: Algebra and Functions

In This Chapter

- Working through expressions with exponents
- Using factoring to find solutions
- Unraveling equations to get to the right answer
- Understanding functions and knowing how to solve them

If x is the value of the present your mom expects for her birthday and y is the amount of money in your piggybank, what equation best represents your chances of staying on her good side this year? Don't worry. This problem won't appear on the EST, but it's a good example of algebra: You play with letters, even though you're solving a math problem.

If you love algebra, or even if you'd prefer to shred the pages of your algebra text, this chapter's for you. Here, you find a quick and dirty review of the basics of EST algebra, plus a spin through functions, where $f(x)$ rules.

Note: Throughout this chapter and this book, the $\times$ multiplication symbol is used in problems that involve two or more numbers; whenever the problem involves variables, the $\cdot$ multiplication symbol is used instead.

Powering Up: Exponents

- **The *base* is the big number (or letter) on the bottom. The *exponent* is the little number (or letter) in the upper-right corner.**
  - In x^5, x is the base; 5 is the exponent.
  - In 3^y, 3 is the base; y is the exponent.
- **A base to the zero power equals one.**
  - $x^0 = 1$
  - $129^0 = 1$

  There is a long, ***soporific*** (sleep-causing) explanation as to why a number to the zero power equals one, but you don't really care, do you? For now, just memorize the rule.
- **A base to the first power is just the base.** In other words, $4^1 = 4$.
- **A base to the second power is *base* $\times$ *base*.**
  - $x^2 = x \cdot x$
  - $5^2 = 5 \times 5 = 25$

- **The same is true for bigger exponents.** The exponent tells you how many times the number repeats. For example, 3^4 means that you write down four 3s and then multiply them all together.
  - $3^4 = 3 \times 3 \times 3 \times 3 = 81$
  - Remember that an exponent tells you to multiply the base times itself as many times as the exponent, so 2^3 does *not* equal 6 ($2^3 = 2 \times 2 \times 2 = 8$).

  On most calculators, you can do powers with either the "y^x" or the "^" button. Just type the base, the appropriate button, the exponent, and the trusty "=" button. However, almost all the exponents you encounter on the EST are simple enough that you don't need a calculator.

- **A base to a negative exponent is the reciprocal of the base to a positive exponent.**

  A *reciprocal* is the upside-down version of a fraction. For example, $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$. An integer (except 0) can also have a reciprocal: $\frac{1}{3}$ is the reciprocal of 3. When you have a negative exponent, just put base and exponent under a 1 and make the exponent positive again.
  - $x^{-4} = \frac{1}{x^4}$
  - $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

  The answer isn't negative. When you flip it, you get the reciprocal, and the negative goes away. Don't fall for the trap of saying that $5^{-3} = -5^3$ or -125.

  Also, if a number or variable with a negative exponent, such as x^{-4}, appears in the denominator of a fraction, such as $\frac{2}{3x^{-4}}$, you can make the exponent positive and move it to the numerator, like this: $\frac{2x^4}{3}$.

- **A base to a fractional exponent is a root of the base.** Ah, more confusion. You're already familiar with the standard square root of a number: $\sqrt{25} = 5$ because $5^2 = 25$. Because it takes two 5s to make 25, you can also write $25^{1/2} = 5$.

- **To multiply like bases, add the exponents.**
  - $x^3 \cdot x^2 = x^{(3+2)} = x^5$
  - $5^4 \times 5^9 = 5^{(4+9)} = 5^{13}$
  - $p^3 \cdot p = p^3 \cdot p^1 = p^{(3+1)} = p^4$
  - $129^3 \times 129^0 = 129^{(3+0)} = 129^3$

  You can't multiply numbers with *unlike* bases. (Actually you can, by making the exponents the same, but that's not something you do on the EST.)
  - $x^2 \cdot y^3$ stays $x^2 \cdot y^3$
  - $5^2 \times 7^3$ stays $5^2 \times 7^3$ (you actually have to work it out)

- **To divide like bases, subtract the exponents.** You can divide two bases that are the same by subtracting the exponents.
  - $x^5 \div x^2 = x^{(5-2)} = x^3$
  - $5^9 \div 5^3 = 5^{(9-3)} = 5^6$
  - $x^3 \div x^7 = x^{(3-7)} = x^{-4} = \frac{1}{x^4}$
  - $129^2 \div 129^0 = 129^{(2-0)} = 129^2$

  (That last one should make sense if you think about it. Any base to the zero power is 1. Any number divided by 1 is itself.)

WARNING!

Did you look at the second example, $5^9 \div 5^3$, and think that it was 5^3? Falling into the trap of dividing instead of subtracting the exponents is easy, especially when you see numbers that just beg to be divided, such as 9 and 3. Keep your guard up.

- **Multiply exponents that appear inside and outside of parentheses, like this:**
  - $\left(x^2\right)^3 = x^{(2\cdot3)} = x^6$
  - $\left(5^4\right)^3 = 5^{(4\times3)} = 5^{12}$
- **You can add and subtract bases with exponents if the bases and exponents are the same.** Remember, the *base* is the number or letter tied to the exponent.
  - $\boldsymbol{x^3}$**:** The base is *x*, and the exponent is 3.
  - $\boldsymbol{x^3 + x^3 = 2x^3}$**:** This works the same way as $x + x = 2x$.
  - $\boldsymbol{37x^3 + 10x^3 = 47x^3}$**:** Because the bases are the same and the exponents are the same, just add the numbers (also known as numerical coefficients) to count the x^3: $37 + 10 = 47$.
  - $\boldsymbol{15y^2 - 10y^2 = 5y^2}$**:** Just subtract the numbers to count the y^2: $15 - 10 = 5$.

WARNING!

You can't count bases with different exponents or different bases. In other words, $13x^3 - 9x^2$ stays $13x^3 - 9x^2$, and $2x^2 + 3y^2$ stays $2x^2 + 3y^2$. The bases and exponents must be the same for you to combine them.

Putting It Together and Taking It Apart: FOIL and Factoring

One of the most common tasks that you probably remember from algebra class is the multiplication of expressions. These expressions come in several varieties:

- **One term times one term:** To multiply two terms, multiply their coefficients and *add* the powers of any common variables being multiplied; for example, $\left(3a^3\right)\left(-2a\right) = \left(3\times-2\right)\left(a^{3+1}\right) = -6a^4$. Check out the earlier section "Powering Up: Exponents" for more details about exponents.
- **One term times two (or more) terms:** Use the familiar distributive law: Multiply the single term by each of the terms in parentheses. Be sure to take your time and work out each product individually before combining them for the final answer.

  To simplify $3b^3\left(2b^2 - 5\right)$, write each multiplication task separately:
  - $\left(3b^3\right)\left(2b^2\right) = 6b^5$
  - $\left(3b^3\right)\left(-5\right) = -15b^3$

  And your answer is $6b^5 - 15b^3$.
- **Two terms times two terms:** Now, you FOIL: Multiply in the order *First, Outer, Inner, Last*.

  To work out $\left(x - 3\right)\left(2x + 5\right)$:

  1. Multiply the ***First*** terms: $\left(x\right)\left(2x\right) = 2x^2$.
  2. Multiply the ***Outer*** terms: $\left(x\right)\left(5\right) = 5x$.
  3. Multiply the ***Inner*** terms: $\left(-3\right)\left(2x\right) = -6x$.
  4. Multiply the ***Last*** terms: $-3\times5 = -15$.
  5. Combine like terms: $5x - 6x = -x$.

  And your solution is $2x^2 - x - 15$.

TIP

Memorize the following three special cases of FOIL. Don't bother to work them out every time you see them. If you know them by heart, you can save valuable EST minutes on test day.

- $(a+b)(a-b)=a^2-b^2$. You can use this shortcut only when the two terms are exactly the same *except for their signs*. For example, $(x+5)(x-5)=x^2-5^2=x^2-25$.
- $(a+b)^2=(a+b)(a+b)=a^2+2ab+b^2$.
- $(a-b)^2=(a-b)(a-b)=a^2-2ab+b^2$. Check out the following example to see the rule in action.

EXAMPLE

The expression $(x-3)^2$ is equivalent to

(A) x^2-9

(B) x^2+9

(C) x^2+6x-9

(D) x^2-6x+9

The correct answer is Choice (D). Choices (A) and (B) are wrong, because you don't just distribute the exponent. If you FOIL it out, you get $(x-3)(x-3)=x^2-3x-3x+9=x^2-6x+9$. Or you could just use the formula: $(x-3)^2=x^2+2(-3)x+(-3)^2=x^2-6x+9$.

Now that you know how to do algebra forward, are you ready to do it backward? You need to be able to factor down a quadratic equation, taking it from its final form back to its original form of two sets of parentheses.

For example, given $x^2+13x+42=0$, solve for x.

Take this problem one step at a time:

1. **Set up your answer by drawing two sets of parentheses.**

   $(\quad)(\quad)=0.$

2. **To get x^2, the *first* terms have to be x and x. Fill those in.**

   $(x\quad)(x\quad)=0.$

3. **Look now at the *last* term in the problem.**

   You need two numbers that multiply together to be +42. Well, there are several possibilities such as 42×1, 21×2, or 6×7. You can even have two negative numbers: -42×-1, -21×-2, or -6×-7. You aren't sure which one to choose, so go on to the next step.

4. **Look at the *middle* term in the problem.**

   You have to add two values to get +13. What's the first thing that springs to mind? Probably $6+7$. Hey, that's one of the possibilities in the preceding step! Plug it in and try it.

   $$(x+6)(x+7)=x^2+7x+6x+42=x^2+13x+42$$

Great, but you're not done, yet. The whole equation equals 0, so you have $(x+6)(x+7)=0$. Because any number times 0 equals 0, either $(x+6)=0$ or $(x+7)=0$. Therefore, x can equal either -6 or -7, not both and *not* +6 or +7. These possible values of x are known as the *roots* of the equation.

You may also have to factor an expression like $y^2 - 49 = 0$. This sort of problem probably looks familiar to you if you remember the formula (discussed earlier in this section): $(a+b)(a-b) = a^2 - b^2$. The expression $y^2 - 49 = 0$ is known as a *difference of squares* because it equals $y^2 - 7^2 = 0$. Any difference of squares can be factored like this: $y^2 - 49 = (y+7)(y-7) = 0$, and just as with $y^2 = 49$, y equals either 7 or –7. This is why $y^2 = 49$ is known as a *square root.* EST factoring is usually simple.

Absolute value

Absolute value presents you with a number, letter, or expression inside of two lines, as explained in Chapter 10. Here is what you do when one pops up in an equation. Check out this problem:

EXAMPLE

In the equation $|x-4| = 3$, x could equal

(A) 7 only

(B) 1 only

(C) 7 or 1

(D) 7 or –1

The correct answer is Choice (C). Because an absolute value symbol turns everything into a positive number, the expression inside the absolute value could equal either 3 or –3. This is the key to solving an equation with an absolute value. If $|something| = n$, then either $something = n$ or $something = -n$. You must solve each of these equations separately to get two answers. But there's a catch: You also must check each answer in the original equation. Only solutions that make the original equation true count in your final answer.

Now back to the preceding example:

$$
\begin{aligned}
&|x-4| = 3 \\
&x-4 = 3 \quad \text{or} \quad x-4 = -3 \\
&x = 7 \qquad\qquad x = 1
\end{aligned}
$$

Check your work:

$$\begin{aligned} |(7)-4| &= 3 \\ |3| &= 3 \\ 3 &= 3 \end{aligned} \quad \text{and} \quad \begin{aligned} |(1)-4| &= 3 \\ |-3| &= 3 \\ 3 &= 3 \end{aligned}$$

Because both checks work, your answer is Choice (C): 7 or 1.

You could, of course, just plug in the choices to solve the problem. Remember those pesky grid-ins, though! Because not every problem is multiple-choice, take the time to figure out how to solve each type of equation from scratch.

REMEMBER

The value of x can't be both 7 *and* 1. x has one value, and that's why the problem says, "x *could* equal."

Radical equations

EXAMPLE

Radical equations are equations that contain square roots. Check out this example:

Find the solution to the equation $3\sqrt{x}+5=17$.

Because this question isn't multiple-choice (yep, it's a grid-in), you have to solve this problem the long way. In a normal linear equation, you start by isolating x; here, you must first isolate $\sqrt{x}$:

$$\begin{aligned} 3\sqrt{x}+5 &= 17 \\ 3\sqrt{x} &= 12 \\ \sqrt{x} &= 4 \end{aligned}$$

Now don't make the mistake of thinking that x should be 2; $\sqrt{2}$ doesn't equal 4. Instead, square both sides, and $x=16$.

Rational equations

Rational equations have fractions in them. Sometimes, when the denominators of the fractions contain only numbers, removing the fractions and dealing with a simpler problem is easier. To solve $\frac{x}{3}+1=\frac{x}{2}-3$, you multiply every term by 6, because that's the smallest number that eliminates both the 2 and the 3 (officially, 6 is the *least common denominator,* or LCD, of 2 and 3). Assuming you cancel correctly, your new equation is $2x+6=3x-18$, and x equals 24. (We're letting you do the steps by yourself; practice makes perfect scores.)

When the denominator contains variables, your best bet is to combine terms with like denominators and then cross-multiply:

$$\begin{aligned} \frac{12}{x}+\frac{15}{x-1} &= \frac{25}{x-1} \\ -\frac{15}{x-1} &\quad -\frac{15}{x-1} \\ \frac{12}{x} &= \frac{10}{x-1} \end{aligned}$$

Cross-multiplying gives you $12(x-1)=10x$. Then $12x-12=10x$, and $x=6$. If you plug back into the original equation, you get

$$\frac{12}{6}+\frac{15}{5}=\frac{25}{5}, \text{ or } 2+3=5$$

So your answer checks out.

Direct and inverse proportion

In a direct or inverse proportion problem, instead of being given an equation to work with, the EST-makers tell you that two quantities "are directly proportional" or "inversely proportional." These expressions represent two specific types of equations that you're already familiar with under other names.

A *direct proportion* problem is just another type of ratio problem. If *a* and *b* are directly proportional, then the ratio $\frac{a}{b}$ is always equal to a certain constant. Thus, you can solve a direct proportion problem by setting up the ratio $\frac{a_1}{b_1}=\frac{a_2}{b_2}$, cross-multiplying, and solving as usual, as you do in this example:

x and y are directly proportional. If $x=10$ when $y=6$, what does x equal when $y=21$?

Let $x_1=10$ and $y_1=6$. Then x_2 is what you're looking for, and $y_2=21$. Set up the ratio $\frac{x_1}{y_1}=\frac{x_2}{y_2}$ or $\frac{10}{6}=\frac{x_2}{21}$, and cross-multiply to get $6x_2=210$, so $x_2=35$.

Notice that as one variable increases, the other variable also increases. This feature of direct proportion problems helps you do a common-sense check of your answer.

When two variables vary *inversely,* their product is always equal to the same number. For example, suppose that p and q vary inversely, and $p=3$ when $q=12$. Because $pq=(3)(12)=36$ in this case, pq must equal 36 for all values of p and q. When $p=2$, $q=18$ (and vice versa); when $p=6$, $q=6$ as well. This strategy works for all inverse proportion problems.

Barely Functioning

By the time you get to functions on the EST, you may think that you can't . . . function, that is. But think of a function as a simple computer program: You give it an input, and it produces an output. For example, $f(x)=2x-1$ is a function. You put in a number for x (5, for example) and get out $2x-1$ or $2(5)-1$ as a result (9, in this case). The input and output can then be written as an ordered pair: $(5, 9)$ is a member of this function, as are the pairs $(1, 1)$ and $(0.5, 0)$, along with infinite others.

Consider the example $f(x)=x-4$: $f(9)=5$, $f(4)=0$, and $f(1)=-3$. In other words, when you put in 9 for *x,* you get out 5; when you put in 4, you get out 0; and when you put in 1, you get out –3. Notice that $f(x)$ and y are the same thing.

Notice that the number replaces x when evaluating the function. When x is 9, for example, just replace x with 9 throughout the equation. So $f(x) = x - 4$ becomes $f(9) = 9 - 4$, and you know that $f(9) = 5$.

On the EST, you may be given a function and be asked what *can't* go into it. Keep in mind two things that you can't do in a function:

- Divide by zero.
- Take the square root of a negative number.

So if you see a function like $y = \sqrt{x-4}$, numbers like 4, 5, 6, and so on are okay, but numbers less than 4 aren't, because then you'd have a negative number under the radical. For a function like $f(x) = \frac{x+2}{(x-4)(x+1)}$, x could be any number except 4 or –1, because plugging in those numbers gives you a denominator of zero, which doesn't work. Notice, by the way, that plugging in –2 is fine, because it's okay for the *numerator* of a fraction to be zero.

Functioning at a Higher Level

At this point, you may be wondering, "Why are functions such a big deal?" After all, functions seem like a pretty abstract concept. However, it turns out that a huge number of real-life situations can be modeled using functions. To do well on the Math section of the EST, you definitely want to be good friends with two of the most common types of functions: linear and quadratic.

Figuring out linear functions

You've probably worked a lot with linear functions, especially in graphing. All linear functions have the form $y = mx + b$ or $f(x) = mx + b$. In graphing terms, m represents the slope of the line being drawn, while b represents its y-intercept. Take a look at this example:

If $f(x)$ is a linear function with a slope of 2, passing through the point $(-2, -3)$, $f(x)$ must also pass through the point

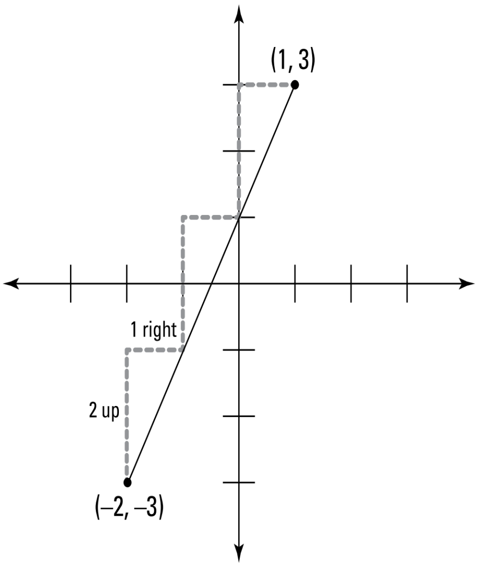
(A) $(1, 2)$

(B) $(1, 3)$

(C) $(2, 2)$

(D) $(2, 3)$

The correct answer is Choice (B). The best way to solve this problem is to draw a graph. To get it right, you have to remember the meaning of slope: Slope $= \frac{\text{rise}}{\text{run}}$. A slope of $\frac{2}{5}$, for example, tells you to move 2 spaces up (the rise) and 5 spaces to the right (the run). You don't have to be a great artist, just count the spaces. The function in this problem has a slope of 2, which is the same as $\frac{2}{1}$. Starting at $(-2,-3)$ and following these directions yields this graph:

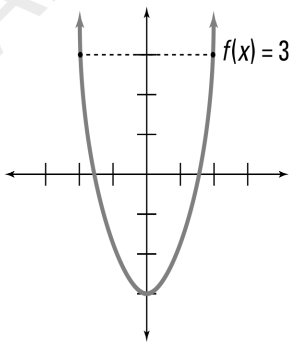


Instead of simply giving you numbers, the EST-writers may present a real-world situation and ask you to model it with a function. For example, if an express mail package costs $1.50 plus $0.40 per pound, you can write $c = 1.50 + 0.40p$, where c is the cost and p represents the number of pounds.

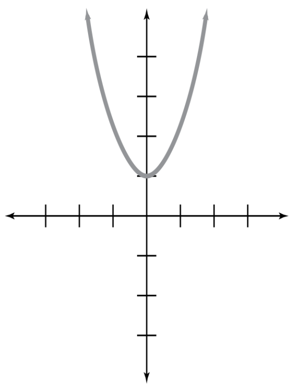
Thinking through quadratic functions

Quadratic functions, on the other hand, have the form $y = ax^2 + bx + c$ or $f(x) = ax^2 + bx + c$. Graphically, they're represented by a *parabola,* a shape that resembles the basic roller-coaster hump. You certainly won't be asked to graph any of these, but you may be asked some graph-based questions. Keep these points in mind as you work with quadratic functions:

- The roots or solutions of a function are the *x*-values that make $f(x) = 0$. On a graph, these roots are the points where the graph crosses the horizontal *x*-axis.
- The number of solutions of $f(x) = a$ is the number of points where the graph has a height of *a.* On the following graph, $f(x) = 3$ twice, at the marked points.

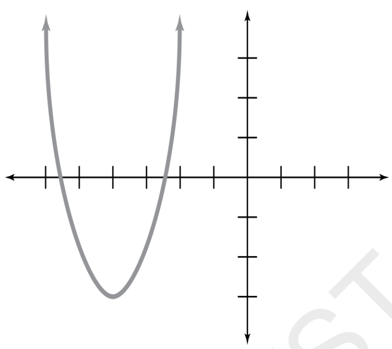


- If a number is added to a function, the graph is moved up that many units. If the function above were changed from $f(x)$ to $f(x)+4$, the new graph would be:



Note that subtracting a number from a function moves the graph down.

- If a number is added to x in a function, the graph is moved that many units *to the left.* This rule is tricky because you may have guessed the graph moved the other way. If the original function were changed to $f(x+4)$, it would look like the following graph. Notice that this rule is used when you're adding to x, not to the whole function. As you may guess, if you were to graph $f(x-4)$, you'd move four units to the right.



Some graphing problems don't involve equations; instead, you may be given a pair of points and be asked about the line connecting them. In these types of problems, three formulas are crucial:

- The slope of the line connecting the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.
- The distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- The midpoint of the line connecting the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$.

You probably learned these formulas some time ago. They're not exciting, but they are useful. Try using them on the points $(-1, 2)$ and $(5, -6)$:

$$\text{Slope} = \frac{(-6)-(2)}{(5)-(-1)} = \frac{-8}{6} = -\frac{4}{3}$$

$$\text{Distance} = \sqrt{\left[(5)-(-1)\right]^2 + \left[(-6)-(2)\right]^2} = \sqrt{(6)^2 + (-8)^2} = \sqrt{36+64} = \sqrt{100} = 10$$

$$\text{Midpoint} = \left(\frac{(5)+(-1)}{2}, \frac{(-6)+(2)}{2}\right) = \left(\frac{4}{2}, \frac{-4}{2}\right) = (2, -2)$$

Thanks, you've been *grat*

Did your favorite grandparent send you a reward to ensure success ? If so, you probably expressed your ***gratitude*** (thanks) because you were ***grateful*** (thankful) and not an ***ingrate*** (someone who thinks the whole world owes him or her a living and, therefore, never appreciates anything). Waiters and bartenders, on the other hand, always appreciate ***gratuities*** (tips). Other *grat* words include ***gratis*** (free — you'll be thankful for the gift, right?) and ***gratuitous*** (given freely but not necessary, like your mom's criticism of your latest dating partner). You may find the *grat* words ***gratifying*** (filling one's needs or desires) when they

Decoding symbolism

If $a \wedge b = 2a - b$, which of the following is equal to $3 \wedge 4$?

(A) 1

(B) 2

(C) 3

(D) 4

Choice (B) is the answer. Just as in a normal function problem, where you plug in a number for x, here you plug in 3 and 4 for a and b. It's actually very simple. To solve $3 \wedge 4$, plug in those numbers, in order:

$$\begin{aligned} a \wedge b &= 2a - b \\ 3 \wedge 4 &= 2(3) - (4) \\ &= 6 - 4 \\ &= 2 \end{aligned}$$

And the answer is Choice (B).

Chapter 13

Practicing Problems in Algebra and Functions

In This Chapter

- Practicing algebra and functions with some guided problems
- Troubleshooting your problem areas with some practice questions

In this chapter, you hone your skills for EST algebra and function problems. Try ten, see how you do, and then try ten more if you're a glutton for punishment (or algebra, which in some people's minds is the same thing).

Be sure to check all your answers. Don't forget the explanations, which may help you understand what went wrong or a faster, better way to work a question you got right.

In the first practice set, each answer immediately follows the question. Don't cheat. Cover the answer with a piece of paper until you're ready to read the explanation. The second set is set up like the real test: You go through all the questions and then check your answers in the next section.

For more information on any of the topics in these questions, check out Chapter 12.

Set One: Getting Started with Some Guided Questions

Note: Question 3 is a grid-in, so you don't get any answers to choose from. See Chapter 9 for more on answering grid-ins correctly.

1. If k is a positive integer, which of the following is a possible value for k^2?

(A) –1

(B) 0

(C) 6

(D) 9

Choice (A) is impossible because any number, when squared, is positive. Choice (B) is 0 squared, but the problem said that the original number had to be positive. Choice (C) isn't a perfect square; no number multiplied by itself gives you 6 as an answer. That leaves you with Choice (D), which is 3^2.

2. If $y = \frac{x+5}{2}$, then increasing the value of y by 2 will increase x by

(A) 1

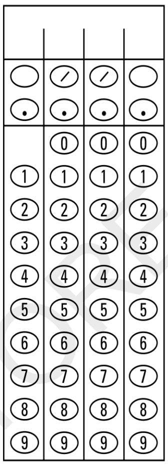
(B) 2

(C) 3

(D) 4

This problem is good for picking your own numbers. For example, say that y was originally 10; then you would have $10 = \frac{x+5}{2}$. Multiplying both sides by 2 gives you $20 = x + 5$, or $x = 15$. Now the problem tells you to increase y by 2, making it 12. If you do the math, you find that x is now 19, so it increased by 4. This result makes sense because the equation tells you that you need to divide $x + 5$ by 2 to get y; y increases half as quickly as x. Thus, your answer is Choice (D).

3. In his will, a man left his land to his three children: $\frac{2}{3}$ of the estate to his oldest child, $\frac{1}{4}$ to his middle child, and 15 acres to his youngest. How many acres were in the original estate?



A word problem? With no multiple-choice answers? And fractions? Okay, deep breaths. Now get to work. In any word problem, list the various things you need to know to solve the problem. Four important things pop up in this one: the original estate and the amount left to the three children. Because the original estate is what you're looking for, call it x. Remembering that the word *of* usually indicates multiplication, you can then make a list:

x = original estate

$\frac{2}{3}x$ = oldest child

$\frac{1}{4}x$ = middle child

15 = youngest child

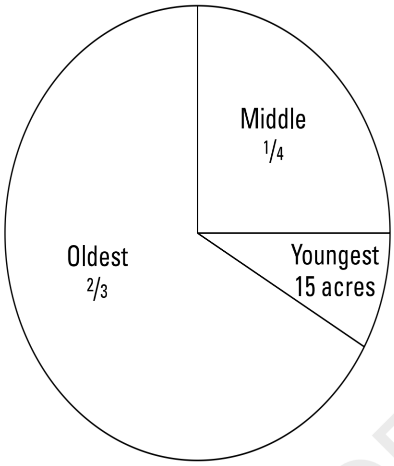
Because the three children's shares made up the whole estate, you can write $\frac{2}{3}x + \frac{1}{4}x + 15 = x$.

With a common denominator of 12:

$$\frac{8}{12}x+\frac{3}{12}x+15=x$$
$$\frac{11}{12}x+15=x$$
$$15=x-\frac{11}{12}x$$
$$15=\frac{1}{12}x$$

Finally, multiplying both sides by 12, you get $x=180$.

That's a lot of work for one problem. If you're a visual person, you may prefer to solve it with a graph:



4. If x and y are both integers, $x>3$, and $y<2$, then $x-y$ could be

(A) 3

(B) 2

(C) 1

(D) −1

After the last problem, this one's a breeze. x is an integer greater than 3, so it must be at least 4. y is an integer less than 2, so it must be at most 1. To find $x-y$, just plug in those numbers: $4-1=3$, so Choice (A) is correct. Notice that making x bigger or y smaller would make $x-y$ greater than 3, so all the other choices are impossible.

5. If n and p are directly proportional, and $n=12$ when $p=9$, which of the following pairs is a possible set of values for n and p?

(A) $n=9$, $p=12$

(B) $n=18$, $p=15$

(C) $n=18$, $p=6$

(D) $n=20$, $p=15$

If n and p are directly proportional, the ratio of n/p must remain the same. That ratio is 12/9, which reduces to 4/3, so find an answer choice with that same ratio. Only Choice (D), at 20/15, reduces to 4/3.

6. If $a^2 - b^2 = 40$ and $a - b = 10$, then $a + b =$

(A) 4

(B) 10

(C) 14

(D) 30

When you see a quadratic expression in a problem, see whether it can be factored. $a^2 - b^2$ should look familiar to you. (If not, turn to Chapter 12.) Keep in mind that $a^2 - b^2$ factors out to $(a - b)(a + b)$. Because $a^2 - b^2 = 40$ and $a - b = 10$, $(10)(a + b) = 40$ so $a + b = 4$. Notice that you didn't even have to figure out what a and b are to solve the problem, which happens a lot on the EST. The correct answer is Choice (A).

7. A copying service charges $2.50 to copy up to 20 pages plus 5 cents per page over 20. Which formula represents the cost, in dollars, of copying c pages, where c is greater than 20?

(A) $2.50 + 5c$

(B) $2.50 + 0.05c$

(C) $(2.50)(20) + 0.05c$

(D) $2.50 + 0.05(c - 20)$

As is often the case, one good approach is to pick a number for c and then see which formula works. Try $c = 28$. (Remember, c has to be greater than 20). The cost for 28 pages would be $2.50 for the first 20, plus $0.05 times the 8 remaining pages, which is $0.40, for a total of $2.90. Plugging 28 into the various formulas yields $142.50 for Choice (A), $3.90 for Choice (B), $51.40 for Choice (C), and $2.90 for Choice (D). Choice (D) is correct.

8. Let *a be defined as one more than a if a is odd and as one less than a if a is even. Which of the following would result in the lowest value of *a?

(A) –2

(B) –1

(C) 0

(D) 1

Test out each answer choice. Choice (A), –2, is even, so one less than –2 is –3. The other choices give you 0, –1, and 2, in that order. Thus, Choice (A) is correct.

9. If $(2g - 3h)^3 = 27$, then $(2g - 3h)^{-2} =$

(A) $\frac{1}{9}$

(B) $\frac{1}{6}$

(C) 6

(D) 9

First, give up on trying to figure out the values of g and h. The key is the expression $(2g - 3h)$, which shows up in both parts of the problem. Replace it with something simpler, like q. (Why use x all the time?) So you know that $q^3 = 27$. A little trial and error (or your calculator) reveals that $q = 3$. Now you need to find $q^{-2} = (3)^{-2} = \frac{1}{(3)^2} = \frac{1}{9}$. Choice (A) is correct.

10. A party supplier charges a flat rate plus a certain amount per person. If supplies for 12 people cost \$140 and supplies for 20 people cost \$180, then supplies for 40 people would cost

(A) \$220

(B) \$280

(C) \$300

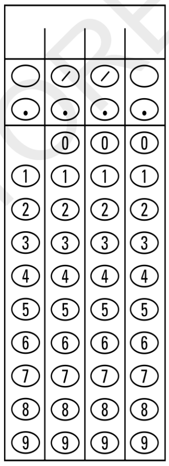
(D) \$360

First, find the cost per person. If 12 people cost \$140 and 20 people cost \$180, then 8 people cost \$40 (because $20-12=8$ and $\$180-\$140=\$40$). Divide \$40 by 8 for a per-person cost of \$5. Now take the 20-person cost and add the cost of 20 more people. You know that 20 people cost \$180 and that 20 additional people cost \$100 (because $20\times\$5=\$100$), so add the two together: $\$180+\$100=\$280$, and the answer is Choice (B).

Set Two: Practicing Some Questions on Your Own

Note: Question 1 is a grid-in. Turn to Chapter 9 for help on answering that type of question.

1. If $(x+2)^2+(x-1)^2=ax^2+bx+c$, find the value of b.



2. A gas's pressure and volume are inversely proportional. If a certain gas has a pressure of 120 kilopascals (kPa) when its volume is 250 cubic centimeters (cc), what is its pressure when its volume is 200 cc?

(A) 170 kPa

(B) 150 kPa

(C) 100 kPa

(D) 96 kPa

3. Given the function $f(x)=\dfrac{5x}{x^2-4x+4}$, which is not a possible value of x?

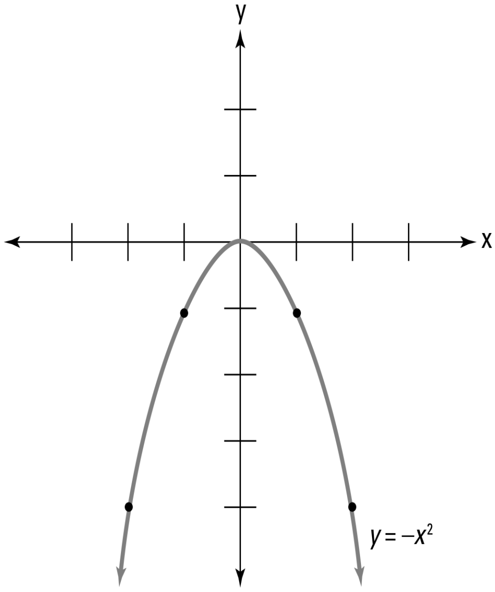
(A) 4

(B) 2

(C) 0

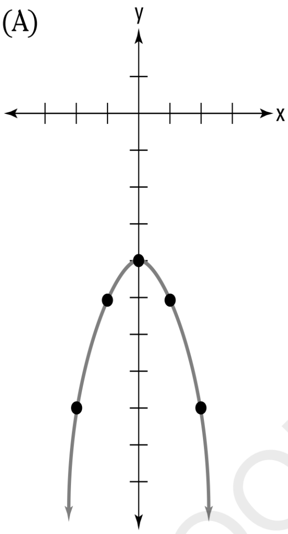
(D) –2

4. Below is the graph of the equation $y = -x^2$.

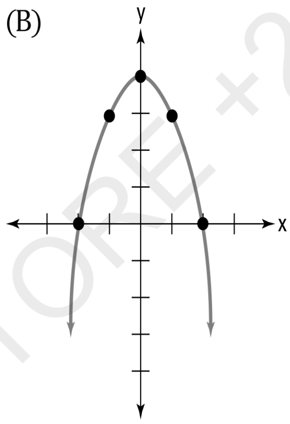


Which of the following choices represents the graph of $y = -x^2 + 4$?

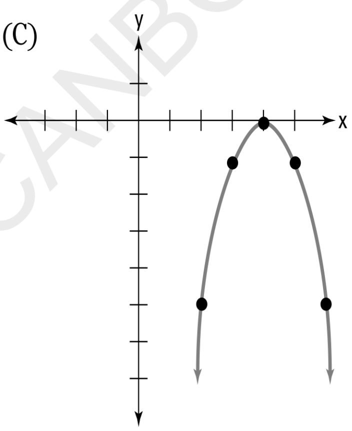
(A)



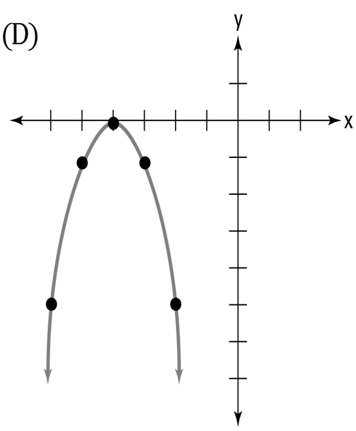
(B)



(C)



(D)



5. The solution set to the equation $|x+3|=5$ is

(A) $\{2\}$

(B) $\{2,-8\}$

(C) $\{-8\}$

(D) $\{-2,-8\}$

6. If $f(x)$ is a linear function passing through the points $(2,5)$ and $(6,3)$, then the y-intercept of $f(x)$ is

(A) 7

(B) 6

(C) 5

(D) 3

7. If $x \otimes y$ is defined as $x^2 - y$ for all integers x and y, which of the following is always true?

(A) $1 \otimes y = y$

(B) $x \otimes 2$ is positive

(C) $x \otimes 3 = x \otimes -3$

(D) $4 \otimes y = (-4) \otimes y$

8. If $k^{1/2} - 3 = 5$, then $k =$

(A) 64

(B) 16

(C) 8

(D) 4

9. The population of a certain city can be modeled by the function $p(y) = 20{,}000\left(2^{y/20}\right)$, where $p(y)$ represents the population and y measures years since 1975. If the city had a population of 40,000 in 1995, then its population in 2015 is

(A) 40,000

(B) 60,000

(C) 80,000

(D) 100,000

10. In the equation $5 - \frac{2x+2}{x+1} = \frac{9}{x+1}$, x is equal to

(A) 0

(B) 1

(C) 2

(D) 3

Answers to Set Two

1. **2.** This problem is an easy one to mess up, but not if you use the formulas from Chapter 12: $(a+b)^2 = a^2 + 2ab + b^2$. It's also fine to just do FOIL, rewriting the problem as $(x+2)(x+2)+(x-1)(x-1)$. Either way, the problem becomes $x^2+4x+4+x^2-2x+1$, which equals $2x^2+2x+5$, so $b=2$.

2. **B.** When two quantities are *inversely proportional,* their product is always the same number. Usually, finding that number is the key to getting the right answer. You're told that a pressure of 120 corresponds to a volume of 250, and $120 \times 250 = 30,000$. Thus, your missing pressure (call it p) times 200 must equal 30,000. Solving $200p = 30,000$ gives you $p = 150$. Common-sense double-check: If quantities vary inversely, one goes up when the other goes down. Notice that the volume went down from 250 cc to 200 cc and that the pressure went up from 120 kPa to 150 kPa. Choice (B) is correct.

3. **B.** Don't bother trying the answer choices. Use your head. You know that dividing by 0 is against the rules. The denominator can be factored to $(x-2)(x-2)$, which means that 2 is the only number that makes the denominator 0, so it's the only number that can't be a value of x. Choice (B) is correct.

4. **B.** Adding 4 to a function raises its graph by four units, so Choice (B) is correct. If you're not sure, eliminate wrong answers by finding the x, y coordinates of one point on each graph, and plugging them into the equation $y = -x^2 + 4$. The equation only works with the correct answer.

5. **B.** You could just plug in all the choices, but, for practice, go through the official steps:

   Create two equations:

   $x+3=5$ and $x+3=-5$

   Solve them separately:

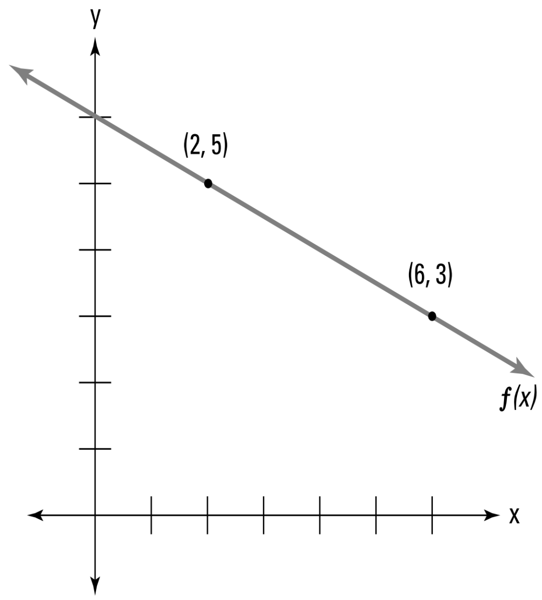
   $$x+3=5 \qquad x+3=-5$$
   $$x=2 \quad \text{and} \quad x=-8$$

   Check your answers:

   $$|x+3|=5 \qquad |x+3|=5$$
   $$|(2)+3|=5 \quad \text{and} \quad |(-8)+3|=5$$
   $$|5|=5 \qquad |-5|=5$$
   $$5=5 \qquad 5=5$$

   So the answers that work are 2 and –8. Choice (B) is correct.

6. **B.** A sketch can help you see what's going on here, so draw something like this:



The graph suggests that the answer is either Choice (A) or Choice (B). To be sure, use the formula for linear equations, $y = mx + b$. First, find the slope of the line:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{6-2} = \frac{-2}{4} = -\frac{1}{2}$$

So the equation is $y = -\frac{1}{2}x + b$. Plug in $(2, 5)$ to find b:

$$\begin{aligned} (5) &= -\tfrac{1}{2}(2) + b \\ 5 &= -1 + b \\ 6 &= b \end{aligned}$$

The y-intercept is 6, and the answer is Choice (B).

7. **D.** No cool solving methods here. You just have to check all the possibilities.

Choice (A): $1 \otimes y = 1^2 - y = 1 - y$, which equals y only if y is $1/2$. Moving on to Choice (B): $x \otimes 2 = x^2 - 2$. This is positive if x is 2 or more, but if x is 1 or 0, then it's negative. Now for Choice (C): $x \otimes 3 = x^2 - 3$ and $x \otimes -3 = x^2 - (-3) = x^2 + 3$. They're not equal. Which leaves Choice (D): $4 \otimes y = 4^2 - y = 16 - y$ and $(-4) \otimes y = (-4)^2 - y = 16 - y$. At last.

8. **A.** Start by isolating the $k^{1/2}$:

$$\begin{aligned} k^{1/2} - 3 &= 5 \\ k^{1/2} &= 8 \end{aligned}$$

Because $k^{1/2}$ means $\sqrt{k}$, square both sides:

$$k = 8^2 = 64$$

So Choice (A) is correct.

9. **E.** Put away your calculator. EST functions are simple, even if they look mad. The expression $2^{y/20}$ isn't so bad when y is a multiple of 20, and in this case it's 40 (the number of years from 1975 to 2015). This means that $2^{y/20}$ is really just $2^{40/20}$, or 2^2, which of course equals 4. And $p(40)$ is simply $20{,}000(4)$, which equals 80,000. It's really no fun when it's that simple.

10. **D.** The best way to do this problem is to start by noticing that the two fractions have common denominators. Therefore, they can be combined if you get them on the same side:

$$5 = \frac{9}{x+1} + \frac{2x+2}{x+1}$$
$$5 = \frac{9+2x+2}{x+1}$$
$$5 = \frac{11+2x}{x+1}$$

Now, multiply $x+1$ on both sides and solve for x:

$$(x+1)5 = 11+2x$$
$$5x+5 = 11+2x$$
$$3x = 6$$
$$x = 2$$

Choice (C) is correct.

Chapter 14

Checking More Figures Than an IRS Agent: Geometry and Trigonometry

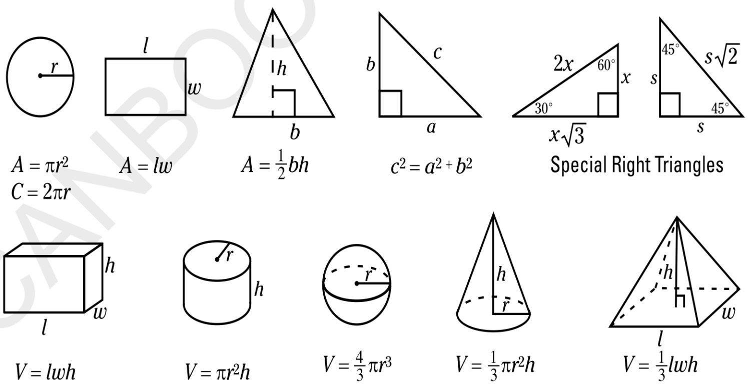
In This Chapter

- Getting acquainted with angles
- Solving problems containing triangles, quadrilaterals, and polygons
- Zeroing in on circles: Calculating circumference, area, and more
- Thinking in 3-D: Looking at volume and surface area
- Working with trigonometry

Think of geometry as a mini art lesson. The key to doing well on geometry problems is this: Draw a picture for every geometry problem that you face. Include every measurement that you know on the diagram, and get a sense of what the problem is describing.

After you get a clear illustration on paper, you're ready to get through the problem. Even if a problem seems to be incredibly easy, draw a quick diagram anyway. It doesn't have to be perfect, but it will get you to the right answer.

Fair warning: This chapter has a lot of information to memorize, but don't panic. Much of what's covered in this chapter appears in the box of formulas printed at the beginning of each Math section on the EST. Here's what the box looks like:



There are 360 degrees of arc in a circle. The number of radians of arc in a circle is 2π.

There are 180 degrees in the sum of the interior angles of a triangle.

Knowing What Makes One Angle Different from Another

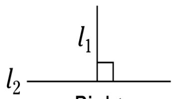
Angles are a big part of the EST geometry problems, so pay attention to this section. Finding an angle is usually a matter of simple addition or subtraction, provided you remember these key facts.

When it comes to angles, these three rules pretty much apply to the EST math questions:

- There are no negative angles.
- There are no zero angles.
- Fractional angles rarely appear on the test. For example, an angle is unlikely to measure $19\frac{7}{8}$ degrees or $23\frac{4}{5}$ degrees.

Now that you know what you won't find in the EST geometry questions, take a look at the important facts you need to remember for the problems you will find:

- **Angles that equal 90 degrees are called *right angles*.** They're formed by perpendicular lines and are indicated by a box in the corner of the two intersecting lines.

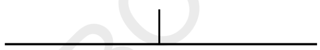


Right

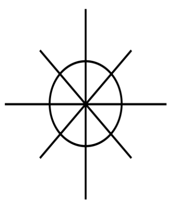
A common EST trap is to have two lines appear to be perpendicular. Don't assume you're looking at a right angle unless you see one of the following:

  - The words "This is a right angle" in the question
  - The perpendicular symbol ⊥, which indicates that the two lines form a 90-degree angle
  - The box in the corner of the two intersecting lines

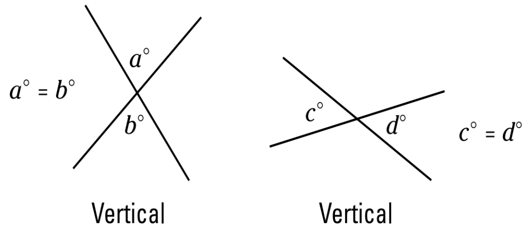
If you don't see one of these three notes, don't assume that the angle is 90 degrees.


Not necessarily a right angle

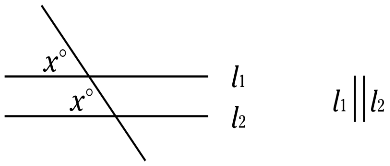
- **A circle measures 360 degrees.** Remember that 360 degrees always creates a complete circle.

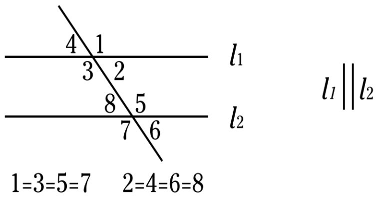

360 degrees

- **Angles that are opposite each other are *congruent* (they have equal measures) and are called *vertical angles*.** The name sticks even if the angles aren't positioned vertically. Just remember that vertical angles are across from each other, whether they're up and down or side by side.

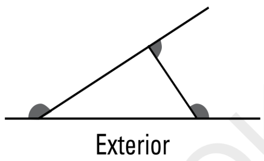


- **A line crossing two parallel lines creates two sets of four angles having the same measures.** This actually creates four sets of vertical angles, with the odd-numbered angles having one measure and the even-numbered angles having another.



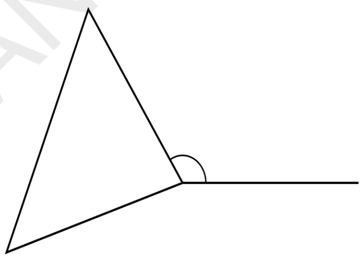


- **The *exterior angles* of any figure are supplementary to the interior angles and sum up to 360 degrees.** Exterior angles can be very confusing; keep in mind that they *always* sum up to 360 degrees, no matter what type of figure you have.



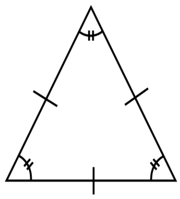
REMEMBER

An exterior angle is *supplementary* to an interior angle; in other words, the two angles must form a straight line with a side of the figure. The following isn't an exterior angle:



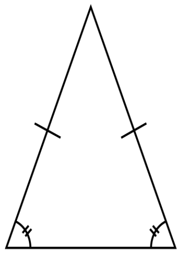
Figuring out what you need to know about triangles

- **A triangle with three equal sides and three equal angles is called *equilateral*.**



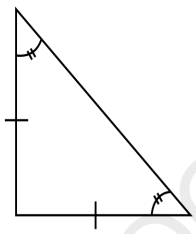
Equilateral

- **A triangle with two equal sides and two equal angles is called *isosceles*.**



Isosceles

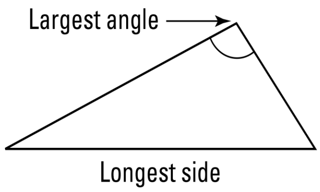
- **Angles opposite equal sides in an isosceles triangle are also equal.**



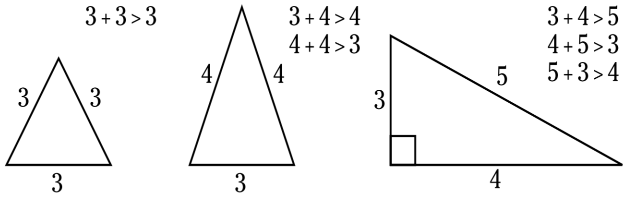
Isosceles

- **In any triangle, the largest angle is opposite the longest side.** Similarly, the smallest angle is opposite the shortest side, and the medium angle is opposite the medium-length side.

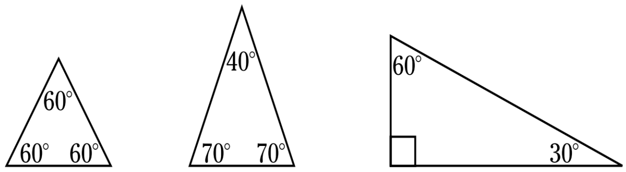
  Note: In a right triangle, this largest angle is the right angle because the other two angles total 90 degrees. The longest side is the *hypotenuse,* which is always opposite the right angle.



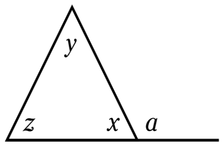
- **In any triangle, the sum of the lengths of two sides must be greater than the length of the third side.** This is written as $a+b>c$, where *a, b,* and *c* are the sides of the triangle.



- **In any type of triangle, the sum of the interior angles is 180 degrees.**



- **The measure of an exterior angle of a triangle is equal to the sum of the two opposite interior angles.**

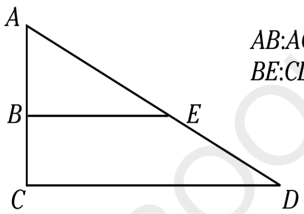


$a = y + z$

When you think about this rule logically, it makes sense. The sum of supplementary angles is 180 degrees, and the sum of the triangle angles is 180 degrees. In the preceding triangle, angle $x = 180 - (y + z)$ and angle $x = 180 - a$. Thus, $a = y + z$.

Identifying what makes two triangles (or other figures) similar

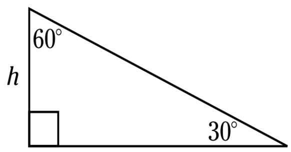
Two triangles are *similar* if they are exactly the same shape but different sizes. This also means that their sides are in proportion. For example, if the heights of two similar triangles are in a ratio of 2:3, the bases of those triangles are in a ratio of 2:3 as well.



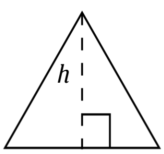
$AB:AC = 2:3$
$BE:CD = 2:3$

Calculating the area of triangles

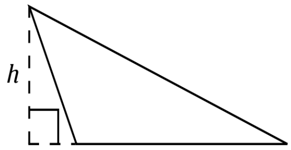
The formula for finding the area of a triangle is: $A = \frac{1}{2} base \times height$. The height (also known as the *altitude*) is always a line perpendicular from any vertex to the opposite base. It may be a side of the triangle, as in a right triangle:



The height may be inside the triangle, in which case it's often represented by a dashed line and a small 90-degree box:



The height may also be outside the triangle. This configuration is very confusing, but you may find it in trick questions. On the plus side, the problem will usually show you the altitude, as in this drawing:



Using the Pythagorean theorem

You've probably studied the *Pythagorean theorem* (PT) at some point in your math career. The theorem says that in any right triangle, you can find the lengths of the sides using the formula $a^2 + b^2 = c^2$, where a and b are the legs of the triangle and c is the hypotenuse. The hypotenuse is always opposite the 90-degree angle and is always the longest side of the triangle.

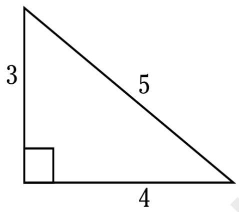
WARNING!

Keep in mind that the Pythagorean theorem works only on right triangles. If a triangle doesn't have a 90-degree angle, you can't use it.

Simplifying things with Pythagorean triples

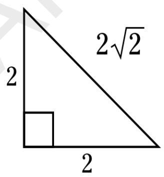
Going through the whole PT formula every time you want to find the length of a side in a right triangle can be a pain. To help you simplify your work, memorize the following three common PT ratios:

- **Ratio 3:4:5.** In this ratio, if one leg of the triangle is 3, the other leg is 4, and the hypotenuse is 5.

  Because this is a ratio, the sides can be in any multiple of these numbers, such as 6:8:10 (two times 3:4:5), 9:12:15 (three times 3:4:5), 27:36:45 (nine times 3:4:5), and so on.

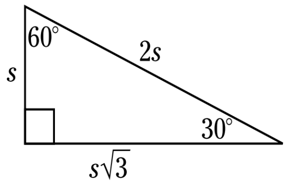
- **$s : s : s\sqrt{2}$, where s stands for the side of the figure.** Because two sides are congruent, this formula applies to an isosceles right triangle, also known as a 45-45-90 triangle. If one side is 2, then the other leg is also 2, and the hypotenuse is $2\sqrt{2}$.

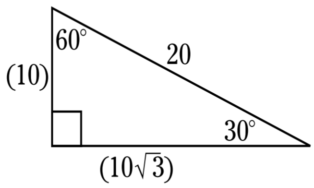
  TIP

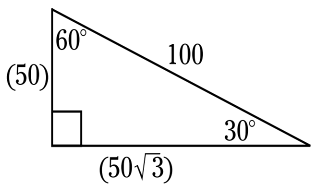
  This formula is great to know for squares. If a question tells you that the side of a square is 5 and wants to know the diagonal of the square, you know immediately that it is $5\sqrt{2}$. Why? A square's diagonal cuts the square into two isosceles right triangles (isosceles because all sides of the square are equal; right because all angles in a square are right angles). What is the diagonal of a square with sides of 64? $64\sqrt{2}$. How about a square with sides of 12,984? $12{,}984\sqrt{2}$.

- **$s : s\sqrt{3} : 2s$.** This ratio is a special formula for the sides of a 30-60-90 triangle.



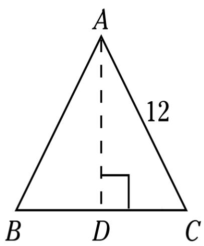
This type of triangle is a favorite of the test-makers. The important thing to keep in mind here is that the hypotenuse is twice the length of the smallest side, which is opposite the 30-degree angle. If you get a word problem saying, “Given a 30-60-90 triangle of hypotenuse 20, find the area” or “Given a 30-60-90 triangle of hypotenuse 100, find the perimeter,” you can do so because you can find the lengths of the other sides:





Two 30-60-90 triangles are formed whenever an equilateral triangle is cut in half. If an EST question mentions the altitude of an equilateral triangle, you almost always have to use a 30-60-90 triangle to solve it.

Time to stretch those mental triangular muscles. Try this sample problem:



In this equilateral triangle, the length of altitude *AD* is

(A) 6

(B) 9

(C) $6\sqrt{2}$

(D) $6\sqrt{3}$

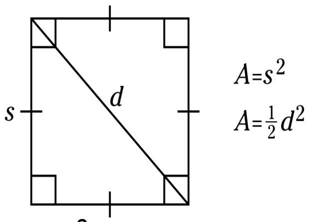
The answer is Choice (D). Look at the 30-60-90 triangle formed by *ABD.* The hypotenuse is 12, the original side of the equilateral triangle. The base is 6 because it’s half the hypotenuse. That makes the altitude $6\sqrt{3}$, according to the ratio.

Remember that the 45-45-90 and 30-60-90 triangle patterns are included in the formula box at the beginning of each Math section, in case you forget them. Don’t hesitate to refer to this formula box as you move through the Math sections.

Taking a quick look at quadrilaterals

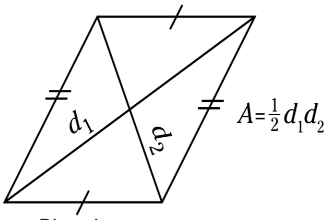
Think you know everything there is to know about squares and rectangles? Well, think again! We’re here to give you a few more oh-so-interesting rules to add to your geometry toolbox. These rules all have to do with four-sided figures, or *quadrilaterals:*

- **Any four-sided figure is called a *quadrilateral.*** The sum of the interior angles of any quadrilateral equals 360 degrees.
- **A *square* is a quadrilateral with four equal sides and four right angles.** The area of a square is $side^2$.



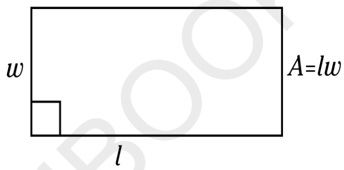
Square

- **A *rhombus* is a quadrilateral with four equal sides and four angles that aren't necessarily right angles.** A rhombus looks like a square that's slipping sideways (unless it is a square — every squares is a rhombus but not every rhombus is a square). The area of a rhombus is $\frac{1}{2}d_1d_2\left(\text{that is, } \frac{1}{2}diagonal_1 \times diagonal_2\right)$.



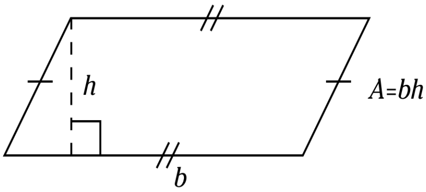
Rhombus

- **A *rectangle* is a quadrilateral with four equal angles, all of which are right angles.** The top and bottom sides are equal, and the right and left sides are equal. All angles in a rectangle are right angles. (The word *rectangle* means "right angle.") The area of a rectangle is $length \times width$ (which is the same as $base \times height$).



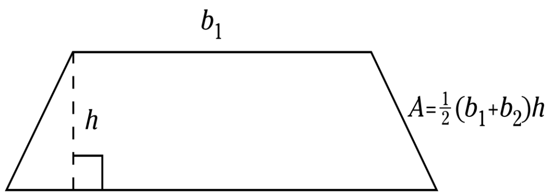
Rectangle

- **A *parallelogram* is a quadrilateral with two opposite and equal pairs of sides.** The top and bottom sides are equal, and the right and left sides are equal. Opposite angles are equal but not necessarily right. The area of a parallelogram is $base \times height$. (***Note:*** The height is a perpendicular line from the tallest point of the figure down to the base.)



Parallelogram

- **A *trapezoid* is a quadrilateral with two parallel sides and two nonparallel sides.** The area of a trapezoid is $\frac{1}{2}(base_1 + base_2) \times height$. The bases are the two parallel sides, and the height is the perpendicular distance between them.



Trapezoid

Considering some other polygons

Triangles and quadrilaterals are the most common polygons tested on the EST; however, they're certainly not the only ones out there. Table 14-1 notes a few other polygons you may see on the test.

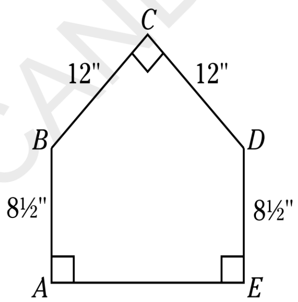
Table 14-1 Some Polygons

Number of Sides	*Name*
5	Pentagon
6	Hexagon (think of *x* in six and *x* in hex)
8	Octagon (like a stop sign)

A polygon with all equal sides and all equal angles is called *regular.* For example, an equilateral triangle is a regular triangle, and a square is a regular quadrilateral.

TIP

The EST-writers won't ask you to find the area of any of these polygons, but they may ask you to find the *perimeter,* which is just the sum of the lengths of all the sides. They may also ask you to find the exterior angle measure, which is always 360 degrees. If they ask you about other angles, divide the shape into triangles, as in the following figure. Then try your hand at the sample grid-in question that follows the illustration.



EXAMPLE

As the diagram shows, an official major league home plate has the shape of a pentagon. Given the measurements shown, the length of *AE,* to the nearest inch, must be

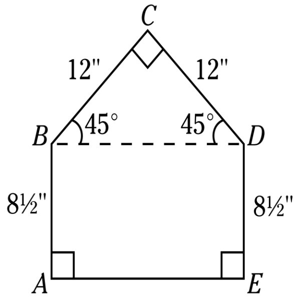
(A) 10

(B) 12

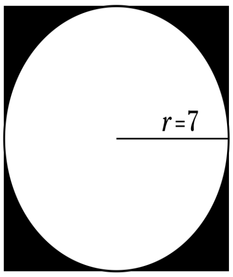
(C) 17

(D) 20

The answer is Choice (C). The key to solving this problem is in shape *BCD*. Because angle *C* is a right angle, and the two sides adjacent to *C* are the same length, *BCD* is an isosceles right triangle, also known as a 45-45-90 triangle, with the side-length ratio $s : s : s\sqrt{2}$. Therefore, the hypotenuse, or *BD*, must be $12\sqrt{2}$, which multiplies out to 16.97, or 17. Because *ABDE* is a rectangle, *AE* has the same length.



Sometimes you have to use several different shapes to solve a problem, especially when the EST throws a strange diagram at you and asks you to find the area of a shaded section, a very popular question (popular with the test-makers, not with the test-takers). In the following diagram, a circle of radius 7 is surrounded by a square. How would you find the area of the shaded section?

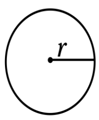


In any problem like this one, the shaded area is equal to the area of the larger shape minus the area of the smaller shape — in this case, the area of the square minus the area of the circle. Because the circle has a radius of 7, its diameter is 14, which must be the same as the side of the square. The area of the square, then, equals $14 \times 14 = 196$. You can find the circle's area by using the formula $\pi r^2 = \pi(7)^2 = 49\pi$. The shaded area, then, equals $196 - 49\pi$. (This calculates to 42.06, but you can usually leave the answer as $196 - 49\pi$.)

Getting the Lowdown on Circles

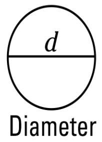
TheEST`-makers love asking about circles, and they toss in enough for a three-ring circus. But don't panic. Problems involving circles are easy to do as long as you keep in mind the following points:

- **A *radius (r)* goes from the center of a circle to its outer edge.** (The plural of *radius* is *radii,* in case you're curious.)

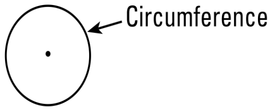


Radius

- **A *diameter (d)* connects two points on the outside or edge of the circle, going through the center.** A diameter is equal to two radii.



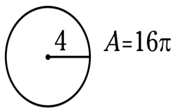
- **The perimeter of a circle is called the *circumference*.** The formula for circumference (C) is $C = 2\pi r$, but you can also use πd, because 2 radii = 1 diameter.



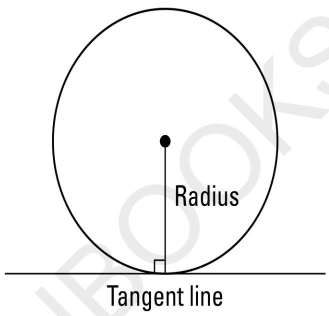
TIP

You may encounter a wheel question in which you're asked how much distance a wheel covers or how many times a wheel revolves. The key to solving this type of question is knowing that one rotation of a wheel equals one circumference of that wheel. There's an example problem like this later in this section.

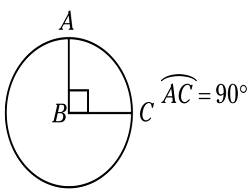
- **The formula for the area of a circle is $A = \pi r^2$.**



- **A *chord* is a straight line segment that connects any two points on a circle.** The longest chord in a circle is the diameter.
- **A *tangent* is a line that touches the circle at exactly one point.** When a tangent line meets a radius of the circle, a 90-degree angle is formed.



- **An *arc* is a portion of the circumference of a circle.** The degree measure of an arc is the same as its central angle. (A *central angle* is an angle with endpoints on the circumference of the circle and its vertex at the center of the circle.)



The EST may ask you to find the length of an arc. To do so, follow these steps:

1. **Find the circumference of the entire circle.**
2. **Put the degree measure of the arc over 360 and reduce the fraction.**
3. **Multiply the circumference by the fraction.**

- **A *sector* is a portion of the area of a circle.** To find the area of a sector, do the following:

  1. **Find the area of the entire circle.**
  2. **Put the degree measure of the sector over 360 and reduce the fraction.**
  3. **Multiply the area by the fraction.**

  Finding the area of a sector is very similar to finding the length of an arc. The only difference is in the first step. Whereas an arc is a part of the circle's circumference, a sector is a part of the circle's area.

Now that all these rules are circling in your head (so to speak), try your hand at a few example problems.

A child's wagon has a wheel of radius 6 inches. If the wagon wheel travels 100 revolutions, approximately how many feet has the wagon rolled?

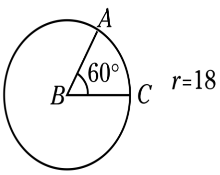
(A) 325

(B) 314

(C) 255

(D) 201

The answer is Choice (B). One revolution of the wheel is equal to its circumference: $C = 2\pi r = 2\pi(6) = 12\pi$ = approximately 37.68 inches, which is 37.68 inches $\div 12 = 3.14$ feet. Multiply that by 100, and $3.14 \times 100 = 314$ feet.



Find the length of minor arc AC.

(A) 36π

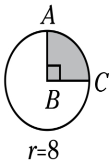
(B) 60

(C) 18π

(D) 6π

The answer is Choice (D). Take the steps one at a time. First, find the circumference of the entire circle: $C = 2\pi r = 36\pi$. Don't multiply 36π out; EST math problems usually leave it in that form. Next, put the degree measure of the arc over 360. The degree measure of the arc is the same as its central angle, $60° = 60/360 = 1/6$. The arc is 1/6 of the circumference of the circle. Multiply the circumference by the fraction: $36\pi \times 1/6 = 6\pi$.

Be very careful not to confuse the degree measure of the arc with the length of the arc. The length is always a portion of the circumference, typically with π in it, and always in linear units. If you chose Choice (B) in this example, you found the degree measure of the arc rather than its length.



If point B is at the center of the circle, what's the area of the shaded sector?

(A) $\frac{1}{4}\pi$

(B) 16π

(C) 64π

(D) 90π

The answer is Choice (B). To do this problem, first, find the area of the entire circle: $A = \pi r^2 = 64\pi$. Second, put the degree measure of the sector over 360. The sector is 90 degrees, the same as its central angle: $90/360 = 1/4$. Third, multiply the area by the fraction: $64\pi \times 1/4 = 16\pi$.

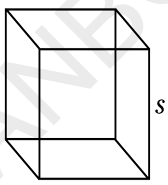
Avoiding Two-Dimensional Thinking: Solid Geometry

Almost every EST has a couple of problems dealing with a box, a cylinder, a sphere, a cone, or a prism. The key formulas you need to know regarding these 3-D figures are included in the direction text at the start of each Math section, but we also offer you a quick review here.

Volume

The volume of most EST 3-D shapes is area of the *base* × *height*. This should help you memorize the following more-specific formulas:

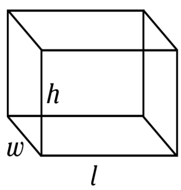
- **Volume of a cube:** $V = s^3$

  Cube

  A cube is a 3-D square. Think of a die (one of a pair of dice). All a cube's dimensions are the same; that is, *length* = *width* = *height*. In a cube, these dimensions are called *edges* or *sides*. The volume of a cube is edge cubed: $V = side \times side \times side = side^3 = s^3$.

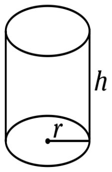
- **Volume of a rectangular solid:** $V = l \cdot w \cdot h$

  Rectangular solid

A rectangular solid is a box, sometimes called a prism. The base of a box is a rectangle, which has an area of $length \times width$. Multiply that area by height to fit the original volume formula: *Volume* = area of $base \times height$ or $V = l \cdot w \cdot h$.

- **Volume of a cylinder:** $V = \pi r^2 h$



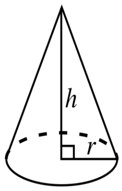
Cylinder

Think of a cylinder as a can of soup. The base of a cylinder is a circle. The area of a circle is πr^2. Multiply that by the height of the cylinder to get this formula: *Volume* = area of $base \times height$, or $V = \pi r^2 h$. Note that the top and bottom of a cylinder are identical circles. If you know the radius of either the top base or the bottom base, you can find the area of the circle.

- **Volume of a sphere:** $V = \frac{4}{3}\pi r^3$

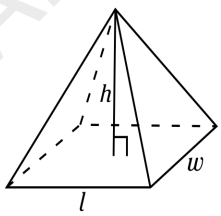
A sphere is a perfectly round ball, like a basketball. Like a circle, it has a radius. Simply plug the radius into the equation, and you have the volume.

- **Volume of a cone:** $V = \frac{1}{3}\pi r^2 h$



A cone has a circular base and sides that taper toward a point. Like a cylinder, you find the area by multiplying the area of the circle by the height, but with the cone, you divide this product by 3.

- **Volume of a pyramid:** $V = \frac{1}{3}lwh$



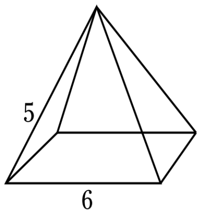
A pyramid has a square base and four identical triangular sides. Like a rectangular solid, you find the area by multiplying the length, width, and height, but with a pyramid, you divide this product by 3.

Although it may be helpful to memorize these formulas, fortunately, you don't have to. They are always right there in the formula bar that starts each Math section.

Surface area

On rare occasions, the EST-writers may ask you to find the surface area of a solid object. The surface area is, sensibly enough, the total area of all the sides (surfaces) of the object. To find the surface area of a box with six sides, calculate the area of each of the rectangles that form a side, and then add them all up. If the test-makers were in a particularly bad mood when they wrote the test, you might see a problem like the following example:

Find the surface area of the square-based pyramid shown below:



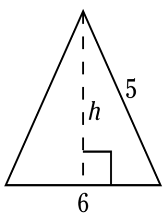
(A) 60

(B) 84

(C) 96

(D) 120

Choice (B) is the right answer. The area of the bottom square is $6 \times 6 = 36$. You know that the area of one of the triangular sides can be found by using the formula area $= 1/2\ \textit{base} \times \textit{height}$, but you don't yet know the height of each triangle. So take a moment to draw one of the triangular sides by itself:



The dashed line is the height, which makes a right angle with the base and cuts it in half. Thus, you have a right triangle with a leg of length 3 and a hypotenuse of 5. Does that sound familiar? Of course: It's a 3-4-5 triangle! So the height is 4, making the area of one triangle $1/2\ \times 6 \times 4 = 12$. And because there are four such triangles, their total area is $4 \times 12 = 48$, which you can now add to the 36 from the base to get Choice (B), or 84.

Trying Trigonometry

Many students clamp their teeth when they hear that the EST has trigonometry questions. If you're clamping right now, relax and breathe easy. The EST has only a few trig questions, and this section covers what you need to know to answer them, even if you've never stepped foot in a trigonometry classroom.

SOH CAH TOA: The trigonometric ratios

Most EST trigonometry questions are based on *trigonometric ratios,* which are the relationships between the angles and sides of a right triangle in terms of one of its acute angles. (An *acute angle* is less than 90 degrees, so the ratio questions aren't based on the right angle.) You can answer almost every EST trig question by using the mnemonic device for the three basic trigonometric ratios: SOH CAH TOA.

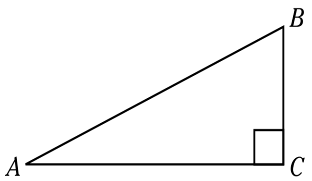
SOH CAH TOA stands for

$$\text{Sine} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\text{Cosine} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\text{Tangent} = \frac{\text{Opposite}}{\text{Adjacent}}$$

Opposite, adjacent, and *hypotenuse* refer to the sides of a right triangle in relation to one of the acute angles. Take a look at this right triangle:



As you know, side *AB* is the *hypotenuse* of the triangle. (The hypotenuse is opposite the right angle and the longest side of the right triangle.) For angle *A,* side *BC* is *opposite,* and side *AC* is *adjacent.* For angle *B,* the opposite and adjacent switch: Side *AC* is opposite, and side *BC* is adjacent.

Use SOH CAH TOA to quickly find the sine, cosine, or tangent of any acute angle in the right triangle.

- **To find sin *A* (the sine of angle *A*), use the SOH part of SOH CAH TOA.** Place the length of the side opposite angle *A* (in this case, side *BC*) over the hypotenuse (side *AB*).

$$\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\sin A = \frac{\overline{BC}}{\overline{AB}}$$

- **To find cos *A* (the cosine of angle *A*), use the CAH part of SOH CAH TOA.** Place the length of the side adjacent angle *A* (in this case, side *AC*) over the hypotenuse (side *AB*).

$$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\overline{AC}}{\overline{AB}}$$

- **To find tan *A* (the tangent of angle *A*), use the TOA part of SOH CAH TOA.** Place the length of the side opposite angle *A* over the side adjacent angle *A*.

$$\tan A = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\tan A = \frac{\overline{BC}}{\overline{AC}}$$

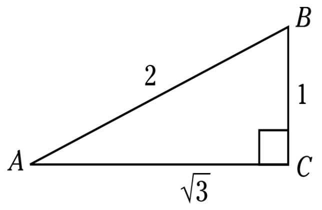
SOH CAH TOA applies only to a *right triangle* and only to an *acute angle,* never the right angle.

When you work with sine and cosine, keep this simple rule in mind: Sine and cosine can never be greater than 1.

$\sin 90^\circ = 1$ and $\sin 0^\circ = 0$

$\cos 90^\circ = 0$ and $\cos 0^\circ = 1$

Find the sine of angle A in right triangle shown below:



(A) 2

(B) $\sqrt{3}$

(C) 1

(D) $\frac{1}{2}$

Choice (D) is the right answer. Using the SOH in SOH CAH TOA, you know that the sine of angle A comes from the opposite (1) over the hypotenuse (2), for an answer of 1/2.

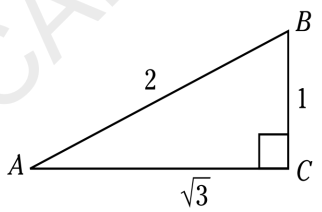
Three additional ratios aren't directly covered by SOH CAH TOA. They appear less frequently than sine, cosine, and tangent but are just as easy to find. These are cosecant (csc), secant (sec), and cotangent (cot). Basically, you find the sine, cosine, or tangent by using SOH CAH TOA and take the reciprocal to find cosecant, secant, or cotangent. The angle is usually represented by the Greek letter theta, θ:

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$

Using the same right triangle, find the cotangent of angle A:



(A) 3

(B) 2

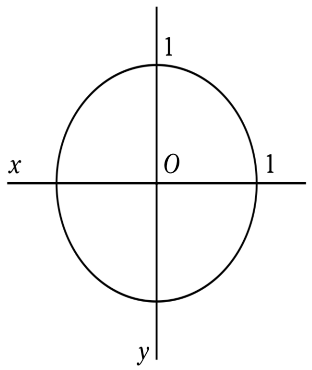
(C) $\sqrt{3}$

(D) 1

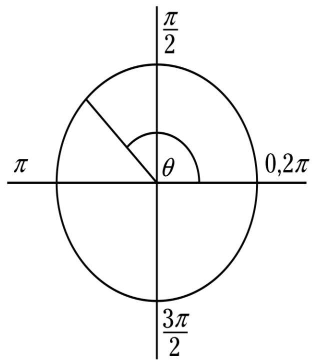
Choice (C) is the right answer. Using the TOA in SOH CAH TOA, you know that the tangent of angle A comes from the opposite (1) over the adjacent ($\sqrt{3}$), for a tangent of $1/\sqrt{3}$. Take the reciprocal of this to find the cotangent of the angle A: $\sqrt{3}$.

Going around in unit circles

The *unit circle* is a circle drawn on the *x-y* graph. The circle has a center at the origin of the graph — that is, coordinates $(0, 0)$ — and a radius of 1.



Starting with the radius of the circle at $(1, 0)$, the angle θ is measured going counterclockwise.

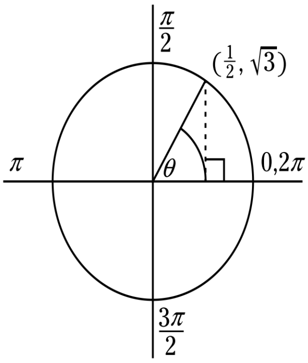


In this drawing, $\theta = 135°$. However, the angle isn't always measured in degrees; rather, it's in *radians,* which means that it's in terms of π, where $\pi = 180°$.

An angle measuring 45° also measures $\pi/4$ radians. An angle measuring 270° also measures $3\pi/2$ radians. More importantly, or the way it's used on the EST, you can tell which quadrant an angle is in from the range of radians. In other words, an angle between $\pi/2$ and π is in the second quadrant. For the angle θ,

- $0 < \theta < \frac{\pi}{2}$ places the angle in the first quadrant.
- $\frac{\pi}{2} < \theta < \pi$ places the angle in the second quadrant.
- $\pi < \theta < \frac{3\pi}{2}$ places the angle in the third quadrant.
- $\frac{3\pi}{2} < \theta < 2\pi$ places the angle in the fourth quadrant.

When SOH CAH TOA is applied to an angle in the unit circle, it's always for the angle θ, which comes from the radius of the circle. The *hypotenuse* is this radius, the *adjacent* is the *x*-value, and the *opposite* is the *y*-value. Consider this example, where $\theta = 60°$ and the radius meets the circle at $\left(\frac{1}{2}, \sqrt{3}\right)$, where on the *xy* graph, $x = \frac{1}{2}$ and $y = \sqrt{3}$.



The sine of θ is the opposite over the hypotenuse, which in this case is $\frac{\sqrt{3}}{2}$. The cosine of θ is the adjacent over the hypotenuse, $\frac{1/2}{1}$, or $\frac{1}{2}$. The tangent of θ is the opposite over the adjacent, which is $\frac{\sqrt{3}}{1}$, or $\sqrt{3}$.

Knowing the quadrant and the sine, cosine, or tangent of an angle on the unit circle, you can find exactly where the angle is and solve almost any problem about it.

EXAMPLE

If $\frac{\pi}{2} < \theta < \pi$ and $\cos\theta = -\frac{3}{5}$, what is $\sin\theta$?

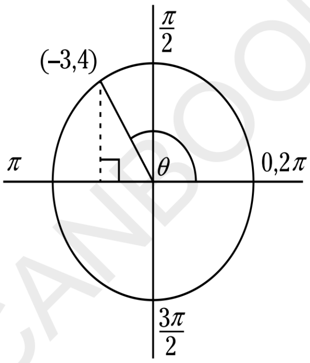
(A) $-\frac{4}{5}$

(B) $-\frac{3}{5}$

(C) $\frac{3}{5}$

(D) $\frac{4}{5}$

The answer is Choice (D). The first expression, $\frac{\pi}{2} < \theta < \pi$, places the angle in the second quadrant, and $\cos\theta = -\frac{3}{5}$ means the ratio of the *x*-value of the endpoint to the radius is $-\frac{3}{5}$. Because the hypotenuse (or radius) is always positive, the *x*-value is negative.



The $\sin\theta$, being the opposite over hypotenuse, is therefore $\frac{4}{5}$.

Chapter 15

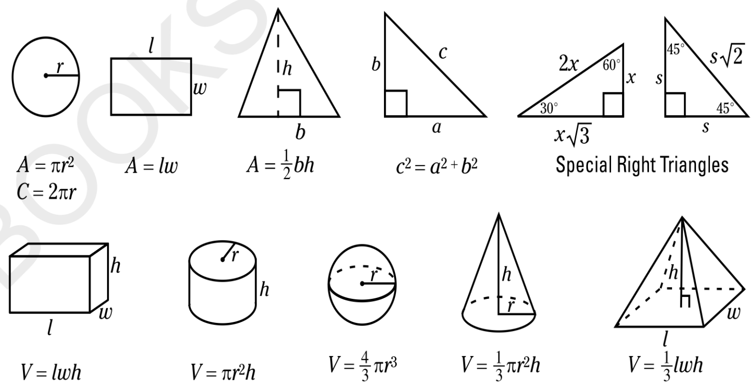
Practicing Problems in Geometry and Trigonometry

In This Chapter

- Practicing a few guided geometry and trigonometry problems
- Focusing on angles, shapes, and distances in some sample questions

Even if you'd rather squash a polygon than calculate its measurements, bite the bullet and check out the practice questions in this chapter. You get two sets. In the first set, move through each problem by solving it and then immediately checking your answer using the explanation that follows. Don't cheat: Cover the answers with a blank sheet of paper until you come up with your own solutions. Then, try your hand at the second set, which is set up like the actual test. In that set, solve all the questions *before* checking your answers with the explanations in the section that follows the questions. (Turn to Chapter 14 for more information on any of the topics covered in these practice questions.)

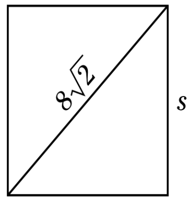
The following diagrams appear at the beginning of each Math section of the EST to help you work through the geometry problems. Don't hesitate to use them as you answer the practice problems in this chapter.



Set One: Getting Started with Some Guided Questions

Note: Questions 3 and 6 are grid-ins, so you don't get any answers to choose from. See Chapter 9 for a quick review of grid-ins.

1. In the following square, what is the length of side s?



(A) 8

(B) $8\sqrt{2}$

(C) $8\sqrt{3}$

(D) 16

Choice (A) is correct. When you cut a square in half, you get a 45-45-90 triangle, with the square's diagonal as the hypotenuse. The freebie information at the beginning of each Math section (nice of them to help you, don't you think?) tells you that in a 45-45-90 triangle, the length of the hypotenuse equals $\sqrt{2}s$, where s is the length of a side of the square. Because the hypotenuse equals $8\sqrt{2}$, the side equals 8.

2. In triangle *ABC,* if the distance between points *A* and *B* is 5 and the distance between points *B* and *C* is 7, then the distance between points *A* and *C* may not equal

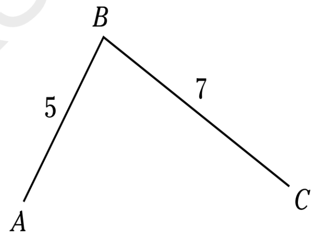
(A) 1

(B) 3

(C) 4

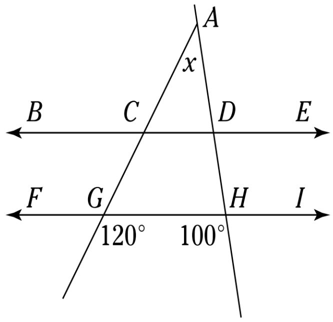
(D) 6

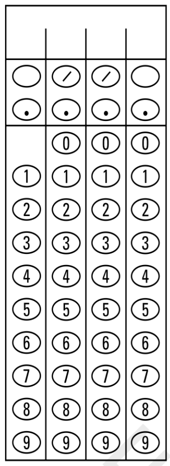
Choice (A) is correct. To answer this question, draw a line connecting *A* and *B* and another one connecting *B* and *C,* like so:



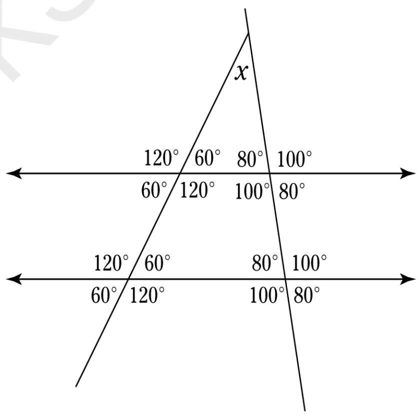
Now you can use a little thing called the *triangle inequality.* The distance from *A* to *C* forms the third side of a triangle, and the sum of two sides of a triangle must be greater than the third side. This makes it impossible for *AC* to equal 1, because $1+5=6$, which isn't bigger than 7. Before moving on, take a minute to make sure the other three answers do satisfy the triangle inequality.

3. In the following drawing, $\overline{BE} \parallel \overline{FI}$. Find the measure, in degrees, of the angle marked x.





Because this drawing contains parallel lines cut by transversals (the two lines meeting at point *A*), you can fill in a whole lot of angles right off the bat. Each transversal creates eight angles, and these angles come in two groups of four pairs of vertical and supplementary angles. (Remember, a pair of *vertical angles* is two angles opposite each other and equal to each other. Supplementary angles total 180 degrees.) Here they are, filled in:

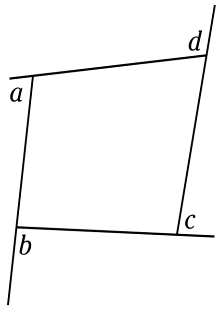


REMEMBER

After you determine the angles, the problem becomes simpler. Because *ACD* is a triangle, its angles must add up to 180 degrees. With a 60-degree and an 80-degree angle already accounted for, the missing angle must be 40 degrees — your correct answer.

Don't grid-in the degree symbol, just the number.

4. What is the sum of the angles marked *a*, *b*, *c*, and *d* in the following diagram?



(A) 180 degrees

(B) 360 degrees

(C) 540 degrees

(D) 720 degrees

Choice (B) is correct. This one you just have to memorize. The sum of the exterior angles of any shape is always 360 degrees. Remember that fact.

5. If an equilateral triangle has sides of length 6, then its altitude has a length of

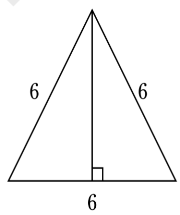
(A) 3

(B) $2\sqrt{3}$

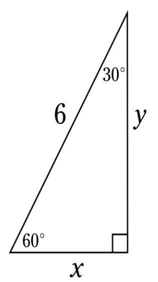
(C) $3\sqrt{2}$

(D) $3\sqrt{3}$

Choice (D) is correct. This one's a special-triangle problem in disguise. Here's the equilateral triangle with its altitude drawn:

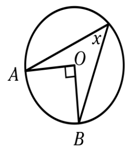


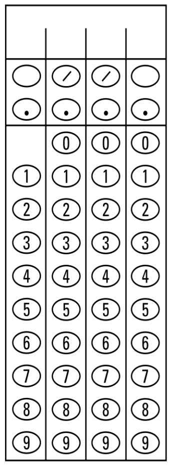
(Of course, you drew this triangle as soon as you were done reading the problem, right?) Each half of the original triangle forms a 30-60-90 triangle. Making a second drawing just to be clear is worth your time.



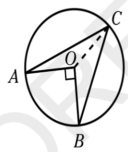
From the box of formulas and diagrams at the beginning of this chapter, you know that the side marked x must be half of 6, or 3, which means that y, the altitude, must equal $3\sqrt{3}$. Thus, Choice (D) is correct.

6. In the following diagram, O is the center of the circle, and angles A and B have the same measure. Find the measure, in degrees, of the angle marked x.

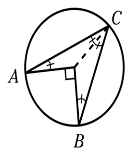




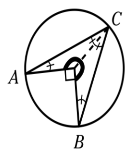
The easiest way to think about this problem is to cut the arrowhead shape into two triangles, like so:



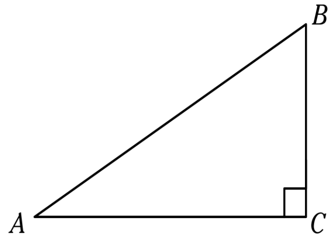
Notice that the line segments OA, OB, and OC are all radii, making them all the same length. That means both triangles are isosceles. In any isosceles triangle, the base angles must have equal measures; because you were told that A and B have the same measure, all the angles marked in the following figure are congruent:



You're almost there. Because angle AOB is 90 degrees, the other side of the angle (dark line in the following figure) must measure 270 degrees to make 360 degrees around a point. That means that half of 270, or 135 degrees, is the top angle of the isosceles triangle. That leaves 45 degrees for the other two angles. And, because two of these angles together made up x, x must equal 45. Whew!



7. For this right triangle, if $\tan B = \frac{4}{3}$, find $\cos A$.



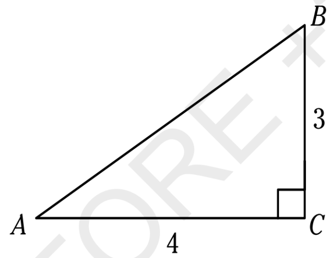
(A) $\frac{4}{3}$

(B) $\frac{5}{4}$

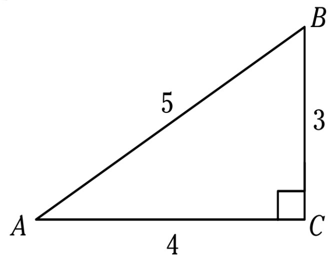
(C) $\frac{3}{4}$

(D) $\frac{4}{5}$

Choice (D) is the answer. If $\tan B = \frac{4}{3}$ and tangent is opposite over adjacent (the TOA from SOH CAH TOA), then draw the triangle like this:



Spotting this as one of the Pythagorean triples (more on these in Chapter 14), you automatically throw the 5 down onto the hypotenuse:

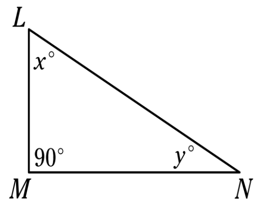


Cosine is CAH, which is adjacent over hypotenuse. Therefore, $\cos A = \frac{4}{5}$.

Set Two: Practicing Some Questions on Your Own

Note: Refer to the box of formulas and diagrams at the beginning of this chapter to help you answer these questions. Remember that those diagrams appear at the beginning of the Math sections on the real EST.

1. In this triangle, the measure of angle x is greater than the measure of angle y. Which of the following statements must be false?



(A) $MN > LM$

(B) $(LN)^2 - (LM)^2 = (MN)^2$

(C) $LN > MN$

(D) $LN - LM = MN$

2. A car has wheels with radii of 1.5 feet. If the car is backed down a driveway that is 95 feet long, about how many complete turns will the wheels make?

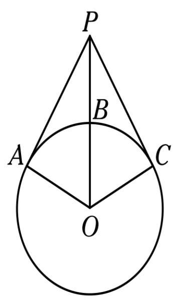
(A) 10

(B) 13

(C) 15

(D) 20

3. In the following diagram, O is the center of the circle, and $\overline{AP}$ and $\overline{CP}$ are tangents. If $OA = 8$ and $BP = 9$, find CP.



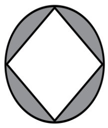
(A) $\sqrt{17}$

(B) 9

(C) $\sqrt{145}$

(D) 15

4. In the following diagram, a square is inscribed in a circle. If one side of the square has a length of 10, then the shaded area equals



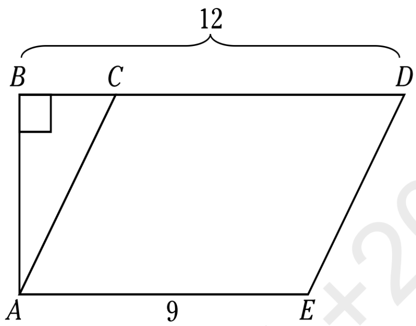
(A) $100\pi - 100$

(B) $50\pi - 100$

(C) $100\pi - 50$

(D) $50\pi - 50$

5. In the following drawing, *ACDE* is a parallelogram with an area of 36. Find the length of *AC*.



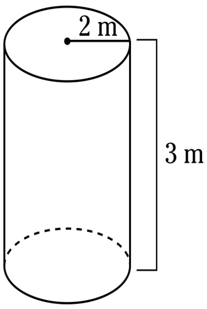
(A) 3

(B) 4

(C) 5

(D) 6

6. This cylindrical gas tank, originally empty, has a radius of 2 meters and a height of 3 meters. At 11 a.m., gas starts being added to the tank at a rate of 10 meters^3 per hour. The tank will be completely full closest to



(A) 2 p.m.

(B) 2:30 p.m.

(C) 3 p.m.

(D) 3:30 p.m.

7. If $\frac{3\pi}{2} < \theta < 2\pi$ and $\sin\theta = -\frac{4}{5}$, what is $\cos\theta$?

(A) $\frac{4}{5}$

(B) $\frac{3}{5}$

(C) $-\frac{3}{5}$

(D) $-\frac{4}{5}$

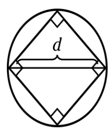
Answers to Set Two

1. **D.** As you often should do in this type of problem, go through the answers one by one. Choice (A) is true because, in any triangle, the shortest side is opposite the smallest angle. Because *y* is smaller than *x* and both of them must be smaller than 90 degrees, *y* is the smallest angle and *LM* is the shortest side. Choice (B) is just a fancy way of writing the Pythagorean theorem. Because *LN* is the hypotenuse of a right triangle, $(LM)^2 + (MN)^2 = (LN)^2$, so $(LN)^2 - (LM)^2 = (MN)^2$. Choice (C) is true for essentially the same reason as Choice (A): *LN* must be the longest side of the triangle because it's across from 90 degrees, the largest angle. Choice (D) wins the "False Award" because of the triangle inequality. In any triangle, the sum of the two short sides must be *greater* than the longest side. That fact means that $LM + MN > LN$, so $LN - LM$ can't equal *MN*. The correct answer — the false statement — is Choice (D).

2. **A.** This one is a classic EST' problem. The key is knowing that one complete rotation equals the circumference of the wheel. Because circumference = $2\pi r$, you have $2(3.14)(1.5) \approx 9.42$ feet. Dividing 95 by 9.42 gives you 10.08, so your answer is 10. The correct answer is Choice (A).

3. **D.** Because $\overline{AP}$ and $\overline{CP}$ are tangents, the angles at *A* and *C* must be right angles. (If this fact is a surprise, turn back to Chapter 14. We can almost guarantee that this concept will show up in some form on the test). Triangles *OPC* and *OPA* are right triangles, so the Pythagorean theorem comes into play (and hits the ball out of the park). Because $OA = 8$, *OB* and *OC* are also 8 because all radii are equal. That makes $OP = 8 + 9 = 17$. *OP* is the hypotenuse, *OC* is a leg, and *CP* is a leg. So $(CP)^2 + (8)^2 = (17)^2$; $(CP)^2 + 64 = 289$; $(CP)^2 = 225$; and $CP = 15$. Choice (D) is correct.

4. **B.** This one is a shaded-area problem, so your answer must be the circle's area minus the square's area. The square's area is pretty simple to figure out: It's $(10)^2 = 100$. To find the circle's area, you need to know its radius. You can make a diameter by drawing the diagonal of the square, like so:



Look familiar? The diagonal of a square creates a 45-45-90 triangle, so the length of the diagonal is $10\sqrt{2}$. (The EST-makers love special triangles.) The radius is half of the diameter, $\frac{10\sqrt{2}}{2}$, or $5\sqrt{2}$. Bingo. The area of the circle is $\pi(5\sqrt{2})^2 = \pi(25 \times 2) = 50\pi$. So your answer is $50\pi - 100$, Choice (B).

Circling in on a better vocabulary

Lots of vocabulary words — the kind you may find in reading-comprehension passages or even in normal conversation — pop out of math lessons and land in the real world. For example, the line that touches a circle but doesn't pierce it (a *tangent*) gives rise to the expression *going off on a tangent* (moving away from the main topic to something that is only marginally related), as in "Coop went off on a tangent about eggs when he was supposed to be discussing feather boas." A related word, *tangential,* shows up in sentences such as "The dry cleaners' association and the United Featherworkers of America criticized Coop's tangential remarks."

You probably know how to find the *circumference* of a circle (the distance around the edge). Well, the following words are all in the same family:

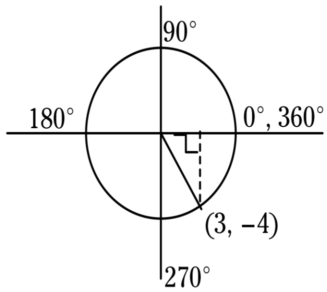
- ***Circumlocution:*** To talk around by speaking indirectly and avoiding a clear statement, as in "Politicians use more circumlocutions than usual during an election year."
- ***Circumnavigate:*** To sail around, as in "Some say Phileas circumnavigated the globe in a hot air balloon."
- ***Circumscribe:*** To limit; picture a warden drawing a circle around someone and forbidding him or her to cross the line.
- ***Circumspect:*** Cautious; think of an imaginary circle around yourself that you venture beyond only with extreme care.
- ***Circumvent:*** To go around, as in "Bruckner circumvented the door alarm by breaking through the wall."

A distant cousin of the *circum* family is ***circuitous,*** an adjective that may describe the sort of route taxi drivers take with tourists in the back seat (round and round, just to drive up the fare).

5. **C.** The area of a parallelogram uses the same formula as a rectangle: *base* × *height*. Because the base, *AE,* is 9, and the area is 36, the height, *AB,* must be 4. (Don't be fooled into thinking that *AC* is the height. The height is always perpendicular to the base, never slanted.) Meanwhile, $BC = 12 - 9 = 3$. This is yet another right triangle, so you can use the Pythagorean theorem to get $AC = 5$. Even better, if you remember the 3-4-5 right triangle, you just know that the answer is 5 without having to do all the work. Choice (C) is the correct answer.

6. **C.** The volume of a 3-D figure equals the area of its base times its height. Because the base is a circle, its area is $\pi r^2 = \pi(2)^2 = 4\pi \approx 12.56 \text{ m}^2$. Multiplying by 3, the height, gives you a volume of 37.68 m^3. Dividing 37.68 m^3 by 10 m^3 per hour gives you 3.768 hours to fill the tank. This answer is a little bit closer to four hours than to 3.5 hours (3.75 would be exactly halfway), so you can round up to 4 hours. Four hours after 11 a.m. is 3 p.m. Choice (C) is correct.

7. **B.** The expression $\frac{3\pi}{2} < \theta < 2\pi$ places the angle in the fourth quadrant, and $\sin\theta = -\frac{4}{5}$ means that the ratio of the *y*-value to the hypotenuse is $-\frac{4}{5}$, or –4 to 5, because the hypotenuse is always positive. This means that drawn on the unit circle, and completing the 3-4-5 triangle, angle θ looks like this:



To find $\cos\theta$, place the adjacent (*x*-value) over the hypotenuse: $\frac{3}{5}$.

Chapter 16

Playing the Odds: Statistics and Probability

In This Chapter

- Understanding probability and answering multiple-probability questions
- Solving problems that deal with geometric probability
- Using mean, median, and mode
- Interpreting scatter plots and other graphs
- Thinking logically to solve logic-based EST questions

If you take the EST 25 times, what are the odds that you'll die of boredom before entering college? Your EST proctor, even more bored than the 25 test-takers in front of him, launches an ink-filled balloon into the 30-square-foot classroom. What is the probability that it will miss you and land on the mouth-breather in the next row?

Questions like these — similar but humorless — confront you on the EST. To increase the odds that you'll ace the topic of statistics and probability, read on. Also peruse this chapter to get the lowdown on the three *m*'s (mean, median, and mode) and scatter plots and other graphs as well as logic questions.

Working with the Odds: Probability

The *probability* of an event (or the odds that it will occur) is almost always defined as a fraction. So in many probability situations, you have to compute two separate numbers, one for the numerator and one for the denominator of the fraction. What do these numbers stand for? Well, here's the formula:

$$\text{the probability of an event} = \frac{\text{the number of ways for the event to happen}}{\text{the total number of possible outcomes}}$$

Say that you have a jar containing 6 red, 4 yellow, and 8 blue marbles. The probability of randomly picking a blue marble is $\frac{8 \text{ blue marbles}}{6+4+8=18 \text{ total marbles}}$, which can be reduced to $\frac{4}{9}$.

Probability can also be written as a percentage. The easiest way to compute the percentage is with your calculator. Suppose that the probability that a major label will sign your garage band is $\frac{2}{5}$. (In your dreams, by the way. The real probability is $\frac{1}{10{,}000{,}000}$.) Enter $2 \div 5$ on your calculator to get 0.4. Now move the decimal two places to the right. Bingo. The probability that you and your bandmates will ride to the MTV studio in a limo is 40%.

An event that is certain to happen has a probability of 1, or 100%. An event that is impossible has a probability of 0. Nothing can ever have a probability greater than 1 or less than 0. Another way to say the second fact: Negative probability doesn't exist.

When you calculate probability, remember the number 1. All the possible events must have probabilities that add up to 1 (or 100%). That fact leads to a useful rule, which may be stated in three ways:

- The probability that an event won't happen equals 1 minus the (decimal or fractional) probability of the event.
- The probability that an event won't happen equals 100% minus the (percent) probability of the event.
- The probability that an event won't happen equals

$$\frac{\text{total number of possibilities} - \text{the number of ways for the event to happen}}{\text{the total number of possible outcomes}}$$

Imagine that you're sitting in class with 19 other students, and your teacher decides to pick one student at random to stay after school for a round of eraser cleaning. What's the chance that she picks you? There's only 1 of you, and she's choosing from 20 students. The probability is $\frac{1}{20}$, or 5%.

So what's the chance that she *doesn't* pick you? Because there are 20 total possibilities (20 students), $\frac{20-1}{20} = \frac{19}{20} = 95\%$ or $100\% - 5\% = 95\%$.

The following sections discuss two variations of the typical EST probability problem.

Psyching out multiple-probability questions

Not surprisingly, the EST-writers have found plenty of ways to make probability problems harder. One of their favorite torture devices is to ask you about a probability involving multiple events. When a problem involves multiple events, the total number of possibilities is the product of the number of possibilities for each event. If, for example, you open your closet on laundry day and find two clean shirts and three pairs of pants, the total number of outfits you can make is $2 \times 3 = 6$ (assuming that you're not a fashionista and don't care about little things like complementary colors). This rule is known as the *counting principle,* although the "multiplication principle" may be a better name for it. This method works whether you're using whole numbers, percentages, or fractions.

Multiple-probability questions on the EST may resemble the following example.

Jenny arranges interviews with three potential employers. If each employer has a 50% probability of offering her a job, what's the probability that she gets offered all three?

(A) 10%

(B) 12.5%

(C) 100%

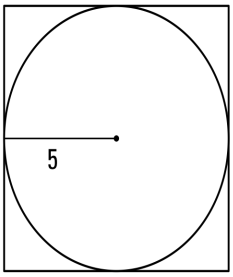
(D) 150%

The answer is Choice (B). Applying the counting principle to Jenny's situation, you can say that the probability of her being offered all three jobs is $50\% \times 50\% \times 50\%$, or $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ (12.5%).

Surviving geometric probability

Unbelievably, those EST writers sometimes expect you to combine your knowledge of two different areas of mathematics in a single question! In this example, the areas are *geometry* and *probability*. (Turn to Chapter 14 for a geometry review.) Check out the following example.

A dart is thrown at the dartboard below. If the radius of the circle is 5 inches, then the probability that the dart lands in the square but not in the circle is closest to



(A) 15%

(B) 21%

(C) 50%

(D) 78%

The answer is Choice (B). You may recognize this question as a variation of the shaded-area problem. (Check out Chapter 14 for more info on shaded areas.) Because the problem asks about the four corner regions of the diagram, first you have to figure out the area of these regions. The area of the circle is $\pi r^2 = \pi(5)^2 = 25\pi$ square inches. Because a side of the square equals the circle's diameter, which is 10, the square's area is $10^2 = 100$ square inches. That makes the total area of the corner regions equal to $100 - 25\pi \approx 100 - 78.54 = 21.46$ $100 - 25\pi \approx 100 - 78.54 = 21.46$. The probability of the dart hitting somewhere in one of the corner areas is found by dividing the area of the corners by the total area of the square. Because the square's area is 100, the probability that the dart lands in one of the corners is $\frac{21.46}{100} = 21.46\%$.

Finding the Three Ms: Mean, Median, and Mode

Sometimes the EST gives you a group of numbers and asks you to find the *average* (officially called the *mean* or *arithmetic mean*). This sort of problem is probably familiar to you, especially if you're into computing your grade-point average or your favorite baseball player's batting average. To find the average, just add up the numbers and divide the total by the number of numbers you just added. For example, to find the average of 2, 4, and 9, add those three numbers (total = 15) and divide by 3. The average is 5.

If a group of numbers is evenly spaced, the mean is the middle number or the average of the two middle numbers. Suppose that you're asked to find the arithmetic mean of the numbers from 1 to 19. Even with a calculator, adding all the numbers and then dividing is time-consuming, not to mention easy to mess up. But the 19 numbers are evenly spaced (all 1 apart), and 10 is the middle number. No matter which way you start, from 1 or from 19, you find nine numbers evenly spaced on either side of 10. Therefore, 10 is the average.

Remember that this trick works only when the numbers are evenly spaced, such as 5, 10, 15, 20, and 25 having an average of 15. If you're told to find the average of 3, 5, 7, 12, and 18, you have to do the math: Add them up and divide by the number of numbers.

Moving beyond *mean*, the EST also asks about *median.* The *median,* as those of you with drivers' licenses already know, is the strip down the middle of a road. In math, the *median* is defined as the middle number in a list, when the list is in numerical order. If you have a list such as 5, 3, 8, 7, 2, and need to find the median, put the numbers in order: 2, 3, 5, 7, 8. The middle number, or median, is 5.

If you have an even number of numbers (say, 3, 5, 6, 7, 8, 10), the list has no middle number, so take the mean of the two numbers closest to the middle. In this example, the two numbers in question are 6 and 7, so the median is 6.5.

The last of the three *m*'s is the *mode,* the easiest to find. In a mixed bag of numbers, the *mode* is the number or numbers that pop up most frequently. So if you have a set with two 4s and two 8s, plus a bunch of other single numbers, you have two modes, 4 and 8, in that set. You can also have a set with no mode at all if everything shows up the same number of times.

Which of the following is true for the set of numbers 3, 4, 4, 5, 6, 8?

(A) mean $>$ mode

(B) median $>$ mean

(C) median $=$ mode

(D) median $=$ mean

The answer is Choice (A). If you average the terms, you get $30 \div 6 = 5$, which is the mean. The median is 4.5 (halfway between the third and fourth terms), and the mode is 4. So Choice (A) is the only one that fits.

Reading Graphs

Some of the math questions on the EST are called *data interpretation.* Sounds important, huh? Actually, it's just a pompous name for "reading a graph," something you've been doing for years. Don't let graph problems intimidate you. Here are the three most common types of graphs you're likely to see on the EST:

- Bar graph
- Circle or pie graph
- Two-axes line graph

We explain these graphs in more detail in the following sections. Because the EST-writers sometimes try to trip you up by asking you to compare statistics in two different graphs, we cover that topic here as well.

Bar graphs

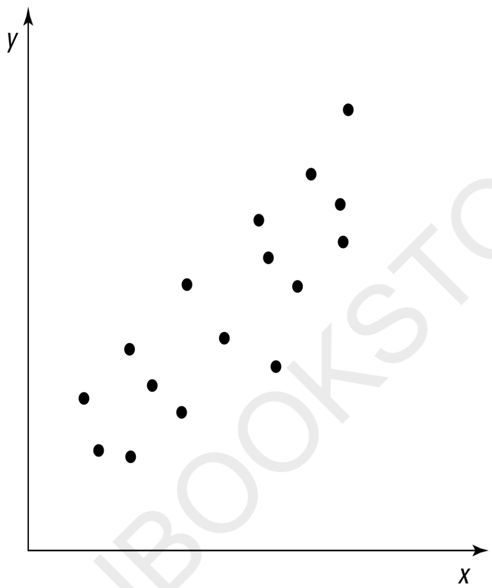
A *bar graph* has vertical or horizontal bars. The bars may represent actual numbers or percentages. If a bar goes all the way from one side of the graph to the other, it usually represents 100 percent.

Circle or pie graphs

The *circle* or *pie graph* represents 100 percent. The key to this graph is determining the total that the percentages are part of. Below the graph you may be told that in 1994, 5,000 students graduated with PhDs. If a 25 percent segment on the circle graph is labeled "PhDs in history," you know that the number of history PhDs is 25 percent of 5,000, or 1,250.

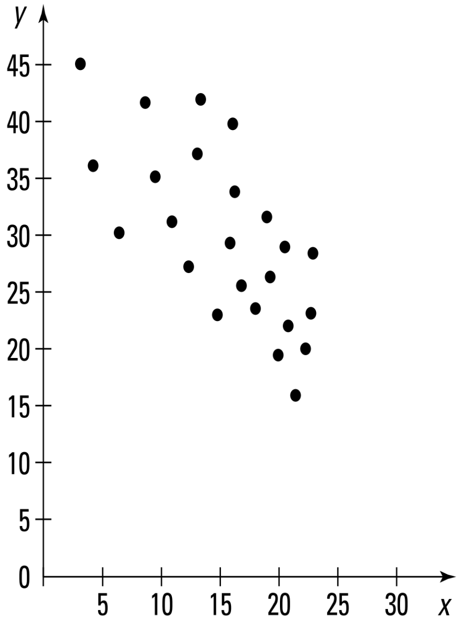
Two-axes line graphs and scatter plots

A typical *line graph* has a bottom and a side axis. You plot a point or read a point from the two axes. A special kind of two-axes graph is the *scatter plot.* A scatter plot contains a bunch of dots scattered around a two-line graph. Here's an example:



Notice how the points seem to follow a certain trend, going higher as they go to the right. When a trend is present, you can draw a line that estimates the behavior of the points. This line is known as a *trend line.* On the test, you may be given a scatter plot and have to estimate where the points are going based on the trend line.

For the following data set, the trend line has a slope closest to



(A) –2

(B) –1

(C) 1

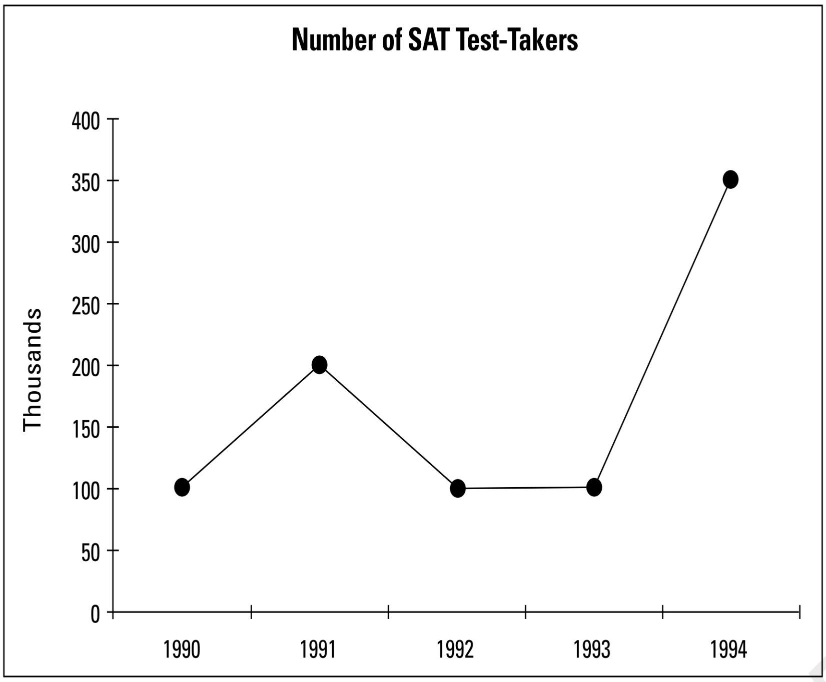
(D) 2

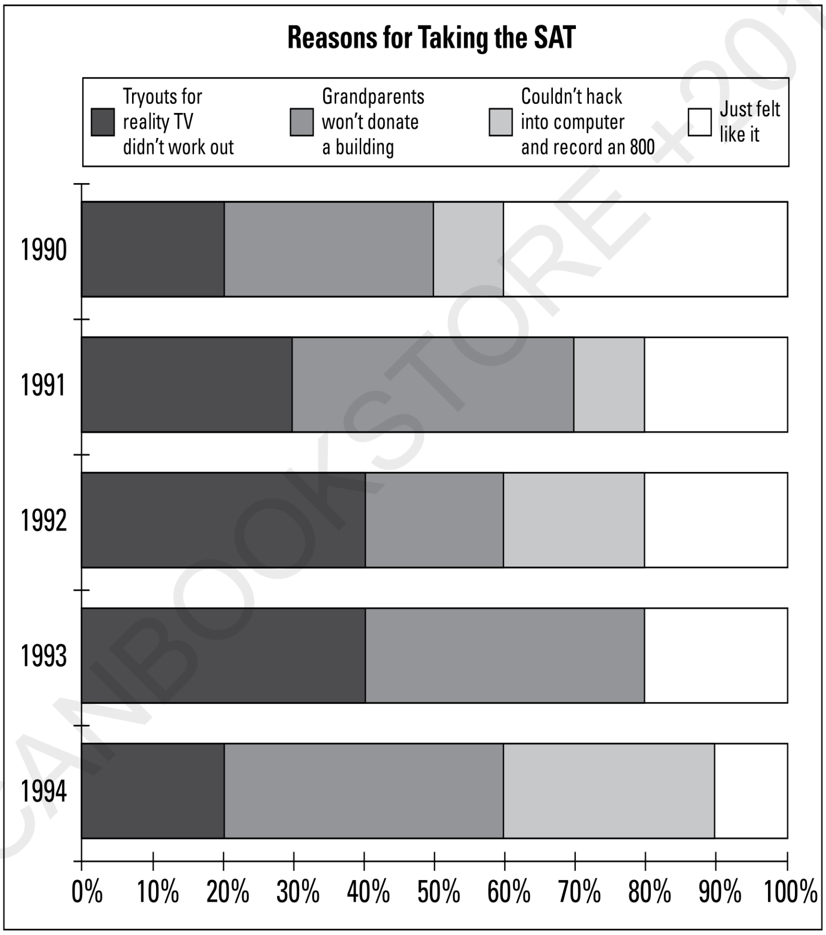
The correct answer is Choice (A). Because the data points flow downward as they go to the right, it must be Choice (A) or (B). If you look at the top left point, you can estimate its coordinates as $(5, 45)$. The bottom right point is around $(20, 15)$. The slope of the line connecting these points is $\frac{15-45}{20-5} = \frac{-30}{15} = -2$.

Multiple graphs

Some questions use two graphs in one problem. No need to fret — there is a simple art to answering multiple-graph questions. To get started, take a look at the two graphs that follow.

You must read these graphs together. The second graph is a bar graph going from 0 to 100 percent. Read the graph by subtracting to find the appropriate percentage. For example, in 1990, "Grandparents won't donate a building" begins at 20 percent and goes to 50 percent, a difference of 30 percent. You've fallen into a trap if you say that "Grandparents won't donate a building" was 50 percent. In 1993, "Just felt like it" goes from 80 percent to 100 percent, which means it was actually 20 percent.





The first graph gives you the number of EST test-takers in thousands. (By the way, these aren't real numbers.) Be sure to look at the labels of the axes. For example, *Thousands* along the side axis tells you that in 1990, there weren't 100 test-takers but 100,000. Using the two graphs together, you can find out the number of test-takers who took the EST for a particular reason. For example, in 1991, 200,000 students took the test. Also in 1991, "Couldn't hack into computer and record an 800" (from 70 to 80, or 10 percent) made up 10 percent of the reasons for taking the EST. Multiply 10 percent or $0.10 \times 200,000 = 20,000$ test-takers.

On the test, you may encounter two to three questions about a particular graph. Answer the following question based on the two practice graphs that deal with EST test-takers and appear earlier in this section.

The number of students who took the EST in 1994 because their grandparents wouldn't donate a building was how much greater than the number of students who took the EST in 1992 because they couldn't hack into the computer and record an 800?

(A) 250,000

(B) 140,000

(C) 120,000

(D) 100,000

The answer is Choice (C). In 1994, "Grandparents won't donate a building" accounted for 40 percent of test-taking reasons (from 20 to 60). Because 1994 had 350,000 test-takers, multiply $0.40 \times 350{,}000 = 140{,}000$. In 1992, "Couldn't hack into computer and record an 800" counted for 20 percent of test-taking reasons (60 to 80). In 1992, 100,000 students took the test. Multiply $0.20 \times 100{,}000 = 20{,}000$. The correct answer is $140{,}000 - 20{,}000 = 120{,}000$, or Choice (C).

Analyzing Logic Questions

The EST occasionally tosses you a logic question, disguised as a simple math question. This type of question has two parts. First is the set of statements or conditions, sometimes called the facts. These statements describe the relationship between or among people, items, or events. You may, for example, be given statements about students at a school and then be asked which ones can be assigned to the same classes. You may be told facts about events that can happen on certain days of the week or about what different combinations of items are possible.

A logic question often takes a long time to solve. Make the decision whether you have the time — and the patience! — to do it properly. If not, skip the question and come back to it later, if you can. Don't rush yourself.

Before you start doodling to solve a logic problem, be sure you know all the people or items involved. Make a "program" of all the players by writing down the pool of people or events. For example, if the question talks about five teachers, Mahaffey, Negy, O'Leary, Plotnitz, and Quivera, use initials and jot down M, N, O, P, and Q on the test booklet.

What not to call the umpire

Mathematicians love baseball — or at least baseball stats. So take a moment to rest from your math labors and watch your favorite team. You can call the ump ***disinterested*** (fair) but not ***uninterested*** (bored out of his skull). Unless you want to go down on strikes, also avoid calling him ***pusillanimous*** (cowardly) and ***mendacious*** (lying). Whatever you think of the pay scale, stay away from ***mercenary*** (in it only for the money, as opposed to in it for the love of the game) and ***partial*** or ***partisan*** (biased). Nor would an umpire appreciate being labeled ***iniquitous*** (evil) or ***intemperate*** (excessive, extreme).

To gain the ump's favor, try calling him ***judicious*** (showing good judgment) and ***discerning*** (sharp, perceptive). You may also call him ***equanimous*** (level headed or even tempered) to stay on his good side.

Next, use a diagram to show the relationship between people or events. Here are a few of the most common diagrams.

- **Calendar:** Draw a simple calendar and fill in the events that happen on particular days.
- **Ordering or sequencing:** You may have a relationship problem in which some people are taller or heavier than others. Write a line of people, with A above B if A is taller than B, C at the bottom if she is the shortest, and so on.
- **Grouping or membership:** This problem asks you which items or people could belong to which group. For example, membership in a club may require four out of five characteristics. Often this type of question doesn't require a graph, but it does require a lot of *if . . . then* statements, such as "If A is in the group, then B isn't."

Try your hand at this logic-based example:

EXAMPLE

Five spices — lemon pepper, marjoram, nutmeg, oregano, and paprika — are aligned next to one another between the left and right sides of a kitchen cabinet. Their arrangement must conform to the following conditions:

The marjoram is immediately to the right of the paprika.

The oregano is either all the way to the left or second from the left.

The lemon pepper is farther left than the nutmeg.

Which of the following could *not* be true?

(A) The paprika is second from the left.

(B) The marjoram is to the right of the lemon pepper.

(C) The nutmeg is exactly in the middle.

(D) The lemon pepper is exactly in the middle.

The answer is Choice (D). To help keep track of the information, write out initials for the roster of spices — L, M, N, O, and P — and make five simple dashes to represent the five positions of the spices:

___ ___ ___ ___ ___

The easiest condition to accommodate is the one that indicates that the oregano must be first or second from the left. Draw these two possibilities:

O ___ ___ ___ ___

___ O ___ ___ ___

The next thing to note is that the paprika and marjoram must always move together. So test out the answer choices, making sure to also fulfill the third condition. Choice (A) is fine, because you can write O, P, M, L, N and meet all conditions. Choice (B) also works, because you can write O, L, P, M, N. And for Choice (C), you can write either O, L, N, P, M or L, O, N, P, M.

Choice (D) is no good, though. If L is in the middle, you have to put P and M to its right, because they always travel together. But that doesn't leave room to put N to the right of L, so you can't fulfill the third condition. Choice (D) is the only option that doesn't work, so that's your answer.

Chapter 17

Practicing Problems in Probability, Statistics, and Logic

In This Chapter

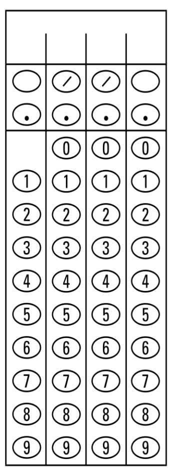
- Practicing some guided questions about probability, statistics, and logic
- Poring over some sample questions on your own

You can count on at least a couple of probability, statistics, and logic problems showing up on your particular version of the EST. Don't freak out! Now's the time to practice with the help of the two sets in this chapter. After you complete each question in the first set, check your answers and read the explanations of any problem you answered incorrectly. (The answers immediately follow each question.) Don't cheat, though! Use a piece of paper to cover the answers as you work through the problems. Then hit set two, which is set up more like the real test, with the answers coming in a section separate from the questions themselves. Turn to Chapter 16 for a refresher course in any topic that stumps you in either set.

Set One: Trying Your Hand at Some Guided Questions

Note: Questions 1 and 7 are grid-ins, which means you don't get answers to choose from. See Chapter 9 for tips on answering grid-ins.

1. A school cafeteria offers two soups, three main dishes, and four desserts. Find the total number of possible meals consisting of one soup, one main dish, and one dessert.



The correct answer is 24. Using the counting principle, $2 \times 3 \times 4 = 24$.

2. The chance of rain tomorrow is 25%. What is the probability that it will *not* rain tomorrow?

(A) 4%

(B) 25%

(C) 40%

(D) 75%

The probability of an event not happening equals 100% minus the probability of it happening: $100\% - 25\% = 75\%$. Choice (D) is correct.

3. In a special deck of 20 cards, 8 cards are red on both sides, 7 cards are blue on both sides, and the other 5 cards are red on one side and blue on the other side. If a student picks a card and places it on his desk, what is the probability that the side facing up is blue?

(A) $\frac{19}{40}$

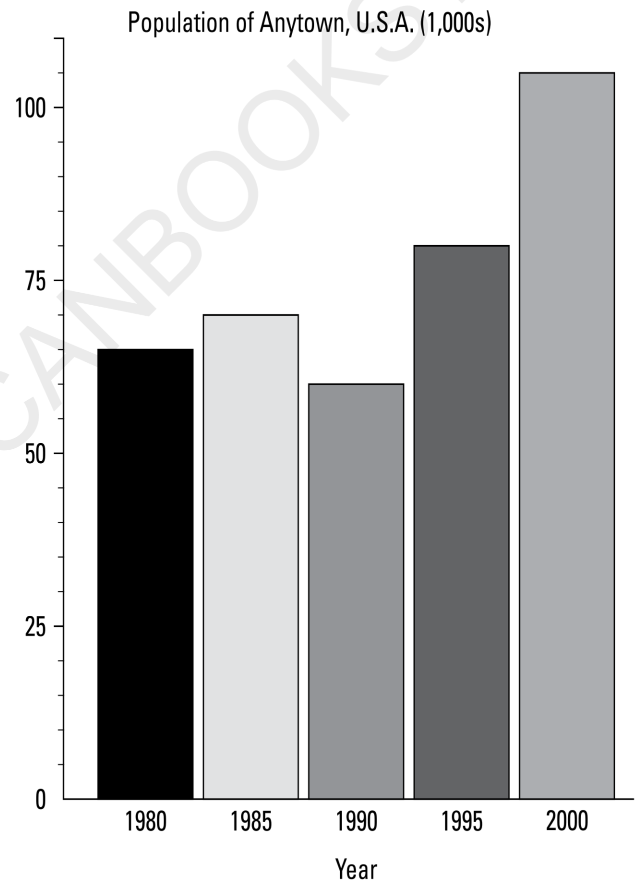
(B) $\frac{6}{10}$

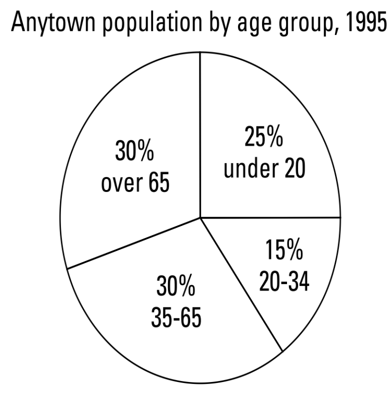
(C) $\frac{7}{40}$

(D) $\frac{7}{20}$

This one's a little tricky. Even though there are 20 cards, the question asks only about the side of the card facing up, and there are $20 \times 2 = 40$ possible sides. The 7 cards that are blue on both sides represent $7 \times 2 = 14$ blue sides, and there are 5 cards with one blue side. Add those together and you get $14 + 5 = 19$, so the probability is $\frac{19}{40}$. Choice (A) is correct.

Problems 4, 5, and 6 use the following graphs.





4. In 1990, what was the approximate number of Anytown residents over the age of 65?

(A) 55,000

(B) 25,000

(C) 14,000

(D) It cannot be determined from the graphs.

Don't trip over this one. You can tell from the bar graph that in 1990 Anytown had approximately 60,000 total residents, but the pie graph tells you only about the ages of the residents in 1995. You have no way to determine anything about the ages of Anytown residents in 1990, so Choice (D) is correct.

5. During which five-year period did Anytown have the greatest percent increase in population?

(A) 1980–1985

(B) 1985–1990

(C) 1990–1995

(D) It cannot be determined from the graphs.

You can throw out Choice (B) right away, because the population decreased. You can also throw out Choice (D), because you can use the graphs to determine the answer. That leaves Choices (A) and (C). Take a quick look at the bar graph, and the gap between 1980 and 1985, Choice (A), is far less than the gap between 1990 and 1995, Choice (C). Thus, Choice (C) is correct.

6. In 1995, roughly how many Anytown residents were between the ages of 20 and 65?

(A) 45

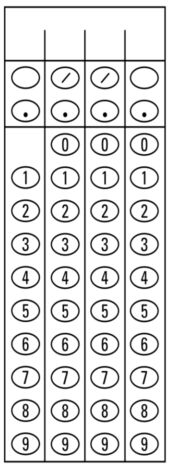
(B) 15,000

(C) 36,000

(D) 45,000

A look at the pie chart tells you that $30+15$, or 45%, of the residents were between 20 and 65 in 1995. Because there were 80,000 residents, change 45% into 0.45, and multiply: $0.45\times 80,000 = 36,000$. The correct answer is Choice (C).

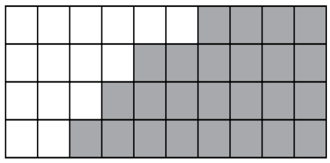
7. A bag contains red, blue, and green marbles. The probability of picking a red marble is $\frac{1}{2}$ and the probability of picking a blue marble is $\frac{1}{3}$. If the bag holds seven green marbles, find the total number of marbles in the bag.



Your answer here is 42. To do this one, you need a little algebra. Because the probability of picking a red marble is $\frac{1}{2}$, half of the marbles are red. Similarly, one-third of the marbles are blue. So one-half of the marbles, plus one-third of the marbles, plus the seven green marbles, is the number you're looking for. If you let x represent the total number of marbles, you can write $\frac{1}{2}x + \frac{1}{3}x + 7 = x$.

Because fractions are annoying, multiply everything by 6 to get $3x + 2x + 42 = 6x$. This equation gives you $5x + 42 = 6x$, so $x = 42$.

8. If a student picks a square at random on the following grid, what is the probability that he picks a square that is not shaded?



(A) $\frac{3}{5}$

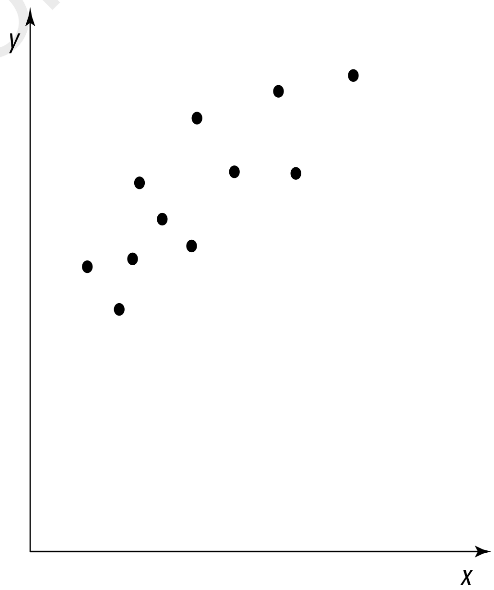
(B) $\frac{3}{8}$

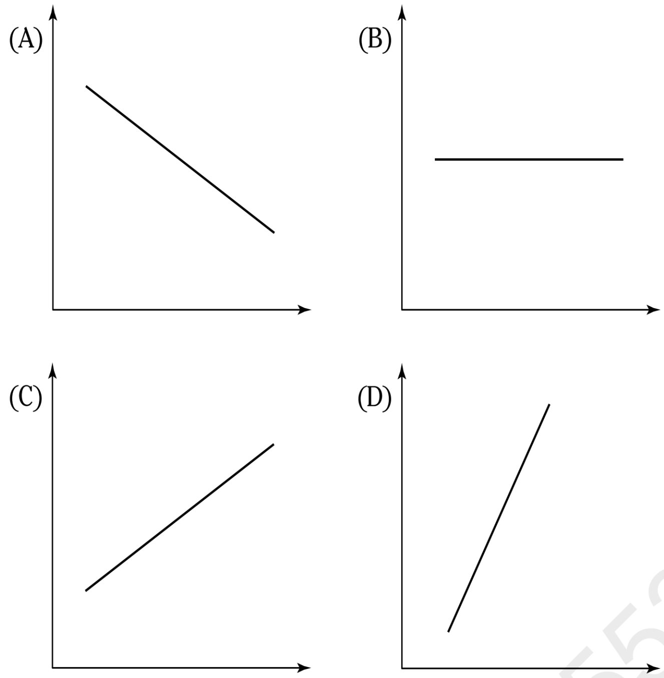
(C) $\frac{2}{5}$

(D) $\frac{5}{8}$

Fifteen of the 40 squares are not shaded. Put the 15 over 40, $\frac{15}{40}$, which reduces to $\frac{3}{8}$. Choice (B) is correct.

9. Which graph could represent the trend line for this scatter plot?





As long as you remember what the trend line is, this one is easy (see Chapter 16 if you don't). Choice (C) is correct.

10. Alison, Bob, Chris, and Darrell all arrive at school between 7:30 and 8:00.

    Chris was late to school, but Bob was not.

    Darrell arrived ten minutes after Alison.

    Bob didn't see Alison when he came into school.

Based on this information, which student(s) could have arrived at exactly 7:30?

(A) only Alison

(B) only Bob

(C) either Alison or Bob

(D) only Chris

You don't know what time the school day starts, but the first clue tells you that Chris must have arrived later than Bob, so Chris couldn't have arrived at 7:30, but (from this clue) Bob could have. The same logic applies to the second clue: Darrell arrived after Alison, so Darrell couldn't have been there at 7:30, but Alison could have. The third clue is just the TEST-makers (okay, us) messing with you; Alison might have already been in the building, or she might not have been, but you just can't tell. So Choice (C) is correct, because either of them might have arrived at 7:30; in fact, they might have both arrived then, if the school has more than one door.

Set Two: Practicing Some Questions on Your Own

1. A certain set of numbers has a mean of 20, a median of 21, and a mode of 22. Which number *must* be in the data set?

   (A) 19

   (B) 20

   (C) 21

   (D) 22

2. A student has a median score of 83 on five tests. If she scores 97 and 62 on her next two tests, her median score will

   (A) increase to 90

   (B) decrease to 82

   (C) decrease to 79.5

   (D) remain the same

3. Alicia picks a number from the set $\{1, 2, 3, 4, 5, 6\}$. Michelle picks a number from the set $\{3, 4, 5, 6, 7, 8\}$. What is the probability that they select the same number?

   (A) $\frac{1}{6}$

   (B) $\frac{1}{9}$

   (C) $\frac{2}{7}$

   (D) $\frac{3}{8}$

4. If a two-digit number is picked at random, what is the probability that the number chosen is a perfect square?

   (A) $\frac{1}{16}$

   (B) $\frac{1}{15}$

   (C) $\frac{1}{9}$

   (D) $\frac{1}{4}$

5. A class contains five boys and seven girls. In how many ways can a teacher line up two boys and two girls, in that order?

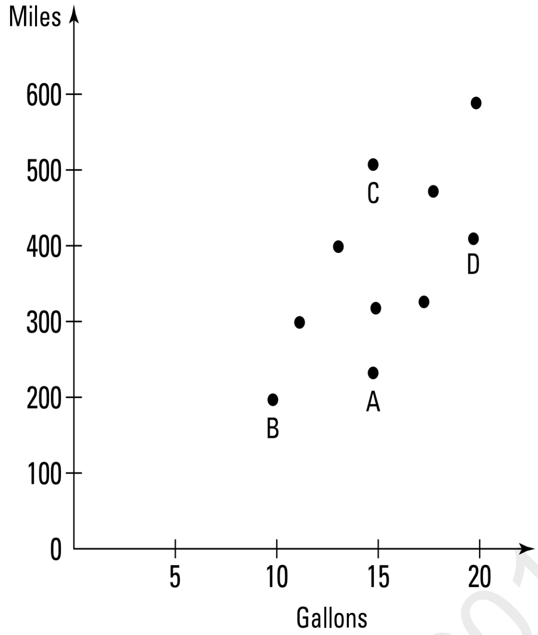
   (A) 35

   (B) 140

   (C) 210

   (D) 840

6. A magazine did a study of ten cars, comparing the number of miles each car could go on a full tank of gas. Their results are shown below. Of the labeled points, which one represents the car that goes the farthest per gallon of gas?



(A) A

(B) B

(C) C

(D) D

7. A junior is choosing her classes for senior year. If she takes calculus, she can also take either history or English, but not both. If she takes psychology in the first semester, she cannot take sociology or creative writing. If she takes psychology in the second semester, she cannot take calculus but can take any elective she wants during the first semester. Based only on this information, which of the following is *not* a possible choice of courses for her?

(A) English, calculus, psychology, statistics

(B) History, sociology, psychology, English

(C) Creative writing, psychology, history, English

(D) Calculus, psychology, creative writing, history

8. A student has taken three tests, with an average (arithmetic mean) of 82. What grade must he receive on his next test in order to have an overall average of 85?

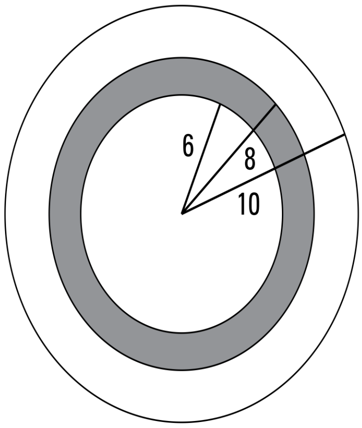
(A) 85

(B) 88

(C) 90

(D) 94

9. The following dartboard consists of three circles with the same center and radii of 6, 8, and 10 inches. If someone throws a dart at the board without aiming for a particular part of the board, what is the probability that the dart lands in the shaded ring?



(A) $\frac{7}{25}$

(B) $\frac{4}{5}$

(C) $\frac{3}{4}$

(D) $\frac{9}{16}$

Answers to Set Two

1. **D.** If you remember how to compute the three *m*'s, you'll realize that the mean and median don't have to be in the data set. (Look in Chapter 16 for more on mean, median, and mode.) But because the mode is the most common measurement, it must be in the set. Thus, Choice (D) is correct.

2. **D.** The median is the score in the middle. If 83 is in the middle, adding a 97 on one side and a 62 on the other side doesn't change where the middle is. The correct answer is Choice (D).

3. **B.** First determine the total number of possibilities. Using the counting principle, you know you have $6 \times 6 = 36$ possibilities. Because the two sets overlap at 3, 4, 5, and 6, the girls may pick the same number in only four ways. Hence, the answer is $4/36 = 1/9$. Choice (B) is correct.

4. **B.** As usual in a probability question, you need to start by determining the total number of two-digit numbers. The two-digit numbers run from 10 to 99. The formula you can use for the number of these is as follows: The total is one more than the difference of the two numbers, or $99 - 10 + 1 = 90$. Of these, only six are perfect squares: 16, 25, 36, 49, 64, and 81. So that gives you a probability of $6/90 = 1/15$, Choice (B).

5. **D.** The teacher can't pick the same person twice, so he has five choices for the first boy but only four left for the second. Similarly, he has seven choices for the first girl and six for the second. Using the counting principle, you know the answer is $5 \times 4 \times 7 \times 6 = 840$. Choice (D) is correct.

6. **C.** Car C travels approximately 500 miles on 15 gallons, for approximately 500/15 or 33 miles per gallon. The other cars all travel fewer than 30 miles per gallon. Choice (C) is correct.

7. **D.** Start by making a list of combinations that are impossible:

   Calculus, English, history

   First-semester psychology, sociology

   First-semester psychology, creative writing

   Second-semester psychology, calculus

   Now consider the choices: Choice (A) is fine, if she takes psychology in the first semester. Choice (B) is okay if she takes psychology in the second semester. Choice (C) is fine, if she takes psychology in the second semester. Choice (D) is a problem. If she takes psychology in the first semester, then creative writing is out. But if she takes it in the second semester, then calculus is out. Choice (D) is the correct answer.

8. **D.** The formula average score $\times$ number of scores $=$ total score helps you answer this problem. The student wants to end up with an average score of 85 on four tests, for a total score of $85 \times 4 = 340$. The student's total number of points on the first three tests is $82 \times 3 = 246$. So to make up the difference, he needs $340 - 246 = 94$ points on his next test to get (1) the car keys, and (2) permission to stay out past 7 p.m. Choice (D) is correct.

9. **A.** The shaded ring has an outer radius of 8 and an inner radius of 6. So the area of the shaded ring is the area of the radius-8 circle minus the area of the radius-6 circle, or $\pi(8)^2 - \pi(6)^2 = 64\pi - 36\pi = 28\pi$. The entire dartboard has a radius of 10, so its area is $\pi(10)^2 = 100\pi$, and the probability equals $28\pi/100\pi = 28/100 = 7/25$. Choice (A) is correct.